

## Sustainable Assessment of Construction Site by Applying Game Theory

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*Selection of a proper construction site is of major importance, as practice is intrinsically collaborative, within knowledge-rich and multi-functional working environments. Success of a construction site selection is an abstract concept. It highly determines whether a project is a success or a failure. The information required for a given site varies considerably according to the size and importance of the designed building, its location, and whether the facilities to be provided are in an unmapped area or just mere extensions of existing facilities. Different methods for the solution of problems are known. Decision theory usually analyses decision-making processes from player's point of view, while game theory emphasizes its analysis in the interaction among many players. Much of the game theory is concerned with finite, discrete games, which have a finite number of players, movements, events, outcomes, etc. This research presents the theory of the two-person zero-sum games with a practical example. This research shows that the applications to construction site selections can be provided and the game theory can be applied for the supporting decision in a competitive environment.*

Keywords: *sustainable, construction, site, assessment, decision making, operation research, game theory, equilibrium point.*

### Introduction

*The research problem.* The project preparation and realization processes, based on theoretical and empirical studies, creation of goods, services and technologies, are the most important human activities. Today it has become a common practice that many organizations work with not only separate and complex but very often with several or even more projects. (Neverauskas, Stankevicius, 2008). Under the development of market economics the requirements for information are changing and there should be a significant step towards the improvement of methodology of its preparation, processing and issue to users. All new ideas and effectiveness of possible decision variants must be compared according to many attributes (Domeika, 2008; Plebankiewicz, 2009; Turskis et al., 2009). Most decisions are made based on knowledge-based techniques and taking into account economic sustainability (Krisciunas, Greblikaite, 2007). Decision-making (Sivilevicius

et al., 2008; Buoziute-Rafanaviciene et al., 2009; Zavadskas et al., 2009b) in projects is a confusing and complex problem, characterized by trade-offs between economic, environmental, and socio-political impacts (Lakstutiene, 2008; Ginevicius, Podvezko, 2008a, b, 2009; Kaplinski, 2008; Boguslauskas, Kvedaraviciene, 2009; Smaiziene, Jucevicius, 2009; Ulubeyli, Kazaz, 2009; Juan et al., 2009). Over the last several decades assessment of possible alternatives and strategies (Ciegis et al., 2009) have become more sophisticated and complex. Operation research and multi-criteria decision making (MCDM) provides useful tools for many complicated assessment problems (Selih et al., 2008; Kalibatas, Turskis, 2008; Zavadskas et al., 2007, 2008a, b, c, d; 2009a) as they are based on criteria values, criteria weights, and allows incorporation of stakeholders' opinions in the ranking of alternatives (Turskis, 2008).

The concept of project success has remained ambiguously defined in the construction industry. The construction sector plays a major role in the development of society. However, it also has a substantial impact on the natural environment, the effects of which are evident across the world. One of the main principal challenges for the modern construction is supporting the sustainable development of cities. Increasing population, life standards and construction causes fossil fuel and raw materials consumption to increase (Aydin et al., 2010). Increasing consumption has major negative impacts on the environment. While the world energy demand is steadily growing, the concern for the environmental aspects and natural resource use and exploitation has increased (Grilo and Jardim-Goncalves, 2009). Land and property development processes obviously can be seen as a social situation in which the interaction of individuals or groups of individuals is one of the essential elements. In order to pursue sustainability in the construction industry, existing development-focused construction activities must be transformed via a focusing on sustainable development through the adoption of sustainable policies by the government and the development and dissemination of sustainable construction technologies (Tae and Shin, 2009). However, the relationship between economic development and environmental sustainability is complex. The decision-making on approval of sustainable assessment is an intrinsically complex multi-dimensional process because it does not only consider scientific facts

but also reflect subjective values. The use of decision-support methods to balance facts and values can be beneficial for decision makers (Liu et al., 2009).

*Newness of the research* is a creative application of game theory methods for sustainable construction site selection. There is proposed to use two-person zero-sum matrix games of strategy.

*The research object* is the concept of the sustainable multiple criteria assessment of construction site.

*The aim of this research* is to propose and apply two-person zero-sum game theory methods for sustainable assessment of alternatives in construction.

*The tasks.* In order to fulfil objectives, the following research tasks had to be accomplished:

- to analyse the methods of construction site selection;
- to determinate success criteria of a construction site selection;
- assessment of feasible alternatives of construction site by applying game theory.

*The method of the research* was an analytic research method based on the case study.

### Construction site selection

A project is a unique endeavour to produce a set of deliverables within clearly specified time, cost and quality constraints. Selecting of a proper construction site is of major importance, as practice is intrinsically collaborative, within knowledge-rich, multi-functional working environments. Success of a construction site selection is an abstract concept, and determining whether a project is a success or a failure is highly. The conventional approaches to construction site selection problem tend to be less effective in dealing with the linguistic assessment. Under many situations, managers are commonly faced with sophisticated decisions, such as choosing the location of a new facility subject to multiple conflicting criteria.

Ruiz and Fernández (2009) proposed a Spatial Decision Support System based on the GIS to evaluate the environmental performance in construction. The multi-criteria evaluation method developed in the Green Building Challenge and implemented in the software SbTool has been used as a reference.

Fan (2009) proposed spatial data mining method to solve site selection of the service centre. On the basis of GIS functions, he designed analytical function to take spatial obstacle factors, spatial environmental factors, spatial terrain factors, spatial traffic factors and cost factors into account.

Vahidnia et al. (2009) developed a Multi-Criteria Decision Analysis process that combines Geographical Information System analysis with the Fuzzy Analytical Hierarchy Process, and used this process to determine the optimum site for a new hospital in the Tehran urban area. The GIS was used to calculate and classify governing criteria, while Fuzzy Analytical Hierarchy Process was used to evaluate the decision factors and their impacts on alternative sites.

Ning et al. (2010) pointed that construction site layout planning is a dynamic multi-objective optimization

problem as there are different facilities employed in the different construction phases of a construction project. They described a method using continuous dynamic searching scheme to guide the max-min ant system algorithm, which is one of the ant colony optimization algorithms, to solve the dynamic construction site layout planning problem under the two congruent objective functions of minimizing safety concerns and reducing construction cost is proposed.

### Methods of a construction site selection

Decision making is a process involving activities that starts with recognition of a decision problem and ends with recommendation for a decision. The quality of the decision depends on the sequence and quality of the activities that are carried out. Two major approaches for the organization of activities can be distinguished:

- a) Alternative-focused. The alternative-focused approach starts with the development of alternative options, proceeds with the specification of values and criteria and then ends with the evaluation and recommendation of an option.
- b) Value-focused. The value-focused approach considers the values as the fundamental element in the decision analysis. Decision alternatives are to be generated so that the values specified for a decision situation are best achieved.

The value-focused approach applied to decision problems, can be much more effective. However, if the decision problem starts with a choice of options, the alternative-focused approach seems more relevant.

For the solution of alternative-focused problems different methods are known. Samsura et al. (2009) proposed and demonstrated that game theoretical modelling could for analysis and predicting the behaviour of actors help us to identify the key strategic decisions of land and property development projects by showing the different payoffs for stakeholders of their chosen strategies and selecting the equilibrium in which all stakeholders involved are best of.

Decision theory usually analyses decision-making processes from one player's point of view, while game theory emphasizes its analysis in the interaction among many players. Because game theory focuses on situations in which interactions and interdependency play a role, it can be seen as an extension of decision theory.

### Success criteria of a construction site selection

Success of a construction site selection is an abstract concept, and determining whether a project is a success or a failure is highly. Actually, owners, designers, architects, consultants, as well as contractors and each project team or individual has a definition of selection success. Success of construction site selection should be viewed from different perspectives. However, the client may have different views on own selection of a construction site objectives and criteria for measuring success. Moreover, even the same person's perception of success changes from project to project. Definitions on the selection of a construction site

success may change according to a project type, size and sophistication, project participants and experience of owners, etc. However, the concept of selection of a construction site success can be evaluated through performance measures. Selection of relevant feasible criteria may subsequently improve the acceptability of the decision. A selection of a construction site can be considered as the achievement of a specified objective, which involves a series of activities and tasks that consume resources. While some authors consider time, costs, quality and labour characteristics, as the predominant targets, others suggest that success is something more complex and consider availability of labour force, quality and reliability of modes of transportation, infrastructure, markets, and proximity to customers. Construction site selection is a long-term decision and is influenced by many quantitative and qualitative criteria. The selection also includes some sub-criteria because of the hierarchical structure of the problem. The type of data to be collected for the selection of a construction site is essentially the same as that used for engineering design, but more detailed information is essential.

Success of selection of a construction site depends on applied construction technology for project implementation. Holistic criteria are needed to assist with the selection of an appropriate construction method in buildings during early project stages (Chen et al., 2009).

Over recent decades, initiatives have proposed environmentally friendly buildings and sustainable construction has centred on residential and office buildings. San-José Lombera and Aprea (2009) presented an Integrated Value Model for Sustainable Assessment that applies a set of six study scopes to define the sustainability criteria of industrial buildings. The system uses a requirements tree to quantify sustainability at various hierarchical levels, in order to assess the behaviour of industrial buildings and compliance with the criteria. Effective and proper evaluation for selection may accelerate the implementation of sustainable construction.

However, varied concerns among related people of different positions make selection problems become a difficult task (Wang and Zeng, 2010). The criteria, which are relevant to the selection can be identified through fuzzy decision making method and used to construct an analytic network process model.

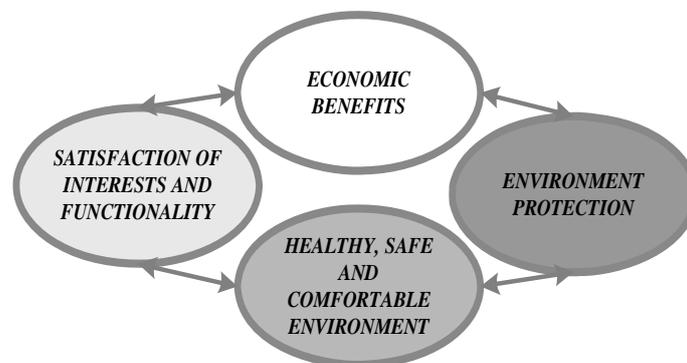
Each project's high-quality development has its own full life cycle (Figure 1). While it is common practice to integrate economic, ecological and social (triple bottom line) criteria, explicit geoscientific factors are relatively rarely considered (Lamellas et al., 2009). If a planned land use involves an interaction with the geosphere, geoscientific aspects should be playing a more important role in the process.

Selection criteria of a construction site success can be defined as the set of principles or standards by which favourable outcomes can be completed within a set specification. Two categories: the macro and micro viewpoints of construction site selection success can be noticed. Figure 2 shows the micro and macro viewpoints of construction site selection success.

The project life cycle must satisfy different requirements in all stages of project's life cycle (Figure 3). Life cycle assessment within the construction industry (Figure 3) is an important methodology for evaluating buildings from the extraction. There is complicated criteria system for life cycle evaluation (Figure 4).

Zucca et al. (2008) discussed a site selection process. It was supported by a value-focused approach and spatial multi-criteria evaluation techniques dealt with a large number of environmental factors and socio-economic constraints. Chen et al. (2009) classified the total number of 66 sustainable performance criteria according to economic, social, and environmental aspects. They created a valuable base for the development of sustainable performance criteria of raw materials, construction, operation and maintenance through to final disposal or demolition, and also life cycle assessment has been gaining attention in the last decade as a means of evaluating building materials explicitly dedicate to residential dwellings (Ortiz et al., 2009a). Life cycle management can be applied to the whole construction process, thus making it possible to improve sustainability criteria and also minimize the environmental loads of the full building life cycle (Ortiz et al. 2009b).

Ortiz et al. (2009b) modelled typical Spanish house located in Barcelona and stated that in terms of this dwelling's environmental loads, the operation phase is the most critical.



**Figure 1.** The main aspects of the development of high-quality, full project life cycle

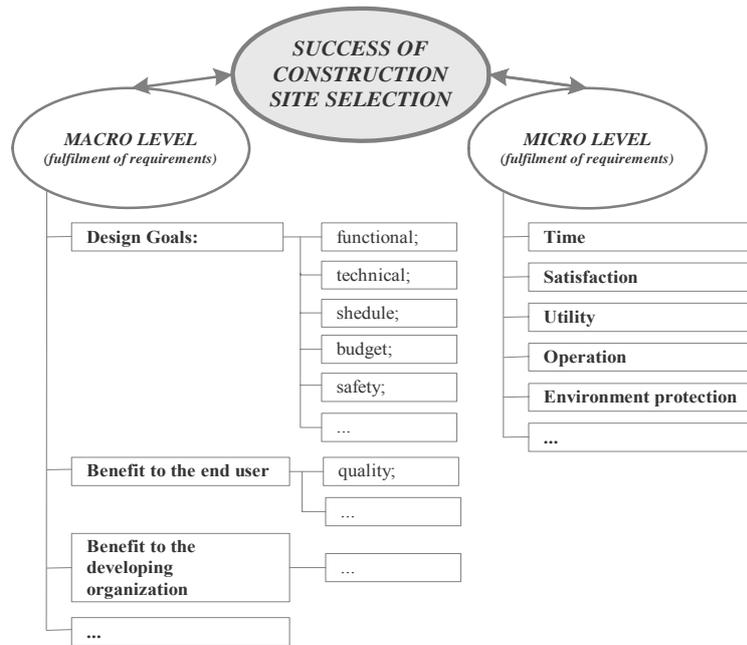


Figure 2. Requirements of micro and macro levels to success of a construction site selection

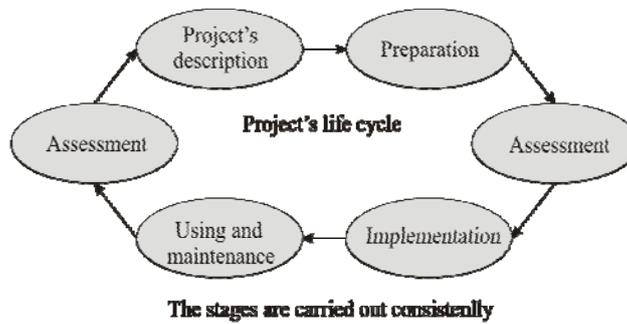


Figure 3. The stages of project life cycle must be carried out consistently

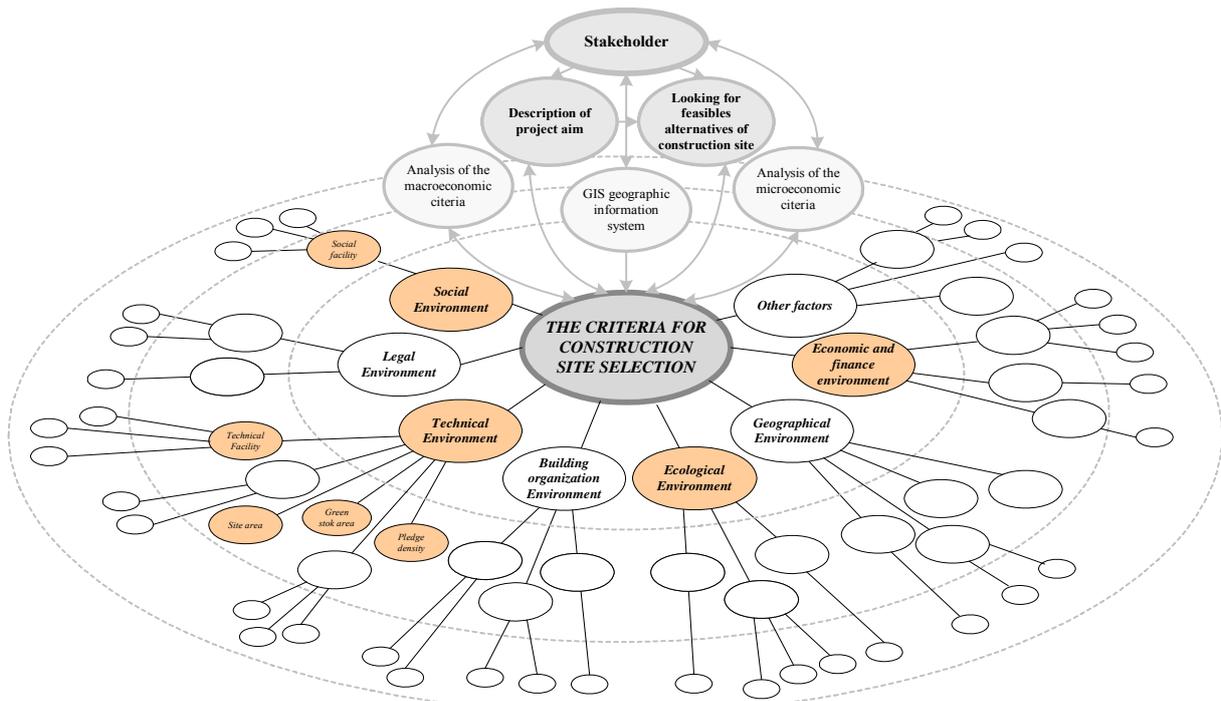


Figure 4. Complicated structure of criteria selection

## Solution methods

There are numerous descriptions of the mathematical theory. In the last decades a variety of applications of multi-criteria evaluation and spatial analysis to site selection have been made. Many of them concerned the search of the most suitable location.

MCDM techniques provide a considerable enrichment to the poor rationality of the single objective optimisation problems (Pomerol and Barba-Romero, 2000; Rogers et al., 2000; Vincke, 1992; Zionts, 1990). Available MCDM methods so far differ from each other in the quality and quantity of additional information they request, the methodology they use, their user-friendliness, the sensitivity tools they offer, and the mathematical properties they verify.

In discrete case of MCDM, the problem is defined by a finite number of alternatives and a family of performance measures (arising from different perspectives of the DM) on which the alternatives are evaluated. The problem could also have a third dimension if it involves multiple DMs and/or uncertainties in the performance measure evaluations (Mareschal, 1986). The outranking methods belonging to the discrete MCDM category have seen a rapid development during the last decade because of their adaptability to the poor structure of most real decision situations, and are becoming more popular among the DMs due to the greater potential for interaction and negotiation.

However, the fullness of the mathematical solution attempts and ideas poses certain problems for the practical uses, so that at this point a choice should be met from the point of the use to the construction technology. If this choice is also subjective and orientates itself by the present state of the application possibilities, it should grant in the area of the construction technology to active engineers an insight into the mathematical demands and serve for the better understanding of the put together use examples.

Hajkovicz (2005) presented a framework for constructing multi-attributed indices to measure welfare derived by a region's citizenry from environmental resources. The process of index construction is based on stakeholder consultation combined with multi-attribute utility theory (used SAW method).

## Game theory and application

Land and property development, spatial planning, and construction processes are characteristically very much context-driven. The issues of complexity and interdependency have become much more important in urban societies (Needham, 2007). Game theory can be used to analyze actual strategic interactions in order to mathematically capture behaviour in strategic situations, in which an individual's success in making choices depends on the choices of others. Interest in game theory has grown, beyond economics to the other social sciences. Theoretical modelling of a game indeed implies, like any modelling exercise, a simplification and abstraction from the real world. However, in economics, the translation of the real world into a formal model is very much accepted and appreciated. Because of its focus on conflicting

preferences, game theory is often defined as a theory of conflict (Luce and Raiffa, 1957) or as a science of strategic decision making. It is concerned with decision making in organisations where the outcome depends on the decisions of two or more autonomous players, one of which may be nature itself, and where no single decision maker has full control over the outcomes. When nature is designated as one of the players, it is assumed that it moves without favour and according to the laws of chance. In the terminology of game theory, nature is not 'counted' as one of the players. Inclusive one can make a distinction for the numerical solution in a one-sided and a two-sided problem. Besides, the calculation of an equilibrium point with a game-theoretical solution counts to the two-side problem.

The first known discussion of game theory occurred in a letter written by James Waldegrave in 1713. In this letter, Waldegrave provides a min-max mixed strategy solution to a two-person version of the card game *le Her*. Game theory in the modern era was ushered in with the publication in 1913, by the German mathematician Ernst Zermelo, of "Über eine Anwendung der Mengenlehre auf die Theorie des Schachspiels". He proved that every competitive two-person game possesses a best strategy for both players, provided both players have complete information about each other's intentions and preferences. In a game of complete information, players know their own strategies and pay-off functions and those of other players. In addition, each player knows that other players have complete information. Interests to modern game theory began growing with the work of Borel (1921) and Von Neumann (1928). After the work of Von Neumann and Morgenstern (1944) game theory became very popular and started as a field of study. The concepts of perfect and imperfect information refer to the information each player has concerning the actions of the other player. A game is one of perfect information if all players know the moves previously made by all other players. Thus, only sequential games can be games of perfect information, since in simultaneous games not every player knows the actions of the others. Much of game theory is concerned with finite, discrete games, which have a finite number of players, moves, events, outcomes, etc. It is only with two or more players that a problem becomes game theoretical.

Games with an infinite number of players are often called *N*-person games (Luce and Raiffa, 1957). While initially developed to analyze competitions in which one individual does better at another's expense (zero sum games), it has been expanded to treat a wide class of interactions, which are classified according to several criteria. In game theory and economic theory, zero-sum describes a situation in which a participant's gain or loss is exactly balanced by the losses or gains of the other participant(s). If the total gains of the participants are added up, and the total losses are subtracted, they will sum to zero. This research usually focuses on particular sets of strategies known as equilibrium in games. From the point of the use a lot of up to now investigated problems of the construction technology became limited by Two-person zero-sum games of strategy modelled, so that the restriction in this special case seems fair-ready.

Zero-sum games are a special case of constant-sum games, in which choices by players can neither increase nor decrease the available resources. In zero-sum games the total benefit to all players in the game, for every combination of strategies, always adds to zero (more informally, a player benefits only at the equal expense of others). The game obeys a law of conservation of utility value, where utility value is never created or destroyed, only transferred from one player to another. The interests of the two players are always strictly opposed and competitive, with no possibility of, or benefit in, cooperation. One player must win and at the expense of the other; a feature known as Pareto-efficiency. More precisely, a Pareto-efficiency is a situation in which the lot of one player cannot be improved without worsening the lot of at least one other player. An efficient outcome cannot be Pareto dominated by any other outcome. In a zero-sum game, all outcomes are Pareto efficient, since at least one player is worse off when others are made better off in choosing another outcome.

The application of game theory was analysed by many authors: Muschik (1975); Muschik, Muller (1986); Peldschus, Zavadskas (1997); Aumann (1989); Hollert (2006); Meszek (2007, 2008); Ginevicius and Krivka (2008); Stain (2010). An introduction to the application of game theory for multiple criteria decision analysis in construction can be found in Peldschus et al. (1983, 2008b). Peldschus (1986, 2007a, b, 2009) analysed the effectiveness of assessments in multi-criteria decision. Multi-criteria optimization system for decision making in construction design and management analysed Turskis et al. (2009).

With respect to construction, property development and management the applications of game theory so far are limited in number (Mu and Ma 2007):

- Zavadskas et al. (2003) developed the software for multiple criteria evaluation;
- Zavadskas et al. (2004) solved problems of construction technology and management;
- Peldschus and Zavadskas (2005) investigated fuzzy matrix game in construction;
- Lo et al. (2006) proposed exit selection model for evacuation based a game theory;
- Su et al. (2007) proposed model of urban public traffic networks;
- Sun and Gao (2007) applied an equilibrium model for urban transit assignment;
- Homburg and Scherpereel (2008) analysed the cost of joint risk capital be allocated for performance measurement;
- Motchenkova (2008) applied a differential game describing the interactions between a firm that might be violating competition law and the antitrust authority. The objective of the authority was to minimize social costs (loss in consumer surplus) induced by an increase in prices above marginal costs;
- Podvezko (2008a) presented the review of the game theory application experience in construction management;
- Peldschus (2008) applied game theory for the problem solution in technology and management of construction;

- Schotanus et al. (2008) analysed unfair allocation of gains under the equal price allocation method in purchasing groups;

- Tamosaitiene et al. (2008) - modelling of contractor selection taking into account different risk level;

- Zavadskas et al. (2008c) developed multi-criteria optimization software LEVI-4.0 based on the game theory;

- Zavadskas and Turskis (2008) developed a new logarithmic normalization method in games theory;

- Westergaard (2008) provided a definition of visualisations, founded in game-theory, which regards visualisations as transition systems synchronised with formal models.

- Gu et al. (2009) analysed Chinese strategies for energy-efficient housing developments from an architect's perspective.

The methodology that was applied in this study for solution of problem is game theory. This process proposed provides a logical and scientific foundation into which the values of decision makers and stakeholders can be integrated. Game theory represents an abstract model of decision making, not the social reality of decision making itself. Therefore, while game theory ensures that a result follows logically from a model, it cannot ensure that the result itself represents reality, except in so far as the model is an accurate one.

The game theory can be used for sustainable decision making. In this case it is very important to find the equilibrium point for the sustainable decision making.

### Definition of matrix games

There are a number of the mathematical theory descriptions. However, the fullness of the mathematical solution attempts and ideas poses certain problems for the practical use, so that at this point a choice should be met from the point of the use for the construction. If this choice is also subjective and orientates itself by the present state of the application possibilities, it should serve in the area of the construction technology and civil engineering activities. There are not determined special solution criteria, as well as they are developed by Muschik (1975) for the Electro engineering. From the point of the use a lot of up to now investigated problems of the construction technology can be assumed as two-person zero-sum game models.

Nash Equilibrium: no player has any incentive to change his or her action, assuming that the other player(s) have chosen their best actions for themselves. In two-player games, Nash equilibrium prescribes strategies that are mutually best response (not universally best responses, as with dominant strategies). At Nash equilibrium, each player is doing the best he/she can, given the strategies of the other players. The Nash equilibrium is not necessarily efficient.

Matrix games are limited by two-person zero-sum games of strategy. The zero-sum payoffs (if one gains, another loses) means that any result of a zero-sum situation is Pareto optimal (generally, any game where all strategies are Pareto optimal is called a conflict game) (Bowles, 2004). They can be derived from  $N$ -person games. The

Table 1

normal (or strategic form) game is usually represented by a matrix which shows the players, strategies, and payoffs. More generally it can be represented by any function that associates a payoff for each player with every possible combination of actions. When an  $N$ -person game is presented in normal form, it is presumed that each player acts simultaneously or, at least, without knowing the actions of the other (if players have some information about the choices of other players, the game is usually presented in an extensive form).

The vector of strategies  $s$  selected by the players determines the outcome for each player, in general; the outcome will be different for different players. To specify the game, it is needed to give, for each player, a preference ordering on these outcomes by giving a complete, transitive, reflexive relation.

Formally an  $N$ -person zero-sum game consists of a set of  $n$  players, ( $i=1, 2, \dots, n$ ). Each player  $i$  has his own nonempty strategy:

$$S_i (i=1, \dots, n). \tag{1}$$

The pay-off functions of the players  $A_i (i=1, 2, \dots, n)$  are  $n$  real-valued functions defined on  $S_1 \times S_2 \times \dots \times S_n$ . Thus  $A(s_1, s_2, \dots, s_n)$  is a real number for each strategy  $s_1 \in S_1, \dots, s_n \in S_n$ .

The strategies product  $S_1 \times S_2 \times \dots \times S_n$  is the amount of all  $(S_1, \dots, S_n)$  with  $s_1 \in S_1, \dots, s_n \in S_n$ . This game is called with  $\{S_1, \dots, S_n, A_1, \dots, A_n\}$ . The strategy  $S_i$  amounts from a structural point of view it seems are pay-offs point in Euclidean space.

The game aim is defined later. For two-person games of strategy can be given as:

$$\Gamma = \{S_1, S_2, A_1, A_2\}, \tag{2}$$

where  $S_1$ - strategies for the first player;  $S_2$ - strategies for the second player;  $A_1$ - pay-off function for the first player;  $A_2$ - pay-off function for the second player.

With the use to construction and technological problems one can assume the strategies  $S_1$  and  $S_2$ , finally, so that is valid:

$$S_1 = \{S_{11}, \dots, S_{1n}\}, S_2 = \{S_{21}, \dots, S_{2n}\}. \tag{3}$$

For the pay-off functions  $A_1$  and  $A_2$  can be shown that  $A_1 = -A_2$ , i.e. the profit of one player is compensated by the loss of the other and vice versa. From it there originated the name two-person zero-sum games of strategy. Games with  $A_1 + A_2 = c$  can be solved without any restrictions:

$$\Gamma' = \{S_1, S_2, A_1, A_2 - c\}. \tag{4}$$

If  $c = 0$ , and if  $A_1 = A$ , then  $A_2 = -A$ . In this case a two-person zero-sum games of strategy in shortened form can be given as matrix games limited by two-person-zero sum games (Hollert, 2006), (Peldschus and Zavadskas, 1997):

$$\Gamma = (S_1, S_2, A), \tag{5}$$

where  $S_{1i}$  are available ( $i = 1, \dots, m$ ) strategies for the first player and  $S_{2j}$  are available ( $j = 1, \dots, n$ ) strategies for the second player and  $A$  the pay-off function for the first and second player: or the game has an ideal saddle point solution (simple min-max principle (Arrow et al., 1949; Arrow, 1951)) or a strategy combination (extended min-max principle) is obtained (In a two-player zero-sum game defined on a continuous space, the equilibrium point is a saddle point) (Table 1).

**Initial decision making matrix**

	$S_{21}$	$S_{22}$	$\dots$	$S_{2n}$
$S_{11}$	$a_{11}$	$a_{12}$	$\dots$	$a_{1n}$
$S_{12}$	$a_{21}$	$a_{22}$	$\dots$	$a_{2n}$
$\vdots$	$\vdots$			
$S_{1m}$	$a_{m1}$	$a_{m2}$	$\dots$	$a_{mn}$

Remark: In Table 1  $i$  is line index and  $j$  is column index.

### Saddle points' equilibrium

After the description of the game is done, it is necessary to analyse the problem to the rational behaviour of the players. It can be assumed that each player seeks maximal profit (and this is also expected from the opponent). For the first player it means the least value of his  $i$ -th aim:

$$\alpha_i = \min_j a_{ij}, \tag{6}$$

Therefore, he will choose those aims for which this profit limit is the biggest:

$$\max_i \alpha_i = \max_i \min_j a_{ij} = \alpha, \tag{7}$$

where  $\alpha$  is named the smallest value of the game. For the second player it means, because  $A_1 = -A_2$ , the choice of his  $j$ -th column:

$$\beta_j = \max_i a_{ij}, \tag{8}$$

where  $\beta$  is highest limit of losses. It must be minimised as follows:

$$\min_j \beta_j = \min_j \max_i a_{ij} = \beta, \tag{9}$$

where  $\beta$  is the biggest value of the player. For an equilibrium point the pay-offs function  $A$  must reach its maximal value  $i$  with regard to a  $j$  minimal values. According to von Neumann there must determined the saddle point that also is named as an equilibrium point. For the game a saddle point will be obtained, if and only if the expressions

$$\max_{s_1 \in S_1} \inf_{s_2 \in S_2} A(s_1, s_2) \text{ and } \minsup_{s_2 \in S_2, s_1 \in S_1} A(s_1, s_2), \tag{10}$$

exists and are equal, i.e., if

$$\begin{aligned} \max_{s_1 \in S_1} \inf_{s_2 \in S_2} A(s_1, s_2) &= \minsup_{s_2 \in S_2, s_1 \in S_1} A(s_1, s_2) = \\ &= A(s_1^*, s_2^*) = v. \end{aligned} \tag{11}$$

In this case both expressions always are right because the strategy sets are finite. The first player's strategy at equilibrium point is those strategies  $s_1 \in S_1$ , for which the  $s_2$  infimum (greatest lower bound) reaches the maximum with regard to  $s_1$ . Similarly, the equilibrium strategies of the second player are those strategies  $s_2 \in S_2$ , for which  $s_1$  supremum (least upper bound) reaches the minimum relative to  $s_2$ . The criterion expressed by equation (15) is a well-known min-max principle (von Neuman and Morgenstern 1944), with the value  $v$  being the game value. This definition expresses the theory of two-person zero-sum; game's rational behaviour has equilibrium point.

A pure strategy for a player is a campaign plan for the entire game, stipulating in advance what the player will do

in response to every eventuality. If a player selects a strategy without knowing which strategies were chosen by the other players, then the player's pure strategies are simply equivalent to his or her choices. With this sentence one has an adequate theory of the rational behaviour for two-person zero-sum games which own an equilibrium point. If no equilibrium point with pure strategies (i.e. to the strategies given on top) is assignable, several strategies with certain frequencies – mixed strategies – are used.

In the case when there is no equilibrium point for the pure strategy for the matrix game with the pure strategies:

$$S_1 = \{S_{11}, \dots, S_{1m}\} \text{ for the first player, and}$$

$$S_2 = \{S_{21}, \dots, S_{2n}\} \text{ for the second player,}$$

and the pay-offs function  $A = [a_{ij}]$ , where  $i = 1, 2, \dots, m$  and  $j = 1, 2, \dots, n$ , is named as distribution  $p$  for the first players mixed strategy  $S_j$ :

$$P = \{p_1, p_2, \dots, p_i, \dots, p_m\} \text{ with } p_i \geq 0 \text{ and } \sum_{i=1}^m p_i = 1, \quad (12)$$

and the similar distribution of possibilities distribution  $q$  for the mixed strategy  $S_2$  of the second player:

$$Q = \{q_1, q_2, \dots, q_i, \dots, q_n\} \text{ with } q_j \geq 0 \text{ and } \sum_{j=1}^n q_j = 1. \quad (13)$$

For every mixed strategy  $p$  and  $q$  of the first player the mathematical expectation  $E$  can be expressed as:

$$E(p, q) = \sum_{i=1}^m \sum_{j=1}^n p_i q_j a_{ij}. \quad (14)$$

The theorem of the mathematical expectation  $(P, Q, E)$  is the mixed extension of the game  $(S_1, S_2, A)$ . The main definition of the min-max theorem is given by von Neumann and Morgenstern (1944).

Every two-person zero-sum game with limited (pure) strategy spaces  $S_1$  and  $S_2$  has its own mixed extension  $(P, Q, E)$ , with value  $V$ , and every player has at least one (mixed) comparable strategy  $p$  or  $q$  with which for him the value  $V$  is guaranteed:

$$v(\Gamma) = \max_{p \in P} \min_{q \in Q} E(p, q) = \min_{p \in P} \max_{q \in Q} E(p, q) = E(p^*, q^*), \text{ and} \quad (15)$$

$$E(p, q^*) \leq E(p^*, q^*) \leq E(p^*, q). \quad (16)$$

This means that in every limited two-person game a pair optimally of mixed strategies  $p^* \in P$  and  $q^* \in Q$  exists, so that  $(p^*, q^*)$  is saddle point from  $\Gamma$ .

### Equilibrium strategies – the min-max-principle

The solution according to the min-max-principle with the rational behaviour can be applied in the cases of direct interest conflicts between two players as well as for decisions under uncertainty (incomplete information) - games against the nature. Muschik distinguished between the simple and extended min-max principles.

In the trivial case – a simple min-max principle – the solution for both players is the same element of matrix and the saddle point is defined for the pure strategies. In this case the condition is valid:

$$\alpha = \beta = v. \quad (17)$$

The optimal strategies are defined as:

$$S_1^* = \{S_{1i} / S_{1i} \in S_1 \cap \max_i \min_j a_{ij}\},$$

$$S_2^* = \{S_{2j} / S_{2j} \in S_2 \cap \min_j \max_i a_{ij}\}, \quad (18)$$

and the value  $V$  is always guaranteed.

For the extended min-max principle a linear system must be solved. For the first player:

$$\sum_{i=1}^m p_i^* a_{ij} \geq v, (j = 1, \dots, n), (i = 1, \dots, m),$$

$$p_i^* \geq 0, \sum_{i=1}^m p_i^* = 1, \quad (19)$$

and for the second player:

$$\sum_{j=1}^n q_j^* a_{ij} \geq v, (j = 1, \dots, n), (i = 1, \dots, m), q_j^* \geq 0, \sum_{j=1}^n q_j^* = 1. \quad (20)$$

For  $v < 0$  the transformation is necessary into an equivalent game.

$$\Gamma_1 = \Gamma + C, C > 0. \quad (21)$$

In case of  $v(\Gamma) > 0$  the game according to the value  $v(\Gamma)$  can be divided to the two comparative systems of inequalities with new variables:

$$\frac{p_i^*}{v} = u_i, \text{ and } \frac{q_j^*}{v} = w_j. \quad (22)$$

If one receives the linear optimisation problems (in which  $V$  is vanished):

for the first player:

$$\sum_{i=1}^m u_i a_{ij} \geq 1, (j = 1, \dots, n), (i = 1, \dots, m), u_i \geq 0,$$

$$\sum_{i=1}^m u_i = \frac{1}{v} \rightarrow \min, \quad (23)$$

for the second player:

$$\sum_{j=1}^n w_j a_{ij} \leq 1, (j = 1, \dots, n), (i = 1, \dots, m), w_j \geq 0,$$

$$\sum_{j=1}^n w_j = \frac{1}{v} \rightarrow \max. \quad (24)$$

Both linear optimisation problems are binary to each other and the solution can be found by applying Simplex method. From the solution results the optimum strategy for both players can be defined:

$$p_i^* = u_{oi} \cdot v, (i = 1, \dots, m), q_j^* = w_{oj} \cdot v, (j = 1, \dots, n). \quad (25)$$

The value  $V$  is the guaranteed average for an enough big number of repetitions:

$$\alpha = \max_i \min_j a_{ij}, \beta = \min_j \max_i a_{ij}. \quad (26)$$

In case of

$$\alpha = \beta = \gamma, \quad (27)$$

a saddle point with pure strategies (only one optimal strategy for each player) is determined as solution – trivial solution.

*Extended min – max principle.* An equilibrium point with mixed strategies is calculated (combination of strategies):

$$\max_i \min_j A(s_1, s_2) = \min_i \max_j A(s_1, s_2) = A(s_1^*, s_2^*) = v. \quad (28)$$

**Case study: Calculation of the equilibrium point for construction site assessment problem** (Zavadskas et al., 2009)

There are available different sites in Vilnius city. For the practical example there were assessed five different feasible alternatives of construction site. To define the site’s selection criteria set for the design, first, it is necessary to identify the main objectives of the project. The formulation of objectives and their related criteria led to the identification of the characteristics and, as a result, to the definition of the construction site.

A strategic game is a model of interacting decision-makers. In recognition of the interaction, in the case study it is referred to the decision-makers as players. A strategic game (with ordinal preferences) consists of: a set of players; for each player, a set of actions; for each player, preferences over the set of action profiles.

The price, area of construction site, and soak density appeared to be one of the main goals. Eight performance criteria were selected for construction sites assessment on the basis of the questioning of 23 high-skilled construction specialists, territory planning specialists, architects, designers, and stakeholders:

- $x_1$  – price of the construction site, [ $10^6 \text{ €}/10^2 \text{ m}^2$ ];
- $x_2$  – area of the construction site, [ $10^2 \text{ m}^2$ ];
- $x_3$  – soak density [%];
- $x_4$  – technical environment of the site, [point];
- $x_5$  – ecological environment of the site, [point];
- $x_6$  – social environment facilities, [point];
- $x_7$  – appeal district, [point];
- $x_8$  – green expansion, [point].

The weights were assigned, using the rank order method, by a group of experts. The weights of the criteria were established on the basis of questioning of the 27 experts. The weights of the criteria and the criteria optimization direction are presented in the Table 2. The initial decision making matrix, the feasible alternatives, the weight of the criteria and the criteria optimization direction are presented in the Table 3. The main application of matrix games is the selection of the alternative. For the description of the problem the alternatives are assigned to the first payer’s strategies and the criteria are assigned to the second player’s strategies. For the pay-off function a

dimensionless evaluation numbers are used. Such dimensionless evaluation numbers describe the situation only coarsely. It is therefore sensible to use real characteristic values. As such characteristic values have different dimensions their impact on the effectiveness is not comparable. Usually, in order to allow a comparability of the characteristic values, they are mapped on the interval [1; 0] or [1; ~0]. Depending on the kind of the problem there are several options for the normalisation of the characteristic values. Generally, a distinction can be made between linear and non-linear transformations (Peldschus, 2007a).

There is no dominated strategy according to all criteria (Table 3). The best performance values are displayed in red, and the worst ones in blue. The best alternative from the set of possible alternatives can be defined according to an equilibrium point. For the decision making there is prepared a normalised matrix of performance (Table 4) and a normalised-weighted matrix of performance (Table 5). Initial decision-making matrix was normalised by applying linear normalisation method (Zavadskas and Turskis, 2008):

$$\bar{x}_{ij} = \frac{x_{ij} - \min_i x_{ij}}{\max_i x_{ij} - \min_i x_{ij}}, \text{ if } \max_i x_{ij} \text{ is preferable,} \tag{28}$$

$$\bar{x}_{ij} = \frac{\max_i x_{ij} - x_{ij}}{\max_i x_{ij} - \min_i x_{ij}}, \text{ if } \min_i x_{ij} \text{ is preferable.}$$

The values  $\alpha = 0.02$  and  $\beta = 0.05$  are determined as a result. This means that there exists no equilibrium point. Therefore, the calculation with extended min – max principle occurs. As a calculation result there are received two following vectors:  $S_1^* = (0.00; 0.00; 0.60; 0.00; 0.40)$ , for the first player and  $S_2^* = (0.00; 0.00; 0.00; 0.00; 0.69; 0.31; 0.00; 0.00)$  for the second player.

The calculation of the equilibrium point shows that the value of the third alternative equals to 60% and the assessment of the fifth alternative equals to 40%. The first, second and fourth alternatives are not involved into the determination of the equilibrium point because its functional influence is lower and, therefore, it is dominated by the other influence.

Table 2

**Weights (w) assigned to each criterion in the evaluation phase**

Criteria	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$x_6$	$x_7$	$x_8$
w	0.21	0.16	0.15	0.05	0.05	0.11	0.12	0.07

Table 3

**Initial decision-making matrix of the problem**

Criteria	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$x_6$	$x_7$	$x_8$
<i>Optimization direction</i>	<i>min</i>	<i>min</i>	<i>max</i>	<i>max</i>	<i>max</i>	<i>max</i>	<i>max</i>	<i>max</i>
Weight/Alternatives	0.21	0.16	0.15	0.13	0.05	0.11	0.12	0.07
$A_1$	0.039	45	12	5	5	2	4	8
$A_2$	0.035	43	46	7	2	5	5	7
$A_3$	0.042	41	38	6	6	3	3	9
$A_4$	0.050	38	67	8	2	8	2	2
$A_5$	0.048	44	24	7	4	6	4	4

Table 4

**Normalised decision-making matrix (linear normalization is applied)**

Criteria	$\bar{x}_1$	$\bar{x}_2$	$\bar{x}_3$	$\bar{x}_4$	$\bar{x}_5$	$\bar{x}_6$	$\bar{x}_7$	$\bar{x}_8$
Weight/Alternatives	0.21	0.16	0.15	0.05	0.05	0.11	0.12	0.07
$A_1$	0.733	1.000	0.000	0.000	0.750	0.000	0.667	0.857
$A_2$	1.000	0.714	0.618	0.667	0.000	0.500	1.000	0.714
$A_3$	0.533	0.429	0.473	0.333	1.000	0.167	0.333	1.000
$A_4$	0.000	0.000	1.000	1.000	0.000	1.000	0.000	0.000
$A_5$	0.133	0.857	0.218	0.667	0.500	0.667	0.667	0.286

Table 5

**Weighted-normalised decision-making matrix**

Criteria	$\hat{x}_1$	$\hat{x}_2$	$\hat{x}_3$	$\hat{x}_4$	$\hat{x}_5$	$\hat{x}_6$	$\hat{x}_7$	$\hat{x}_8$
$A_1$	0.154	0.160	0.000	0.000	0.038	0.000	0.080	0.060
$A_2$	0.210	0.114	0.093	0.033	0.000	0.055	0.120	0.050
$A_3$	0.112	0.069	0.071	0.017	0.050	0.018	0.040	0.070
$A_4$	0.000	0.000	0.150	0.050	0.000	0.110	0.000	0.000
$A_5$	0.028	0.137	0.033	0.033	0.025	0.073	0.080	0.020

As a specific feature with the calculation of the equilibrium point, the assessment of the criteria can be given here. This is represented by the optimum strategy for the second player. Then the criterion 5 (ecology) is valued by 69% and the criterion 6 (social infrastructure) by 31%. The criteria 1, 2, 3, 4, 7 and 8 are not involved into the calculation of the equilibrium point, because they are dominated by the criteria 5 and 6.

**Conclusions**

Game theory, compared with alternative modelling techniques, offers an appropriate approach to modelling construction processes, property development processes, because it takes account of the complexity of the process.

The application of the game theoretical modelling in complex decision-making processes involves, by definition, the simplification of reality in the model. However, these weaknesses can also be seen as strength, as modelling problems are often used in other research fields as decision support tools.

Game theory provides a way to think about the collective decision-making processes.

The calculation of an equilibrium point delivers more information. Beside the assessment of the alternatives, an assessment of the criteria also occurs.

With the calculation of an equilibrium point the same assessment is lifted for all criteria. The meaning of the

criteria comes thus into the result as her effectiveness appears in the transformed matrix. Thus, the effectiveness of the criteria depends on the toe-in of the initial values. Besides, a big toe-in means a big influence and a low toe-in a small influence. Such an explanation seems logical and plausible. An insinuation for the same effectiveness of the criteria cannot be founded unambiguously and stands also in the contradiction to the use of important factor. If one is ready for the meaning of the criteria to change, then one must also accept that its effectiveness on the solution is different.

With the calculated solution one also has to carry out special investigations for the assessment of a complicated problem. If another assessment of the alternative is wished, one can work on the criteria involved in the equilibrium point according to its effectiveness specifically.

The calculation of an equilibrium point is to be carried out by other possibility of multi-attribute decisions for complicated problems.

Game theory assumes that the players possess complete information about the strategies of other players. Unfortunately, in practice, players do not usually hold complete information about the strategies and payoff functions of the other players.

Game theory assumptions that persons act perfectly rational may never match a real-life situation.

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### Darnus statybos vietos parinkimas taikant lošimų teoriją

#### Santrauka

*Aktualumas ir problema.* Parenkant pastato statybos vietą, dalyvauja su statybos planavimo procesais susijusios suinteresuotosios grupės, kurios priima įvairius sprendimus, lemiančius planuojamo statinio ir jo vietos charakteristikas. Uždavinys sprendžiamas įvertinant daugelį tikslų, priklausomai nuo pasirinkamos vietos aplinkos. Sėkmingas statybos vietos pasirinkimas turi didelę įtaką projekto sėkmei arba nesėkmei. Informacija, reikalinga statybos vietai parinkti, yra labai svarbi, ji daro didelę įtaką statinio projektiniams sprendimams, pastato konfigūracijai, statinio naudojimo patogumui.

*Straipsnio tikslas* – pateikti darnų statybos vietos parinkimo modelį, priklausomai nuo vietos aplinkos, taikant lošimų teoriją ir pateikiant praktinį uždavinį.

#### Uždaviniai:

1. Aptarti sprendimų priėmimo metodus, kurie taikyti statybos vietai parinkti.
2. Išanalizuoti statybos vietos parinkimą lemiančius rodiklius.
3. Aptarti lošimų teorijos metodus ir taikymą praktiniams uždaviniams.
4. Pateikti lošimų teorijos, dviejų-lošėjų nulinės sumos matematinį aprašymą.
5. Pateikti praktinį statybos vietos parinkimo uždavinį taikant lošimų teoriją.

*Tyrimo metodai.* Sprendimų priėmimo teorija analizuoja sprendimų priėmimo procesus remiantis lošėjų nuomone. Taikant lošimų teoriją, sprendimas gali būti priimamas, atsižvelgiant į daugelio lošėjų strategijas. Lošimų teorija yra susijusi su baigtiniais, diskrečiais lošimais, baigtiniu dalyvių skaičiumi, rezultatais, ir t. t. Straipsnyje pateikiama dviejų-lošėjų nulinės sumos strateginio lošimo teorija.

*Straipsnio struktūra.* Straipsnį sudaro įvadas, keturios dalys ir išvados. Įvade aptariamas statybos vietos parinkimas, atsižvelgiant į darnią ir konkurencingą aplinką. Pirmoje dalyje pateikiama statybos vietos parinkimo literatūros apžvalga. Antroje ir trečioje dalyje – lošimų teorijos metodų matematinis aprašymas. Ketvirtoje dalyje pateiktas praktinis statybos vietos parinkimo uždavinys taikant lošimų teorijos dviejų-lošėjų nulinės mokėjimo sumos modelį.

*Tiriamasis uždavinys.* Projektų parengimas ir realizavimas, pagrįstas teoriniais ir empiriniais skaičiavimais, prekių, paslaugų ir technologijų kūrimas yra viena iš svarbiausių žmogaus veiklos rūšių. Sprendimų priėmimo procesas yra veikla, kuri prasideda nuo pripažinimo, kad yra uždavinys, kurį reikia išspręsti, ir baigiasi rekomendacijų, grįstų uždavinio sprendiniu, pateikimu.

Kiekvienas statybos projektas yra unikalus. Siekiama pastatyti per apibrėžtą laiką, neviršijant priimtinos kainos ir išpildant kokybinius reikalavimus. Dauguma sprendimų priimami remiantis žiniomis ir atsižvelgiant į tvariąją ekonominę plėtrą. Vienas iš pagrindinių šiuolaikinės statybos uždavinių yra tvari miestų plėtra. Dėl šios plėtos ir gyvenimo standartų augimo didėja iškastinio kuro ir žaliavų poreikis. Todėl aplinkos apsauga ir efektyvus gamtos išteklių naudojimas yra labai svarbūs uždaviniai visai žmonijai. Norint siekti tvarumo statybos srityje, dabartinė plėtra, kuri yra orientuota į statybos plėtrą, turi būti pakeista į tvarią statybos plėtrą plėtojant ir skleidžiant tvarios statybos technologijas. Vis dėlto, santykiai tarp ekonominės plėtos ir aplinkos tvarumo yra labai sudėtingi. Priimti sprendimus dėl statybos tvarumo vertinimo yra sudėtingas ir daugialypis procesas, nes čia įvertinami moksliniai faktai ir subjektyviosios vertybės. Statyboje projekto sėkmė apibrėžiama nevienareikšmiškai. Sprendimų paramos metodų taikymas moksliniams faktams ir subjektyviosioms vertybėms suderinti gali būti naudingas sprendimus priimantiems asmenims. Tinkamas statybvietės pasirinkimas daugiausia grindžiamas bendrąja praktika, įgytomis žiniomis, aplinkos veiksniais. Tiesą sakant, savininkas, projektuotojas, architektas, konsultantas, taip pat rangovas ir kiekvienas projekto komandos narys turi nuosavą sėkmės apibrėžimą. Be to, net to paties asmens sėkmės suvokimas kinta ir priklauso nuo projekto tipo, dydžio ir sudėtingumo, projekto dalyvių ir patirties. Šiuolaikinis galimų veiksmų, strategijų ir priimamų sprendimų vertinimas tapo labai sudėtingas ir painus. Visų naujų idėjų ir galimų sprendimų variantų efektyvumą reikia lyginti pagal daugelį kriterijų. Operacijų tyrimo ir daugiatakliai sprendimų metodai gali būti taikomi tokiems uždaviniams spręsti. Taikant šiuos metodus, galima suderinti socialinius-politinius, aplinkos ir ekonominius poveikius ir projektu suinteresuotųjų šalių nuomones.

*Naujumas ir tyrimai.* Siūlomas dviejų asmenų su nuline mokėjimo suma strateginio lošimo modelis statybos vietai parinkti.

*Tyrimo objektas* – darnios statybvietės parinkimas vertinant kelis efektyvumo kriterijus.

*Šio tyrimo tikslas* – pasiūlyti ir pritaikyti dviejų asmenų lošimo su nuline mokėjimo suma strateginio lošimo teorijos modelį statybos alternatyvų tvarumui vertinti.

*Tyrimo uždaviniai.* Siekiant įgyvendinti tikslus, reikia atlikti tokius mokslinius tyrimus:

- išnagrinėti taikomus statybos vietos parinkimo ir vertinimo metodus;
- nustatyti statybų vietos parinkimo sėkmės kriterijus, atsižvelgiant į tvariąją plėtrą;
- pritaikyti lošimo teorijos metodus įmanomoms statybvietės alternatyvoms vertinti.

*Tyrimo metodas* – analitinis tyrimas, grįstas tiriamuoju uždaviniu.

*Tyrimo pavyzdys.* Sprendžiamas statybvietės parinkimo uždavinys Vilniaus mieste. Sėkmingas statybvietės atrinkimas priklauso nuo įgyvendinamo statybos projekto taikomų technologijų. Duomenys ir jų tipas statybos vietai parinkti iš esmės yra tokie patys, kaip ir taikomi inžineriniame projektavime (2 paveikslas). Projekto gyvavimo ciklas turi atitikti įvairius reikalavimus visose projekto gyvavimo ciklo etapuose (3 pav.). Informacija tam tikrai statybvietei aprašyti (4 paveikslas) skiriasi atsižvelgiant į projektuojamo pastato dydį ir svarbą, įrangą, ir t. t.

Straipsnyje pateiktas galimų penkių skirtingų statybvietės alternatyvų tyrimas. Pagal projekto paskirtį, tipą ir keliamus tikslus, buvo nustatyti statybvietės aprašymo pagrindiniai kriterijai. Aštuoni efektyvumo kriterijai buvo parinkti, apklausus 23 aukštos kvalifikacijos architektus, projektuotojus, suinteresuotųjų šalių atstovus ir statybos ir teritorijų planavimo specialistus, statybvietei įvertinti:

- $x_1$  – statybvietės kaina, [106 € / 10<sup>2</sup> m<sup>2</sup>];
- $x_2$  – statybvietės plotas, [10<sup>2</sup> m<sup>2</sup>];
- $x_3$  – užstatymo tankumas, [%];
- $x_4$  – statybvietės techninė aplinka, [balai];
- $x_5$  – statybvietės ekologinė aplinka, [balai];
- $x_6$  – socialinė aplinka, [balai];
- $x_7$  – rajono aplinka, [balai];
- $x_8$  – žaliąji plėtra, [balai].

Kaina, statybvietės plotas ir užstatymo tankumas pasirodė kaip vieni iš svarbiausių rodiklių. Kriterijų svoriai buvo nustatyti 27 ekspertų apklausos pagrindu. Kriterijų svoriai ir kriterijų optimizavimo kryptys pateikiamos 2 lentelėje. Pradinė sprendimo priėmimo matrica, aprašanti įgyvendinamas alternatyvas, atsižvelgiant į kriterijus, svorius ir kriterijų optimizavimo kryptis, yra pateikiama 3 lentelėje. Nėra dominuojančios strategijos pagal visus kriterijus (3 lentelė).

Kriterijų reikšmės normalizuojamos taikant tiesinį normalizavimo metodą.

Išsprendus uždavinį gauta  $\alpha = 0,02$  ir  $\beta = 0,05$ . Tai reiškia, kad nėra pusiausvyros taško. Todėl pritaikytas išplėstasis min ir max principas. Sprendimo rezultatas yra du vektoriai  $S_1^* = (0,00; 0,00; 0,60; 0,00; 0,40)$  pirmajam lošėjui ir  $S_2^* = (0,00; 0,00; 0,00; 0,00; 0,69; 0,31; 0,00; 0,00)$  antrajam lošėjui. Pusiausvyros taško skaičiavimas parodo, kad trečiosios alternatyvos vertė lygi 60 %, penktosios alternatyvos įvertinimas lygus 40. Pirmą, antrą ir ketvirtą alternatyvas nėra susijusios su nustatytu pusiausvyros tašku. Jų įtaka yra mažesnė, todėl dominuoja kitos. Tai yra antrojo lošėjo atstovaujama strategija. Tuomet vertinamas kriterijus 5 (ekologija), kurio vertė lygi 69 % ir 6 kriterijus (socialinė infrastruktūra), kurio vertė lygi 31 %. 1, 2, 3, 4, 7 ir 8 kriterijai nedalyvauja skaičiuojant pusiausvyros tašką, nes dominuoja 5 ir 6 kriterijai.

*Išvados.* Taikant lošimų teoriją grįstą uždavinio modelį, atsižvelgiama į proceso sudėtingumą. Lošimų teorija pateikia būdą sprendimams pagrįsti, atsižvelgiant į kolektyvinę nuomonę. Pusiausvyros taško apskaičiavimas pateikia daugiau informacijos. Vertinamos alternatyvos, kriterijai ir jų svoriai. Todėl jis, palyginti su kitais modeliavimo metodais, siūlo geresnį būdą, modeliuoti statybos ir nekilnojamojo turto plėtros procesus.

Nustačius uždavinio sprendinius, galima išsamiau tirti sudėtingą uždavinį. Dviejų asmenų strateginio lošimo su nuline mokėjimo suma teorija gali būti sėkmingai taikoma statybvietei parinkti ir kitiems parinkimo uždaviniams spręsti.

Modeliuojant sudėtingus sprendimų priėmimo procesus, taikant lošimų teorijos metodus, taikomas supaprastintas tikrovės modelis. Šis trūkumas kartu yra privalumas, taip pat pagrindinė visų mokslinių tyrimų priemonė sprendimams pagrįsti.

Raktažodžiai: *darna, tvari plėtra, statybos vieta, įvertinimas, operacijų tyrimo metodai, sprendimų priėmimas, lošimų teorija, pusiausvyros taškas.*

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