This paper studies comovement between changes in sovereign bond yields and stock market returns for ten Eurozone countries (Austria, Belgium, Finland, France, Ireland, Italy, Germany, Netherlands, Portugal, and Spain) in the period from January 2000 – end of August 2011, applying the maximal overlap discrete wavelet transform (MODWT) variance and correlation tools.

We found that the for all but one country (namely Portugal) the stock return volatility is higher than the volatility of sovereign bond yield changes across all wavelet scales. The highest volatility in stock indices’ returns and bond yield changes is captured by lower level MODWT wavelet coefficients (i.e., level 1 (j = 1) MODWT wavelet coefficients, associated with dynamics over 2 to 4 days and level 2 (j = 2) MODWT wavelet coefficients associated with dynamics over 4 to 8 days). A practical implication stemming from this finding is that investors with short investment horizons that want to efficiently manage the risk of their portfolio investments have to respond to every fluctuation in realized returns or bond yield changes, while for an investor with a much longer horizon, the need to do this is reduces, as long-run risk is significantly smaller as indicated by the variance.

The results of the paper show also that the estimated wavelet correlation changes with the time scale and is mostly positive for all countries but Portugal. The statistical evidence against hypothesis of no multiscale dependence yet is weak. Thus we cannot statistically claim that the wavelet coefficients at higher scales are significantly different (either higher or lower) that those at lower scales.

For the financial markets of Portugal and Germany we proved that the dependence between stock and sovereign bond market dynamics may not be just a multiscale phenomenon, but may also exhibits time dynamics across scales. The rolling-window wavelet correlation estimates show that at all scales correlation turned negative for Portugal at the start of 2010 – this is when the Eurozone debt crisis started which also hit hard the Portugal sovereign bond market – thus probably causing the “average” wavelet correlation for Portugal for the whole observed period to be negative.

Keywords: Eurozone, stock markets, sovereign bonds, wavelet analysis, Eurozone crisis.

Introduction

The comovement of returns between different asset classes is of major importance for risk management and portfolio management. The (co)movements of stock prices and sovereign bond yields (or returns) are of particular interest to individual and institutional financial investors as stocks and bonds are the two prime investment asset classes for majority of investors. Changes in comovement patterns call for an adjustment of portfolios (Savva & Aslanidis, 2010). Namely, to achieve optimal balance between risk and return many investors use a tactical asset allocation strategy and reallocate their investments when their expectations about risk and returns of particular assets in their portfolios change.

The co-movement between stock prices and sovereign bond yields (or returns) has received considerable attention in the literature. (Campbell & Ammer, 1993; Stivers & Sun, 2002) argue that stock and bond prices should move in the same direction (thus implying that stock market returns and the changes of sovereign bond yields should move in the opposite direction) due to a common discount rate effect or common movements in future expected returns (Stivers et al., 2002; Dajcman, 2012a). More recent studies, investigating the time-varying comovement, however, reported periods of positive correlations between stock market returns and the dynamics of sovereign bond yields (see Dajcman, 2012a). According to (Campbell & Ammer, 1993), theoretically, positive correlations between stock market returns and the dynamics of sovereign bond yields can be explained by variations in expected inflation, since increases in inflation should negatively affect bonds, but not stocks. Financial market dynamics and changes in market participants’ assessments about risk may also have an important impact on the relationship between stock market returns and the dynamics of sovereign bond yields (Gulko, 2002; Baur & Lucey, 2009). In periods of financial market turbulence, the “flight-to-quality” phenomena might be observed (Gulko, 2002).

The comovement of stock price and bond yield changes may not only be time-varying but also dependent on the investment horizon of investors who may hold stocks and/or bond for a shorter or longer time periods. (Candelon et al., 2008) argued that comovement analysis should consider the distinction between short- and long-term investors. From the point of view of portfolio diversification, short-term
investors are more interested in the comovements of asset classes at higher frequencies (short-term movements), while long-term investors focus on lower frequencies comovements (Dajcman, 2012b). Another reason, why the relationship between dynamics of stock prices and bond yields may be a multiscale phenomenon is that the long-run relationship between the two asset classes may be obscured in the short run by financial market volatility or short-term noise which might derive from investors trading in order to rebalance their portfolio or to satisfy their immediate (unexpected) consumption needs (Kim & In, 2007; Harrison & Zhang, 1999).

To study stock and bond yield changes on a multiscale basis one has to resort to a time-frequency domain analysis to obtain insight about comovements at particular time-frequency (scale) level (Lee, 2004; Pakko, 2004; Rua & Nunes, 2009). In such a context, with both the time horizon of economic decisions and the strength and direction of economic relationships between variables that may differ according to the time scale of the analysis, a useful analytical tool may be represented by wavelet analysis (Pinho & Madaleno, 2009).

There are several studies using wavelet tools (variance, wavelet correlation, and wavelet cross-correlation) to investigate interdependence between economic (or financial) variables on different time scales (Kim & In, 2005; In et al., 2008; In & Kim, 2006; Kim & In, 2007; Gencay et al., 2001a; Gallegati, 2008; Conlon et al., 2009; Zhou 2011). These studies confirm that interdependence between financial (or economic) variables is scale dependent, exhibiting different correlation structures at different time scales.

To our knowledge there is only one study (Kim & In, 2007) examining the multiscale relationship between the stock prices and sovereign bond yield changes applying wavelet analysis. (Kim & In, 2007), examining the relationship between stock prices and bond yields in the G7 countries, found that the correlation between changes in stock prices and bond yields can differ from one country to another and can also depend on the time scale.

In this paper we study comovement between changes in sovereign bond yields and stock market returns for ten Eurozone countries (Austria, Belgium, Finland, France, Ireland, Italy, Germany, Netherlands, Portugal, and Spain) in the period from January 2000 – end of August 2011 (different starting periods are used for different countries due to data availability), applying the maximal overlap discrete wavelet transform (MODWT) variance and correlation tools. Unlike the study of (Kim & In, 2007), who use monthly data, we apply the daily data of stock market returns and bond yield changes and also study the dynamics of comovement on a scale-by-scale basis whereas in the study of (Kim & In, 2007) the analysis is static.

**Methodology**

**Wavelet Analysis**

The wavelet analysis\(^1\) involves the projection of the original series onto a sequence of basis functions, known as wavelets. There are two basic wavelet functions: the father wavelet (called also a scaling function), \(\phi\), and the mother wavelet (called also a wavelet function), \(\psi\), which can be scaled and translated to form a basis for the Hilbert space \(L^2(\mathbb{R})\) of square integrable functions. The father and mother wavelets are defined as:

\[
\phi_{j,k}(t) = 2^{-j/2} \phi(2^{-j} t - k) \quad (1a)
\]

and

\[
\psi_{j,k}(t) = 2^{-j/2} \psi(2^{-j} t - k) \quad (1b)
\]

where \(j = 1, \ldots, J\) is the scaling parameter in a \(J\)-level decomposition and \(k\) is a translation parameter (\(j, k \in \mathbb{Z}\)).

The continuous wavelet transform of a square integrable time series \(X(t)\) consists of the scaling, \(\alpha_{j,k}\), and wavelet coefficients, \(\beta_{j,k}\), (Craigmile & Percival, 2002):

\[
\alpha_{j,k}(t) = \int \phi_{j,k}(t)X(t) \quad (2a)
\]

and

\[
\beta_{j,k}(t) = \int \psi_{j,k}(t)X(t) \quad (2b)
\]

It is possible to reconstruct \(X(t)\) from these transform coefficients using:

\[
X(t) = \sum_k \alpha_{j,k} \phi_{j,k}(t) + \sum_k \beta_{j,k} \psi_{j,k}(t) + \sum_k \beta_{j-1,k} \psi_{j-1,k}(t) + \cdots + \sum_k \beta_{1,k} \psi_{1,k}(t) \quad (3)
\]

In practice we observe a time series at finite number of regularly spaced times, so we can make use of a discrete wavelet transform (DWT)\(^2\) or a maximal overlap discrete wavelet transform (MODWT). The MODWT is a linear filtering operation that transforms a series into coefficients related to variations over a set of scales.

**MODWT Wavelet Analysis**

Let\(^3\) \(X\) be a \(N\) dimensional vector whose elements are the real-valued time series \(X_i: t = 0, \ldots, N - 1\), where the sample size \(N\) is any positive integer. For any positive integer, \(J_0\), the level \(J_0\) MODWT of \(X\) is a transform consisting of the \(J_0 + 1\) vectors \(\bar{W}_{J_0}, \ldots, \bar{W}_{J_0}\) and \(\bar{V}_{J_0}\), all of which have the dimension \(N\). The vector \(\bar{W}_{J_0}\) contains the MODWT wavelet coefficients associated with changes at scale \(\tau_j = 2^{J_0 - j}\) (for \(j = 1, \ldots, J_0\)), where \(\bar{V}_{J_0}\) contains MODWT scaling coefficients associated with averages at scale \(\lambda_{J_0} = 2^{J_0}\).

Based upon a definition of MODWT coefficients we can write (Percival & Walden, 2000):

\[
\bar{W}_j = \bar{W}_0 X \quad (4a)
\]

\(^1\) In description of the wavelet analysis we follow (Dajcman et al., 2012; Dajcman, 2013).

\(^2\) For a presentation of DWT, please refer to (Percival & Walden, 2000).

\(^3\) Concepts and notations as in (Percival & Walden, 2000) are used.

\(^4\) (Percival & Walden, 2000) denote scales of MODWT obtained wavelet coefficients with a letter \(\tau\) and scales of scaling coefficients with \(\lambda\). We use the same notations.
\[ \mathbf{V}_j = \mathbf{W}_j \mathbf{X} \quad (4b) \]

where \( \mathbf{W}_j \) and \( \mathbf{V}_j \) are \( N \times N \) matrices. By definition, the elements of \( \mathbf{W}_j \) and \( \mathbf{V}_j \) are outputs obtained by filtering \( \mathbf{X} \), namely:

\[ \mathbf{W}_{j,t} = \sum_{l=0}^{L_j - 1} \tilde{h}_{j,l} X_{t-l \mod N} \quad (5a) \]

\[ \mathbf{V}_{j,t} = \sum_{l=0}^{L_j - 1} \tilde{g}_{j,l} X_{t-l \mod N}, \quad (5b) \]

for \( t = 0, ..., N - 1 \), and where \( \tilde{h}_{j,l} \) and \( \tilde{g}_{j,l} \) are the \( j \)-th level MODWT wavelet and scaling filters defined in terms of the \( j \)-th level equivalent wavelet and scaling filters \( \{ h_{j,l} \} \) and \( \{ g_{j,l} \} \) for a discrete wavelet transform (DWT):

\[ \tilde{h}_{j,l} = h_{j,l}/2^{j/2} \quad (6a) \]

and

\[ \tilde{g}_{j,l} = g_{j,l}/2^{j/2} \quad (6b) \]

Each of the MODWT wavelet filters has width \( L_j \equiv (2^{j} - 1)(L - 1) + 1 \) and can be calculated once basic MODWT wavelet filter \( \tilde{h}_{1,l} \equiv \tilde{h}_1 = h_1/\sqrt{2} \) and MODWT scaling filter \( \tilde{g}_{1,l} \equiv \tilde{g}_1 = (-1)^{l+1}h_{1,-l-1} \) have been specified.

A DWT filter \( \{h_l; l = 0, ..., L - 1\} \) of even width \( L \) is called a wavelet filter if:

\[ \sum_{l=0}^{L-1} h_l = 0 \quad (7a) \]

and

\[ \sum_{l=0}^{L-1} h_l h_{l+2n} = \begin{cases} 1, & \text{if } n = 0 \\ 0, & \text{if } n \text{ is nonzero} \end{cases} \quad (7b) \]

A DWT scaling filter is defined in terms of the wavelet filter:

\[ g_l \equiv (-1)^{l+1}h_{1,-l-1} \quad (8a) \]

and satisfies conditions:

1) \[ \sum_l g_l = \sqrt{2} \quad (8b) \]

2) \[ \sum_{l=0}^{L-1} g_l g_{l+2n} = \begin{cases} 1, & \text{if } n = 0 \\ 0, & \text{if } n \text{ is nonzero} \end{cases} \quad (8c) \]

The MODWT treats the series as if it were periodic, whereby the unobserved samples of the real-valued time series \( X_{-N}, X_{-N+2}, ..., X_{-N}, ..., X_0 \) are assigned the observed values at \( X_{N-1}, X_{N-2}, ..., X_0 \). The MODWT coefficients are thus given by:

\[ \mathbf{W}_{j,t} \equiv \sum_{l=0}^{L_j - 1} \tilde{h}_{j,l} X_{t-l \mod N} \quad (9a) \]

\[ \mathbf{V}_{j,t} \equiv \sum_{l=0}^{L_j - 1} \tilde{g}_{j,l} X_{t-l \mod N} \quad (9b) \]

for \( t = 0, ..., N - 1 \); \( \tilde{h}_{j,l} \) and \( \tilde{g}_{j,l} \) are periodization of \( h_{j,l} \) and \( g_{j,l} \) to circular filters of length \( N \).

This periodic extension of the time series is known as analyzing \( \{X_t\} \) using “circular boundary condition” ( Percival & Walden, 2000; Cornish et al., 2006). There are \( L_j - 1 \) wavelet and scaling coefficients that are influenced by the extension (“the boundary coefficients”). Since \( L_j \) increases with \( j \), the number of boundary coefficients increases with scale. Exclusion of boundary coefficients in the wavelet variance, wavelet correlation and covariance provides unbiased estimates (Cornish et al., 2006).

### Wavelet Variance and Wavelet Correlation

One of the important uses of the MODWT is to decompose the sample variance of a time series on a scale-by-scale basis. Since the MODWT is energy conserving (Percival & Mojeñ, 1997):

\[ ||X||^2 = \sum_{j=0}^{\infty} ||\mathbf{W}_j||^2 + ||\mathbf{V}_j||^2 \quad (10) \]

A scale-dependent analysis of variance from the wavelet and scaling coefficients can be derived (Cornish et al., 2006):

\[ \hat{\sigma}_k^2 = ||X||^2 - \bar{X}^2 = \frac{1}{N} \sum_{j=0}^{\infty} ||\mathbf{W}_j||^2 + \frac{1}{N} ||\mathbf{V}_j||^2 - \bar{X}^2 \quad (11) \]

Wavelet variance is defined for stationary and nonstationary processes with stationary backward differences (Percival & Walden, 2000). Let \( \{X_t; t = -\infty, -1,0,1, ...\} \) be a discrete parameter real-valued stochastic process whose \( d \)-th order differencing (\( d \) may take any nonnegative integer values) gives a stationary process:

\[ Y_t \equiv (1 - B)^d X_t \equiv \sum_{k=0}^{d} (\Delta)^k X_{t-k}. \quad (12) \]

with spectral density function (SDF) \( S_{\chi}(\cdot) \) and mean \( \mu_Y \) (which may not be zero). Let \( S_{\chi}(\cdot) \) represent the SDF for \( \{X_t\} \), for which \( S_{\chi}(f) = S_Y(f)/(d^d(f)) \), where \( D(f) = 4\sin^2(\pi f) \) (if \( \{X_t\} \) is a nonstationary process, then this relationship between \( S_{\chi}(\cdot) \) and \( S_Y(\cdot) \) represents definition for \( S_{\chi}(\cdot) \)). Filtering \( \{X_t\} \) with a MODWT Daubechies wavelet filter \( \{h_{j,l}\} \) of width \( L \geq 2d \), a stationary process of \( j \)-th level MODWT wavelet is obtained:

\[ \tilde{W}_{j,t} \equiv \sum_{l=0}^{L_j - 1} \tilde{h}_{j,l} X_{t-l} \quad t = \ldots, -1,0,1, \ldots \quad (13) \]

where \( \tilde{W}_{j,t} \) is a stochastic process obtained by filtering \( \{X_t\} \) with the MODWT wavelet filter \( \{h_{j,l}\} \) and \( L_j \equiv (2^{j} - 1)(L - 1) + 1 \)

Let us suppose that we are given a time series, which is realization of one segment (segment with values \( X_0, ..., X_{N-1} \)) of the process \( \{X_t\} \). Under condition \( M_j \equiv N - L_j + 1 > 0 \) (i.e. by considering only the non-boundary wavelet coefficients, obtained by filtering stationary time series with MODWT) and that either \( L > 2d \) or \( \mu_X = 0 \) (realization of either of these two conditions implies \( E[\tilde{W}_{j,t}] = 0 \) and therefore \( \hat{\sigma}_k^2 (\tau_j) = E[\tilde{W}^2_{j,t}] \)), an unbiased estimator of wavelet variance of scale \( \tau_j (\hat{\sigma}^2_k (\tau_j)) \) is given by (Percival & Walden, 2000):

\[ \hat{\sigma}_k^2 (\tau_j) = \frac{1}{M_j} \sum_{t=1}^{M_j} \tilde{W}_{j,t} \tilde{W}_{j,t}^*, \quad (14) \]

where \( \tilde{W}_{j,t} \) is the \( j \)-th level MODWT wavelet coefficients for time series \( \tilde{W}_{j,t} = \sum_{l=0}^{L_j - 1} \tilde{h}_{j,l} X_{t-l \mod N}, t = 0,1, ..., N - 1 \).

The estimator \( \hat{\sigma}_k^2 (\tau_j) \) is a random variable and under assumption that \( \{W_{j,t}\} \) is a Gaussian stationary process with mean zero and SDF \( S_{\chi}(\cdot), S^2_Y (f) > 0 \) almost everywhere
and $A_j \equiv \int_2^\infty S_j^p(f)df < \infty$ (where $S_j(f)$ is the spectrum of the wavelet coefficients of scale $j$), then the estimator $\hat{\theta}_j^2(\tau_j)$ of equation (14) is asymptotically normally distributed with the random interval:

$$\left[ \hat{\theta}_j^2(\tau_j) - \Phi^{-1}(1-p)\frac{\sqrt{2j}}{M_j},\hat{\theta}_j^2(\tau_j) + \Phi^{-1}(1-p)\frac{\sqrt{2j}}{M_j} \right]$$  (15)

The interval captures the true variance and corresponds to a $100(1-2p)$% confidence interval for $\theta_j^2(\tau_j)$ (see Percival 1995; Serroukh et al., 2000). However the lower confidence interval given by the equation (15) can be negative. As $\hat{\theta}_j^2(\tau_j)$ is proportional to the sum of squares of $M_j$ zero mean Gaussian random variables (each having the same variance), $\hat{\theta}_j^2(\tau_j)$ could be renormalized to obey a chi-square distribution with $\eta$ degrees of freedom ($\eta = \max \left\{ \frac{M_j}{\tau_j}, 1 \right\}$), from which strictly positive confidence intervals would follow. Using the approximation of (Percival, 1981; Percival & Walden, 2000) show that an approximate $100(1-2p)$% confidence interval for $\hat{\theta}_j^2(\tau_j)$ is given by:

$$\left[ \frac{q_0^2\hat{\theta}_j^2(\tau_j)}{q_0(1-p^2)}, \frac{q_0^2\hat{\theta}_j^2(\tau_j)}{q_0(p)} \right]$$  (16)

where $Q_0(p)$ is the $p \times 100$% percentage point for the $\chi^2_\eta$ distribution, i.e. $P\left[ \chi^2_\eta \leq Q_0(p) \right] = p$.

Given two stationary processes $\{X_t\}$ and $\{Y_t\}$, whose $j$th-level MODWT wavelet coefficients are $\{\hat{W}_{X,j,t}\}$ and $\{\hat{W}_{Y,j,t}\}$, an unbiased covariance estimator $\hat{\theta}_{XY}(\tau_j)$ is given by (Percival, 1995):

$$\hat{\theta}_{XY}(\tau_j) = \frac{1}{M_j} \sum_{j=0}^{N-L_j+1} \hat{W}_{X,j,t}^{(X)} \hat{W}_{Y,j,t}^{(Y)}$$  (17)

where $M_j \equiv N - L_j + 1 > 0$ is the number of non-boundary coefficients at the $j$th-level.

The MODWT correlation estimator for scale $\tau_j$ is obtained by making use of the wavelet covariance and the square root of wavelet variances:

$$\hat{\rho}_{XY}(\tau_j) = \frac{\hat{\theta}_{XY}(\tau_j)}{\hat{\sigma}_X(\tau_j)\hat{\sigma}_Y(\tau_j)}$$  (18)

where $|\hat{\rho}_{XY}(\tau_j)| \leq 1$. The wavelet correlation is analogous to its Fourier equivalent, the complex coherency (Gencay et al., 2002).

Given the inherent non-normality of the wavelet correlation coefficients for small sample sizes, a nonlinear transformation is sometimes required in order to construct a confidence interval (Gencay et al., 2001a). Calculation of confidence intervals of correlation coefficient is based on (Percival, 1995; Percival & Walden, 2000). The random interval:

$$\left[ \tanh\left\{ h[\hat{\rho}_{XY}(\tau_j)] \right\} - \frac{\Phi^{-1}(1-p)}{\sqrt{N_j-1}}, \tanh\left\{ h[\hat{\rho}_{XY}(\tau_j)] \right\} + \frac{\Phi^{-1}(1-p)}{\sqrt{N_j-1}} \right]$$  (19)

Captures the true wavelet correlation and provides an approximate $100(1-2p)$% confidence interval. Function $h(p) = \tanh^{-1}(\rho)$ defines the Fisher’s z-transformation. $N_j$ is the number of wavelet coefficients obtained by $j$th-level of DWT and not by the MODWT transformation. This is because the Fisher’s z-transformation assumes uncorrelated observations and the DWT is known to approximately decorrelate a wide range of power-law processes (Ranta, 2010). The approximate confidence interval for the estimated wavelet correlation does not utilize any information regarding the distribution of the wavelet coefficients. Hence, no adjustment is made regarding the distribution of the incoming wavelet coefficients; they may be Gaussian or non-Gaussian (Gencay et al., 2001a).

Data and Empirical Results

Comovement between stock market returns and the dynamics of sovereign bond yields were calculated for ten Eurozone countries, listed in Table 1. The stock indices returns ($r_{1,t}$) were calculated as the differences in the logarithms of the daily closing prices of indices ($r_{1,t} = \ln(P_t) - \ln(P_{t-1})$, where $P$ is an index value). The following stock market indices were considered: ATX for Austria, BEL20 for Belgium, HEX25 for Finland, CAC40 for France, ISEQ for Ireland, FTSEMIB for Italy, DAX for Germany, AEX for Netherlands, PSI20 for Portugal, and IBEX35 for Spain. Yields ($y$) of the central-government bonds (bullet issues) with 10 year maturity dates were considered. Changes (i.e., dynamics) of sovereign bond yields ($r_{2,t}$) were calculated as $\ln(y_t) - \ln(y_{t-1})$ as suggested by (Durre & Giot, 2005) or by Kim and In (2007).\(^5\) The period of observation is different for individual countries due to data availability. Days where there was no concurrent trading in the national stock and bond market were left out. The data source for the index prices was Yahoo! Finance and for the central government bond yields the Denmark’s central bank. Table 1 presents some descriptive statistics of the data.

The stationarity of time series was tested by the Augmented Dickey-Fuller (ADF) test, Phillips-Perron (PP) and Kwiatkowski-Phillips-Schmidt-Shin (KPSS) tests (Table 2). As the trend was not significant in any of test models, only results for the models with a constant are presented. The rejection of the null hypothesis of ADF and PP test and non-rejection of the null hypothesis of KPSS test leads to the conclusion of no unit-root in the time series. The alternative hypothesis, of stationary time series, can be accepted.

\(^5\) Alternatively, changes (i.e., dynamics) of sovereign bond yields can also be calculated as $\ln(1+y_t) - \ln(1+y_{t-1})$, as also suggested by (Durre & Giot, 2005). The results of the dynamical conditional correlation analysis also for the later definition of bond yield changes however differ only marginally and lead to the same conclusions as with the chosen definition of sovereign bond yield changes.
To obtain MODWT estimates of variance and correlation, a Daubechies least asymmetric filter with a wavelet filter length of 8 (L(8)) was chosen. This is a common wavelet filter used in empirical studies on financial market interdependencies (Gencay et al., 2001b; Gallegati 2005; Ranta 2010, Dajcman et al., 2012). The maximum level of MODWT is 6 (j = 6) to achieve an optimal balance between sample size and the length of the filter. Scale τ₁ (or scale 1, as τ₁ = 2⁻¹⁻¹ = 1) measures the dynamics of returns over 2 to 4 days;² scale τ₂ (scale 2, as τ₂ = 2²⁻ⁱ = 2) over 4 to 8 days; scale τ₃ (scale 4, as τ₃ = 2³⁻¹ = 4) over 8 to 16 days; scale τ₄ (scale 8, τ₄ = 2⁴⁻¹ = 8) over 16 to 32 days; scale τ₅ (or scale 16) over 32 to 64 days; and scale τ₆ (or scale 32) over 64 to 128 days. To obtain unbiased estimates of cross-correlation, only non-boundary wavelet coefficients were considered.

The results of MODWT variance decomposition analysis are presented in Figure 1. Notably, for all but one country (namely Portugal) the stock return volatility is higher than the volatility of sovereign bond yield changes across all wavelet scales.

²¹ To 2¹²⁻¹ days. See, for example (Gencay et al., 2002, 2003; Lee, 2004; Fernandez, 2005; Percival & Walden, 2000).
The highest volatility in stock indices’ returns and bond yield changes is captured by lower level MODWT wavelet coefficients (i.e., level 1 \((j = 1)\) MODWT wavelet coefficients, associated with dynamics over 2 to 4 days and level 2 \((j = 2)\) MODWT wavelet coefficients associated with dynamics over 4 to 8 days). The finding that the variance decreases as the wavelet scale increases has, as (Kim & In, 2005, 2007) note, important implications for the investors in these asset classes. Investors with short investment horizons that want to efficiently manage the risk of their portfolio investments have to respond to every fluctuation in realized returns or bond yield changes, while for an investor with a much longer horizon, the need to do this is reduces, as long-run risk is significantly smaller as indicated by the variance.

The strength of comovement between stock market returns and bond yields changes at different time scales as analyzed by MODWT correlation analysis (the results are presented in Figure 2).

We find that the estimated wavelet correlation changes with the time scale and is mostly positive for all countries. As already noted, from the theoretical perspective the positive correlation can be explained by the different impact of inflation on stock and bond prices (Campbell & Ammer, 1993) or by the “flight-to-quality” argument. As inflation in the observed countries was not particularly high during the observed period, in our opinion the greater role has played the “flight-to-quality”, i.e. the financial market dynamics and the changes in market participants’ assessments about risk of the stocks and sovereign bonds. The negative correlation of
stock market returns and sovereign bond yield changes in Portugal can thus imply that the sovereign bonds on average did not play the "safe haven" role in its financial market throughout the whole observed period. To elaborate this issue, we next calculated the rolling-window correlations between stock market returns and bond yield changes for Portugal and Germany, as the sovereign bonds of the later country are regarded as safe haven in international financial markets. Using this approach, correlation between the two stock indices return series at time $t$ was calculated from $w$ observations (where $w$ is size of the window), centered around time $t$. The window was rolled forward one day at a time, resulting in a time series of wavelet correlation. This way we obtained $N - w$ correlation coefficients. The results are presented in Figure 3.

**Figure 2.** Wavelet correlation between stock returns and sovereign bond yield changes

Notes: The estimated correlation between $j$th-level of MODWT wavelet coefficients for stock market returns and bond yield changes are drawn with a full line. The 95% confidence intervals (calculated by equation (59)) around the wavelet correlation estimates are drawn with dotted lines. The approximate confidence interval for the estimated wavelet correlation does not utilize any information regarding the distribution of the wavelet correlation. Therefore, these confidence intervals are robust to non-Gaussianity.

Source: Own calculations
More findings emerge from Figure 3. Firstly, the correlation between stock market returns and sovereign bond yield changes is not just a multiscale phenomenon, but also exhibits time dynamics. Secondly, the time path of lower wavelet scale correlations is similar, whereas the higher wavelet scale (especially scale \( \tau_6 \)) correlation exhibits more specific time dynamics. Thirdly, the rolling-window correlations at all scales turned negative for Portugal at the start of 2010 – this is when the Eurozone debt crisis started which also hit hard the Portuguese sovereign bond market – thus probably causing the “average” wavelet correlation for Portugal for the whole observed period (as calculated and presented in Figure 2) to be negative.

Conclusion

This paper examined comovement between changes in sovereign bond yields and stock market returns for ten Eurozone countries (Austria, Belgium, Finland, France, Ireland, Italy, Germany, Netherlands, Portugal, and Spain) in the period from January 2000 – end of August 2011 (different starting periods are used for different countries due to data availability), applying the maximal overlap discrete wavelet transform (MODWT) variance and correlation tools.

We found that the for all but one country (namely Portugal) the stock return volatility is higher than the volatility of sovereign bond yield changes across all wavelet scales. The highest volatility in stock indices’ returns and bond yield changes is captured by lower level MODWT wavelet coefficients (i.e., level 1 \((j = 1)\) MODWT wavelet coefficients, associated with dynamics over 2 to 4 days and level 2 \((j = 2)\) MODWT wavelet coefficients associated with dynamics over 4 to 8 days). A practical implication stemming from this finding is that investors with short investment horizons that want to efficiently manage the risk of their portfolio investments have to respond to every fluctuation in realized returns or bond yield changes, while for an investor with a much longer horizon, the need to do this is reduces, as long-run risk is significantly smaller as indicated by the variance.

The results of the paper show also that the estimated wavelet correlation changes with the time scale and is mostly positive for all countries but Portugal. The statistical
evidence against hypothesis of no multiscale dependence yet is weak. Thus we cannot statistically claim that the wavelet coefficients at higher scales are significantly different (either higher or lower) that those at lower scales.

For the financial markets of Portugal and Germany we proved that the dependence between stock and sovereign bond market dynamics may not be just a multiscale phenomenon, but may also exhibits time dynamics across scales. The rolling-window wavelet correlation estimates show that at all scales correlation turned negative for Portugal at the start of 2010 – this is when the Eurozone debt crisis started which also hit hard the Portugal sovereign bond market – thus probably causing the “average” wavelet correlation for Portugal for the whole observed period to be negative.

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