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An Empirical Investigation of Value at Risk (VaR) Forecasting Based on Range-Based Conditional Volatility Models

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Value at Risk (VaR) is a widely used measure of market risk. Precision in the estimation of volatility leads to accurate VaR forecasts. As volatility is time-varying and has a clustering effect, GARCH class of volatility models is helpful in modeling volatility more precisely. Studies have also shown that range-based volatility estimates are more efficient than traditional models that use only closing prices. Therefore, this study uses the GARCH family of volatility models to model and forecast VaR. The study compares conventional models that use closing prices alone, like GARCH and TARCH, with range-based models like RGARCH and RTARCH models, where the range defined as daily high price minus low price is introduced as an exogenous variable, to explore if the latter provides better predictive accuracy. All the models are back-tested using the Kupiec (1995) unconditional coverage and Christoffersen (1998) conditional coverage tests. The data period in the study ranges from 2003–2021, and we consider five BRICS indices and three major developed economies, namely, the USA, the UK, and Germany. An empirical investigation shows that range-based models do a better job in VaR forecasting as it has more information content than daily closing prices, thereby giving more accurate VaR estimates. The study hopes this finding will greatly help stakeholders like financial institutions, regulators, and practitioners in more effective market risk management.

Keywords: *Value-at-Risk (VaR); Volatility Models; Range-Based Estimators; Backtesting; Forecasting.*

Introduction

With the advent of more complexity and openness to the financial markets, the need to have efficient risk management tools and techniques has become imperative. One of the most commonly used measures for risk management to mitigate market risk by financial institutions, portfolio managers, traders, etc., is Value at Risk (VaR). Risk or uncertainty due to the dynamic and volatile nature of asset price movements leads to market risk. Since it can cause potentially substantial monetary losses to the parties involved, mitigating the same becomes essential (Jorion, 2007).

VaR estimates the maximum possible loss an asset or a portfolio can lose within a time horizon at a given confidence level. However, estimating VaR is challenging, as it is difficult to determine the asset returns' quantile or even the time-varying characteristics. Therefore, adopting suitable assumptions and specifications is required to help one accurately compute VaR estimates (Jorion, 2007; Engle & Manganelli, 2004).

One of the essential aspects of VaR estimation is volatility estimation and modeling. Volatility modeling still assumes a significant role in finance research. This becomes more intriguing and challenging because volatility is not constant over time. Further, the conditional nature of volatility, volatility clustering, and leverage effects make this exercise quite complex. Engle (1982) developed the Autoregressive Conditionally Heteroscedastic (ARCH) model for modeling conditional volatility. This was later extended by Bollerslev (1986) as the Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model. Later studies developed these volatility models to predict the time-varying conditional variance of a price series by incorporating the past unpredictable changes in the returns of that price series (Nelson, 1991; Bollerslev *et al.*, 1992; Engle & Patton, 2001; Shin, 2005; Alberg & Shalit, 2008; Shamiri & Isa, 2009; Kalu, 2010). Until Parkinson (1980); Garman & Klass (1980); Ball & Torous (1984); Alizadeh *et al.,* (2002); Brandt & Jones (2006), most volatility modeling studies used only daily closing prices of asset returns for volatility estimation.

Further studies have shown that volatility and co-movement estimators based on extreme values tend to be more efficient, which helps in more accurate volatility modeling (Shaik & Maheswaran, 2019; Shaik & Maheswaran, 2018; Shaik & Maheswaran, 2016; Padmakumari & Maheswaran, 2017). Some studies have also extended the applicability of such extreme values in volatility estimation to the distributional properties of asset returns, which is vital in precise volatility modeling (Shaik, & Maheswaran, 2020). These proved that one could come up with more efficient estimators of volatility based on high-low prices, also called *range-based volatility estimators.* This becomes more helpful as one may not always have access to high-frequency intraday data. Further, these studies also documented that these daily price range data encompassed all the information about the market as compared to closing price data alone. Additionally, these extreme prices are readily available, economical, and easier to handle.

Therefore, since accurate volatility prediction is of prime importance when estimating VaR estimates, studies have tried to use volatility models to estimate and model VaR. The most commonly used volatility model is GARCH (Huang *et al*., 2015; Nieto & Ruiz, 2016). This exercise becomes even more attractive because volatility varies over time with volatility clustering and leverage effects. Some of these above studies considered GARCH-based VaR measures to follow a normal distribution derived from a Brownian motion. However, empirical evidence suggests that this distributional assumption fails to account for price shocks' frequency and true impact, leading to undermining risk. Further, research also showed that using range-based price data to model VaR specifications proved more efficient. (A detailed literature review of the same is given in the next section).

Therefore, under this backdrop, the authors are interested in modeling and predicting Value at Risk (VaR) utilizing the GARCH family of volatility models. The study compares and contrasts by incorporating closing prices and range-based price data to understand if the latter performs similarly to the closing prices. Therefore, the study models VaR are based on traditional volatility models that use closing price data alone, namely GARCH & TARCH models. Next, range-based volatility models like range-based GARCH and Range-based TARCH are used by incorporating range as an exogenous variable into the traditional GARCH and TARCH models to identify if range provides additional predictive capacity. Finally, all the models are back-tested using various statistical measures like Kupiec (1995) unconditional coverage test and Christoffersen (1998) conditional coverage test to determine the different models' statistical accuracy in capturing risks. The study also wants to investigate if there is a difference in the forecasting ability of these models in an emerging economy vs. a developed economy. This is important as geographic diversification of assets is imperative to mitigate portfolio risk exposure. Hence, a comparison between the performance of the VaR models across developing and developed economies will provide valuable insights. Therefore, the sample consists of BRICS economies, namely, Brazil (IBOVESPA), Russia (MOEX), India (Nifty50), China (Shanghai), and South Africa indices, respectively as well as the indices of the three major developed economies, namely, the USA (S&P), the UK (UXX) and Germany (DAX) indices respectively for the period January 2003 - October 2021. Daily data comprising last, high, and low prices are used in the analysis.

The rest of the paper is organized as follows. Section 2 details the relevant literature on VaR estimation based on various volatility models. Section 3 presents the theory of volatility models and the econometric methodology adopted for fitting the different volatility models. Section 4 provides the details on the data used as well as discusses the empirical results. Section 5 presents the VaR back-testing procedure adopted based on the volatility models and the observed effects. Section 6 concludes with a summary of the main findings of the study.

Literature Review

In this section, a bird's eye view of the research in VaR under various volatility modeling specifications is provided

A plethora of research has tried to compare and contrast the VaR forecasting performance of GARCH class models. A study by Huang *et al*., (2015) on MSCI World Index data showed that ARMA (1,1)-GARCHM (1,1) outperforms in terms of violation measures. Interestingly, a study by Nieto & Ruiz (2016) showed that the out-of-sample observation and the time horizon influence the forecasting outputs. Hence, only the asymmetric model like EGARCH assuming skewed Student's t-distribution seemed to fare relatively well. The impact of distributional assumption on forecasting accuracy was also documented by other researchers (Tabasi *et al*., 2019; Altun, 2019). These studies showed that assuming a t-student distribution function instead of the Normal distribution function led to better forecasting accuracy and a lesser violation ratio. Okpara (2015), based on a study of the Nigerian market, reported that the EGARCH model with student t- innovation distribution led to better VaR forecasts.

Persistence or long memory also determines a model's VaR predictive ability. A study on the gold return volatility demonstrated that FIGARCH, which incorporated a long memory process, outperforms other models (Bentes, 2015; Huang *et al.*, 2016). Similar findings were documented by Emenogu *et al.*, (2018); Slim *et al.*, (2017). These studies also showed that accounting for asymmetry is equally essential, especially while studying emerging economies. These studies showed that while FIGARCH outperformed, traditional asymmetrical models like GJR GARCH worked best in emerging economies. Contradicting results were demonstrated by Degiannakis *et. al.,* (2013) based on a study of major stock indices. The author showed that accounting for asymmetry need not always lead to superior forecasts.

Another study considered the forecasting performance of these VaR models over various time horizons and market regimes. It showed that the performance does differ according to these dynamics (Yu *et.al*., 2010). Studies analyzing the forecasting accuracy of such VaR models showcased that changes in market conditions (Elenjical *et al*., 2016), classification of markets (Ng *et al.*, 2014), stock size, liquidity (Bali & Cakici, 2004), etc., may also affect the predictive accuracy of these models. Similar studies also showed that changes in lag structure and forecast horizon chosen could also influence the accuracy of the VaR forecasts (Hafer & Sheehan, 1989).

Maciel & Ballini (2017), while studying various range-based volatility models in the context of Brazilian and American markets, showed that, while range based model outperforms the traditional models, the CARR model (Chou, 2005), an extension of the ACD class of models that incorporates price range, outperforms all models in predictive ability. Kumar (2016) further validated this finding that CARR models outperform traditional and range-based models.

Against this backdrop, the authors are interested in comparing and contrasting various GARCH class models and their predictive accuracy regarding VaR forecasts. Further, even though extensive research has gone into understanding and using volatility models in VaR forecasting, very few studies have attempted to assess the relative efficiency of these range-based models in VaR forecasting. This is especially true in the context of Emerging VS Developed economies. Therefore, this paper aims to understand if using a range-based volatility model will lead to superior forecast accuracy than its traditional counterparts, as seen in the literature. The authors also want to explore if market conditions can affect the result; hence, we compare BRICS and developed economies.

Methodology:

Theory

This section first gives a brief overview of the various volatility models used in this study. The study uses both traditional and range-based volatility models for the analysis. The two most popular conventional volatility models are GARCH and TARCH, which consider daily log returns. The authors then study two range-based volatility models, RGARCH and RTARCH, in which range-based volatility is regarded as an exogenous variable.

Traditional Volatility Models:

In these models, the daily log asset returns are modeled as follows:

$$r_t = \ln\left(\frac{P_t}{P_{t-1}}\right) \qquad (1)$$

Where P_t is the price of the asset as on-time t and P_{t-1} is the price of the asset as on-time $t-1$; $t$he price process is $\epsilon \sim i.i.d\ (0,1)$, with zero mean and σ denoting the time-varying volatility.

GARCH (1, 1) Model

One of the first and most commonly used specifications for time-varying volatility is GARCH(1,1), developed by Bollerslev (1986). The Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model is an extension of the Autoregressive Conditionally Heteroscedastic (ARCH) model, developed by Engle (1982), for modeling conditional volatility. These models help to forecast the time-varying conditional variance of a price series by using past unpredictable changes in the returns of that price series.

The GARCH (1, 1) model can be denoted as:

$$r_t = \sigma_t\, \epsilon_t \qquad (2)$$

The variance equation is denoted as:

$$\sigma^2{}_t = \omega + \beta\sigma^2{}_{t-1} + \alpha r^2{}_{t-1} \qquad (3)$$

Where $\alpha > 0, \beta > 0, \omega > 0$ represent the coefficient for the short-term impact of ϵ_t on the conditional variance, the coefficient for long-term impact, and the constant term, respectively.

TARCH (1, 1) Model

Sometimes, one can observe that a downward movement in the market tends to be more highly volatile than an equally upward movement. In this scenario, a symmetric ARCH model is unsuitable as it does not consider the exact variance process. Engle & Ng (1993) considered including a news impact curve that will have asymmetric responses to good and bad news in the ARCH process to solve this issue. One such asymmetric model is the Threshold ARCH (TARCH) model by Zakoian (1990).

The TARCH (1, 1) model can be denoted as:

$$r_t = \sigma_t\, \epsilon_t \qquad (4)$$

The variance equation is denoted as:

$$\sigma^2{}_t = \omega + \beta\sigma^2{}_{t-1} + \alpha r^2{}_{t-1} + \gamma r^2{}_{t-1} I_{t-1} \qquad (5)$$

Where $\alpha > 0, \beta > 0, \omega > 0$ represent the coefficient for the short-term impact of ϵ_t on the conditional variance, the coefficient for long-term impact, and the constant term, respectively. Additionally, γ captures the asymmetric (leverage effect) and $I_{t-1} = 1$ in the case of negative news and $I_{t-1} = 0$ for any positive news.

Range-Based Volatility Models:

Next, the study considers range-based volatility models RGARCH and RTARCH, in which range-based volatility is regarded as an exogenous variable in the traditional GARCH and TARCH models. This aims to understand if including range will provide more information than daily closing asset returns.

In these models, the range is defined as:

$$R_t = \ln(H_t) - \ln(L_t) \qquad (6)$$

Where H_t is the highest price of the asset as on-time t and L_t is the lowest price of the asset as on-time t.

RGARCH (1,1,1)

The Range GARCH (1, 1,1) model can be denoted by modifying the GARCH (1,1) as below:

$$r_t = \sigma_t\, \epsilon_t \qquad (7)$$

The variance equation is denoted as:

$$\sigma^2{}_t = \omega + \beta\sigma^2{}_{t-1} + \alpha r^2{}_{t-1} + \theta R^2_{t-1} \qquad (8)$$

Where $\alpha > 0, \beta > 0, \omega > 0$ represent the coefficient for the short-term impact of ϵ_t on the conditional variance, the coefficient for long-term impact, and the constant term, respectively. θ capture the effect of range-based volatility on the volatility process.

TARCH (1,1,1)

The Range TARCH (1,1,1) model can be denoted by modifying the TARCH (1,1) as below:

$$r_t = \sigma_t\, \epsilon_t \qquad (9)$$

The variance equation is denoted as:

$$\sigma^2{}_t = \omega + \beta\sigma^2{}_{t-1} + \alpha r^2{}_{t-1} + \gamma r^2{}_{t-1} I_{t-1} + \theta R^2_{t-1} \qquad (10)$$

Where $\alpha > 0, \beta > 0, \omega > 0$ represent the coefficient for the short-term impact of ϵ_t on the conditional variance, the coefficient for long-term impact, and the constant term, respectively. γ captures the asymmetric (leverage effect) and $I_{t-1} = 1$ in the case of negative news and $I_{t-1} = 0$ for any positive news. θ capture the effect of range-based volatility on the volatility process.

Estimation Techniques

In this section, the authors provide details of the various econometric analysis done in this study.

In-Sample Estimation

Firstly, the traditional volatility models (GARCH (1,1) & TARCH (1,1)) as well as the range base volatility models (RGARCH (1,1,1) & RTARCH (1,1,1)) models for the in-sample period on the various indices as described above (Equations 2-10) are run. The study uses daily close-to-close log returns to run the traditional models, and daily log range returns to run the range-based volatility models. Then the study compares the model fitness and suitability based on the log-likelihood functions and AIC & BIC information criteria.

Out-of-Sample Analysis

One cannot observe the actual volatility in asset returns directly. Hence, one often resorts to various proxies for volatility to understand the forecasting capacity of the models. One could use high-frequency data to estimate volatility. Andersen *et al.*, (2001), Zhang *et al*., (2005), etc., used high-frequency return data to estimate the realized volatility. Since the high-frequency data help to capture the time-varying volatility more closely, the estimators based on them are generally considered more efficient. However, for many assets, high-frequency price data is not available.

Moreover, they are more affected by market microstructure errors. This may make the estimators based on the high-frequency data biased and inconsistent. Anderson & Bollerslev (1998) showed that using daily squared return as a proxy for volatility can be noisy. However, comparing the availability of data and the effect of market microstructure, the study uses daily squared close-to-close return as a proxy for realized volatility in this study.

Next, the study uses the estimates from the in-sample analysis and the proxy for realized volatility to understand the models' forecasting performance. The estimation errors in the proxies used for volatility can affect our judgment regarding the competency of the volatility models in forecasting. This is analyzed with the help of statistical loss functions proposed by Patton (2011), namely MSE & QLIKE loss functions. A loss function helps to measure how well our model can predict the expected output. The Mean Squared Error loss function (MSE), a commonly used loss function, penalizes the forecasting error symmetrically. Since one squares the errors, MSE can never be negative. It can be defined as:

$$MSE = E(\sigma_t^2 - \widehat{\sigma_t^2}) \quad (11)$$

Where, σ_t^2 is the forecasted variance and $\widehat{\sigma_t^2}$ is the actual realized variance (daily squared return).

On the other hand, the quasi-likelihood (QLIKE) loss function penalizes the forecasting error asymmetrically by giving more penalty to under-prediction than over-prediction. The QLIKE loss function is defined as:

$$QLIKE = E(\log(\widehat{\sigma_t^2}) + \sigma_t^2 \widehat{\sigma_t^{-2}}) \quad (12)$$

Where, σ_t^2 is the forecasted variance and $\widehat{\sigma_t^2}$ is the actual variance (daily squared return).

Since these are loss functions, the lower the values for MSE & QLIKE, the better the forecasting & predictive capacity of the models.

The study further employs the Diebold-Mariano (DM) test statistic proposed by Diebold & Mariano (1995) to assess the forecasting accuracy of the volatility models econometrically. The null hypothesis of this test is that the models being tested have equal predictive accuracy, thereby providing equally accurate forecasts.

The DM test is defined as:

$$DM = \frac{\bar{d}}{sqrt(Var(\bar{d}))} \quad (13)$$

Where, d_t is the loss differential of the forecasting models $, j$: $L_t^i - L_t^j$. The loss is defined as: $L_t = \sigma_t^2 - \widehat{\sigma_t^2}$. Therefore, $\bar{d} = T^{-1} \sum_{j=1}^{T} d_{t+j}$, where T is the total number of forecasts. The variance of this loss differential is estimated as per Newey & West (1987). Further, DM $\sim N(0,1)$.

Data & Empirical Analysis

Data

This study uses daily price data {opening, closing, highest and lowest} for BRICS economies, namely, Brazil (IBOVESPA), Russia (MOEX), India (Nifty50), China (Shanghai), and South Africa indices respectively as well as three major developed economies, namely, The USA (S&P), UK (UXX) and Germany (DAX) indices respectively for the period January 2003- October 2021. The data is divided into in-sample and out-sample periods, wherein the last 1000 observations are considered for out-of-sample forecasting and accuracy testing procedures. Out-of-sample forecasts are based on re-estimated volatility model parameters using a rolling window method. The in-sample data (up to $N_{1000\ obs}$) is made use of in the volatility modeling. As true volatility cannot be observed, the study uses a commonly used proxy for volatility, "Squared daily return," to enable comparison between the model's predictive performances.

Results & Discussion

This section provides a detailed account of the empirical analysis undertaken to estimate the various volatility models, followed by performance measures.

Data Description[1]

First, the authors present the summary statistics of the asset returns for all the indices for 2003-2021 in Table 1. Panel A depicts the results for the BRICS indices, while Panel B represents the summary statistics for the developed economies.

<INSERT TABLE 1 HERE>

From Table 1, one can see that for all the indices, daily close-to-close and range returns have zero mean and similar variance. The series distribution exhibits excess kurtosis, indicating that the distribution follows non-normality. While daily close-to-close returns have negative skewness, range-based returns exhibit positive skewness, indicating the existence of fat tails in the daily return series. A higher autocorrelation function value (ACF) and the Ljung box Q

[1] All the tables are presented in the Annexure of this paper at the end.

statistics suggest higher persistence in range than return series for both indices. This again validates the argument that range-based estimators are more suitable and efficient than return-based estimators.

In-Sample Analysis

Tables 2-9 present the empirical results of the volatility estimation for the various models, namely, GARCH (1,1), TARCH (1,1), RGARCH (1,1,1), and RTARCH (1,1,1) for the in-sample period for all the indices respectively. While the traditional GARCH (1,1) and TARCH (1,1) make use of daily closing price data alone, range-based models like RGARCH (1,1,1) and RTARCH (1,1,1) incorporate daily price range as an exogenous variable.

BRICS Economies

From Table 2 for the Nifty index, one can see that news (positive or negative) seems to affect only traditional models, as evidenced by the significant ω coefficient. Further, traditional models exhibit long memory in their volatility, as seen by the significant β coefficient. The effect of α is also substantial only for the conventional models, indicating short memory in the volatility process. γ, which captures the leverage effect in TARCH and RTARCH models, is significant, thereby conforming to asymmetry in reactions to shocks. Lastly, θ, which captures range as an exogenous variable in the traditional models, is substantial for both RGARCH and RTARCH models, thereby confirming the claim that range-based volatility models provide more information to volatility modeling. The LLH, AIC, and BIC criteria show that the models are parsimonious, with range-based models being more parsimonious.

<INSERT TABLE 2 HERE>

For an emerging market like China, from Table 3, one can see that only the traditional models seem to capture the effect of news (positive or negative) as evidenced by the significant ω coefficient. Further, GARCH(1,1), TARCH(1,1), and RTARCH(1,1) have a significant β coefficient, and hence these exhibit long memory in their volatility. The effect of α is essential across all the models, indicating short memory in the volatility process. γ, which captures the leverage effect in TARCH and RTARCH model, is insignificant, meaning there is no asymmetry in reactions to shocks. Lastly, θ, which captures range as an exogenous variable in the traditional models, is significant only for the RGARCH model, indicating that RGARCH alone seems to capture range-based information. All the models are more or less equally parsimonious, as seen from the LLH, AIC, and BIC criteria.

<INSERT TABLE 3 HERE>

Empirical findings for the Brazilian index, IBOVESPA, presented in Table 4, paint a similar picture as the Indian index, Nifty, with the empirical deductions being qualitatively identical. From this analysis, one can also observe that these two markets seem to react in the same asymmetric manner to news and shocks. The traditional models can also capture the long and short-memory effects of the asset return volatilities. Here, also, the range-based models seem more parsimonious. The significant θ coefficient that captures range as an exogenous variable in the traditional models is substantial for both RGARCH and RTARCH models, thereby confirming the claim that range-based volatility models provide more information to volatility modeling.

<INSERT TABLE 4 HERE>

From Table 5, even the Russian market, another emerging market, confirms the claim that range-based models capture more information than traditional models,s as seen from the significant θ coefficients. Further, the information criterion shows that all models are parsimonious. Only the conventional models can capture the reaction to the news as validated by the significant ω coefficient. More or less, all the models exhibit long or short memory in their volatility, as seen by the significant β coefficient and α coefficient, respectively. There is a leverage effect as captured by the significant γ coefficient.

<INSERT TABLE 5 HERE>

For the South African index, from Table 6, one can understand that, even though the deductions about all models being able to capture long or short memory along with asymmetric reaction to shocks are qualitatively similar to other emerging BRIC economies, interestingly, the ω coefficient is insignificant for all models. Once again, all models are more or less equally parsimonious, with the range-based models exhibiting higher simplicity.

<INSERT TABLE 6 HERE>

Developed Economies

<INSERT TABLE 7 HERE>

Table 7 presents the estimation results for the SPX index for the in-sample period. The results are qualitatively similar to the results obtained for the Nifty index in Table 2. Traditional models seem to be affected by news, exhibit short and long memory in the volatility process, and also exhibit the leverage effect. A significant θ value for RGARCH and RTARCH models confirms that range as an exogenous variable does add meaningful information content to the traditional volatility models. Once again, based on LLH, AIC, and BIC criteria, all models are parsimonious, with range-based models being simpler with few parameters.

<INSERT TABLE 8 HERE>

One can draw similar conclusions about the UK index (UXX), as seen in Table 8, as the SPX indices. Only the traditional models can account for the reaction and the asymmetry in response to news and shocks. Further, only the conventional models exhibit short and long memory in their return series. One can also confirm that range-based models offer additional information, as evidenced by the significant θ coefficient. Lastly, the relative simplicity of the range models is clear from the information criterion.

<INSERT TABLE 9 HERE>

As seen in Table 9, the German-based DAX index also paints a similar picture as the other indices. One can once again confirm that, while traditional models exhibit memory and asymmetric reaction to shocks, range-based models provide additional information and are more parsimonious.

Out of Sample Analysis

Loss Functions

To determine which volatility model performs better when it comes to forecasting performance, the study computes the MSE and QLIKE loss functions for each of the indices for the out-of-sample period. It uses squared daily closing returns as a proxy for realized volatility.

BRICS Economies

Table 10. presents the results for the BRICS economies. Since MSE and QLIKE are loss functions, the lower their values, the better the model's predictive capacity; one can see that for MSE, for all the models, range-based models perform better, as evidenced by their lower values. As per QLIKE function values, most models have almost equal predictive accuracy, with range-based models scoring better in some cases. This again validates the claim that range-based models perform better than traditional models due to the higher information content encompassed in the former.

<INSERT TABLE 10 HERE>

Developed Economies

One can draw a similar conclusion that even in the case of developed economies, range-based models perform better, as evidenced by their lower MSE values, as depicted in Table 11. Further, range-based GARCH performs better than traditional GARCH, and RTARCH is better than TARCH for both indices with their lower QLIKE function values.

<INSERT TABLE 11 HERE>

DM Test

BRICS Economies

Table 12 presents the DM test statistic results to determine which model statistically performs better in terms of volatility forecasting for the BRICS indices for the out-of-sample period. The null hypothesis is that the comparing models have similar predictive accuracy. At the 5% significance level, one can see that, more or less, both range-based models outperform traditional GARCH & TARCH models with superior predictive accuracy as evidenced by the significant p-values, except in the case of South Africa.

<INSERT TABLE 12 HERE>

From Table 13, one can confirm that, even for developed economies, range-based models have superior predictive accuracy compared to traditional models. Even though both GARCH and TRACH models exhibit similar predictive capacities, RGARCH outperforms TARCH and RTARCH models, respectively, while RTARCH outperforms the TARCH model.

<INSERT TABLE 13 HERE>

Application of GARCH Class of Models in Risk Assessments: Value at Risk (VaR)

Value-at-risk (VaR) is often used as a proxy for market risk by various parties like financial institutions, regulators, business practitioners, and portfolio managers. VaR is the "maximum possible loss that a portfolio could experience at a given level of confidence over a given time horizon" (Kumar, 2016). Using VaR to measure the minimum capital requirement in case of any uncertainties in the market was first adopted by J.P. Morgan (Longerstaey & Spencer, 1996), a financial institution. VaR is also often used in developing risk mitigation policies by traders, financial institutions, and investment banks.

Research has used different methods to estimate VaR, namely non-parametric and parametric. In the parametric approach, one assumes that the underlying distribution of the asset returns is gaussian. Hence, the probability distribution estimation considers the distribution parameters (population mean and population standard deviation). In the non-parametric approach, one makes no such assumptions about the return distribution; hence, empirical distributions are used to determine the population distribution parameters (Ballotta & Fusai, 2017).

One of the significant challenges with VaR is that it does not account for volatility clustering, due to which risk or losses gets undervalued. Therefore, using GARCH class of volatility models to estimate VaR circumvents this issue, as the former accounts for conditional volatility. Further, as mentioned before, range-based volatility measures have been considered more efficient than traditional volatility measures as they encompass more market information than just the closing prices. Therefore, in this study, the authors use the GARCH, TARCH, RGARCH, and RTARCH models for modeling and forecasting VaR to investigate if the range-based volatility models can better model and predict VaR than the traditional models. The study also wants to explore if the conclusion derived differs between an emerging and a developed economy.

Estimating Value at Risk (VaR):

VaR is the potential loss of a financial asset over a period h at a significance level α, and it is not expected to exceed probability $1-\alpha$. VaR can be denoted as:

$$prob(r_{t+h} \leq \quad VaR_{t+h}^{\alpha}) = 1-\alpha \quad (14)$$

Here, $h = 1$ denotes daily frequency.

VaR is the αth quantile of the conditional distribution of asset returns denoted as:

$$VaR_{t+h}^{\alpha} = CDF\ (\alpha) \quad (15)$$

The parametric VaR at time $t+1$ is denoted as:

$$VaR_{t+h}^{\alpha} = \widehat{\sigma_{t+1}} CDF_z^{-1}\ (\alpha) \quad (16)$$

Wherein $\widehat{\sigma_{t+1}}$ is the forecasted volatility at $t+1$, which is obtained by finding σ in fitted values from the model. Here, $h = 1$ denoting daily frequency as analyzing over a daily horizon would give more insights for practical purposes. $CDF_z^{-1}(\alpha)$ denotes the critical value obtained from the normal distribution table at a confidence level of α. Return-based volatility models (GARCH and TARCH) and range-based volatility models (RGARCH and RTARCH) were used to obtain $VaR_{t+h}^{\alpha} = \widehat{\sigma_{t+1}} CDF_z^{-1}(\alpha)$.

The study uses historical simulations by developing CDF for the asset returns over time for calculating non-parametric VaR. Unlike the parametric VaR models, one doesn't make any distributional assumptions in the non-parametric VaR model. This is done to create more generalizable conclusions without making any distributional assumptions.

VaR forecasting Model Performance Evaluation: Backtesting

As mentioned before, one major lacuna in the non-parametric VaR model is the assumption that asset returns are *i.i.d.* However, stylized facts of asset returns have shown that asset returns exhibit volatility clustering and hence are not *i.i.d.* Also, equal weightage is assigned to all asset returns across the return period. All these necessitate checking the accuracy and validity of the VaR forecasts computed above.

The performance and validity of any VaR modeling and forecasting approach are assessed by measuring the number of violations/exceptions. The violation at a given VaR level is calculated by comparing the trading losses with the estimated VaR. It also accounts for fat tails, an essential assumption in VaR modeling (Bollerslev *et al.*, 1992; Pagan, 1996; Palm, 1996). The study also accounts for possible asymmetry in asset return distributions by including asymmetric volatility models (Barndorff & Nielsen, 1997; Giot & Laurent, 2004).

In a back-testing approach, the study compares the estimated VaR with the actual return over the forecasted period. It is considered an exceedance when there is more negative return than what is estimated by VaR, also called "failure rates." The failure rate must match the VaR level specified in a correctly identified model. In this study, two coverage tests, namely, the unconditional coverage test and conditional coverage tests proposed by Kupiec (1995) and Christoffersen (1998), are used to test the same.

In the Unconditional Coverage test by Kupiec (1995), one tests if the realized coverage rate (α) for the back-tested sample consisting of T observations equals the theoretical coverage rate (α). If the loss exceeds the VaR expectation, $I_t\alpha = 1$ & 0 if vice-versa, wherein $\alpha \sim$binomial distribution. The likelihood ratio statistic follows a chi-square distribution with $df = 1$, denoted as:

$$LR_{UC}(\alpha) = -2\ln[(1-\alpha)^{T-N}\alpha^{N}] + 2\ln\left[\left(1-\left(\frac{N}{T}\right)^{T-N}\left(\frac{N}{T}\right)^{N}\right]\right. \quad (17)$$

Here, N denotes the number of violations.

In the Conditional Coverage test by Christoffersen (1998), one additionally checks for the independence of the sequence of violations, thereby combining both frequency and independence properties.

$$LR_{CC}(\alpha) = -2\ln\left[\left(1-\left(\frac{N}{T}\right)^{T-N}\left(\frac{N}{T}\right)^{N}\right] + 2\ln\left[(1-\widehat{\pi_{01}})^{n_{00}}\widehat{\pi_{01}}^{n}_{01}(1-\widehat{\pi_{11}})^{n_{01}}\widehat{\pi_{11}}^{n}_{11}\right] + 2\ln\left[\left(1-\left(\frac{N}{T}\right)^{T-N}\left(\frac{N}{T}\right)^{N}\right] + -2\ln[(1-\alpha)^{T-N}\alpha^{N}] \quad (18)$$

Here, $n_{ij}, i.j = 0{,}1$ is the number of times $I_t(\alpha) = j$ and $It{-}1(\alpha) = i$ with $\widehat{\pi_{01}} = \frac{n_{01}}{n_{00}+n_{01}}$ and $\widehat{\pi_{11}} = \frac{n_{11}}{n_{10}+n_{11}}$.

It is to be noted that it is plausible that one might pass the joint test while failing either the independence or unconditional coverage test; hence, the study reports both results.

Empirical Results & Discussion for VaR Modeling & Back-Testing

In this section, the authors present Tables 14, which provides the results for unconditional (Kupiec, 1995) and conditional coverage tests (Christoffersen, (1998) for the out-of-the-sample period for BRICS indices, respectively. For this, 1-step ahead back testing at a 95% confidence level is performed. This means that, with a probability of 95%, one cannot lose (maximum/threshold loss) more than VaR estimated over one day. Since financial institutions typically use a confidence level of 5%, VaR within the tail end of the distribution curve using a 95% confidence level is chosen for back-testing purposes. It is to be noted that the tail of the curve denotes losses. A rolling window estimation technique refits the estimates every 25th day. Usually, one goes for an entire period analysis assuming that the distribution parameters (mean & standard deviation) will remain constant over the entire sample period. But this is not true. Significantly the more extended the period, the more susceptible the parameters to change. Therefore, in this study, the volatility model parameters are re-estimated for forecasting for a fixed data window of 25 days. Each time, the last observation is removed to maintain uniformity in the data window size. The study also presents a comparative picture of actual exceedances with expected exceedances to identify which models have performed better.

<INSERT TABLE 14 HERE>

From Table 14, at 95% confidence levels, for the Nifty & IBOVESPA indices, all the models (traditional as well as range-based) models can correctly estimate the risk, as in none of the cases, one is unable to reject the null hypothesis of correct exceedances and or independence. Whereas, in the case of Shanghai and South Africa indices, only range-based models perform better than traditional models. Only Russia paints a different picture, with conventional models showcasing slightly better performance. Overall, one can conclude that for the BRICS economies, amongst the models, range-based models perform better, as evidenced by lower exceedance (%).

<INSERT TABLE 15 HERE>

From Table 15 for the developed economies, at a 95% confidence level, all the models except for range-based TARCH seem unrealistically conservative leading to the rejection of the null hypothesis of correct exceedances in the case of the SPX index. For the DAX index, range-based models perform better as only the RGARCH and RTARCH clear the coverage tests. In the case of the UXX index, while the range-based models pass only the unconditional tests, the traditional models show slight superiority in forecasting accuracy. This minor difference in the forecasting ability of VaR models can be attributed to higher noise witnessed in developed nations (He & Wang, 1995), making it easier to account for volatility in these economies. Further, while Arago & Neito (2005) attributed this difference to increased trading volume in an emerging economy, Girard & Biswas (2007) linked it to weaker market efficiency in emerging markets. However, overall, one can still conclude that, amongst the models, range-based TARCH performs better with lower exceedances (%).

Based on the empirical findings, one can conclude that range-based VaR models better provide more accurate VaR forecasts. This could be attributed to the incremental information that the range offers. Further, since geographic diversification of assets is imperative to mitigate portfolio risk exposure, a comparison between the performance of the VaR models across developing and developed economies is warranted. It has also been documented that foreign investors are attracted by developing economies due to the low correlation between the two markets (Sharma *et.al.,* 2016). Also, a financial crisis has a significant impact across markets. Volatility is considered a primary channel for these market linkages. Therefore, a comparative study becomes pertinent to understand if there is any change in the volatility dependence structure over time across markets. The findings from the data show that the range-based measures perform better in developing economies than in developed markets. This finding will help better risk management in a developing economy.

Conclusion & Implications of the Study

Value at risk is one of the most widely used market risk measures by financial institutions, regulators, and market participants. Precision in the estimation of volatility will help in making accurate VaR forecasts. Further, since volatility is time-varying and has a clustering effect, volatility models like GARCH and its various extensions prove helpful in modeling this volatility better. A plethora of research has identified that range-based measures are more efficient regarding volatility estimation than traditional models that use only closing prices.

In this study, the authors combine all these three factors and investigate which volatility model performs better in VaR forecasting. This is done by incorporating closing and range-based price data to explore if the latter performs similarly to the closing prices. For this, the authors compare traditional daily return-based models like GARCH and TARCH with range-based specifications of the same, namely, RGARCH and RTARCH. An empirical investigation of the BRICS and select developed markets shows that range-based models do a better job in VaR forecasting as it has more information content than daily closing prices. Therefore, through this study, it is shown that there exists a simple and efficient alternative to estimate VaR using the range-based GARCH family of volatility models.

This finding is of immense help to market players using VaR to predict their risk and loss. Since VaR is a commonly used measure of risk estimation, incorporating information in the form of high -low data enhances its usefulness to all stakeholders.

The findings will help financial institutions like banks, which can accurately forecast losses using accurate risk estimates. This will assist in allocating capital reserves to cover potential losses. The study's findings will also help practitioners make use of technology rightfully clubbed with theoretical knowledge in implementing VaR estimation using intraday data and developing virtual investment advisors. Further, as seen in the case of the latest Covid-19 pandemic, when markets experience high levels of volatility, it becomes imperative to have in place effective market risk mitigation strategies like range-based VaR.

Further, since range-based VaR models perform better in a developing economy than in a developed economy, risk managers can use range-based VaR in a developing economy and may complement it with other measures like Expected Shortfall in the context of developed economies to accurately measure market risk.

One can extend this research by seeing if the results change due to financial crisis/shock. One can also see if different distributional assumptions of the GARCH class models would yield different results. Further, as proposed by Nieto *&* Ruiz (2016), one can incorporate bias correction to learn if it will lead to a better predictive ability of the GARCH-based VaR models. For better forecasting accuracy, one can also account for bias correction in volatility estimation using range data as proposed by Shaik & Maheswaran (2020). One can also explore the possibility of higher accuracy in VaR estimates by using a more dynamic high-frequency data set to compute realized volatility from this intraday data set.

TABLES [2]

Table 1

Summary Statistics for the Period 2003-2021

Panel A: Descriptive statistics for BRICS indices for the period 2003-2021

Statistics	Nifty close-to-close returns	Nifty Range based returns	IBOVESPA close-to-close returns	IBOVESPA Range based returns	MOEX close-to-close returns	MOEX Range based returns	Shanghai close-to-close returns	Shanghai Range based returns	South Africa index close to close returns	South Africa index Range based returns
Mean	0.0006	0.0165	0.0005	0.0210	0.0005	0.0208	0.0002	0.0178	0.0004	0.0161
Std Deviation	0.0142	0.0131	0.0173	0.0138	0.0188	0.0173	0.0154	0.0120	0.0131	0.0096
Skewness	-0.4951	4.2973	-0.4273	4.4021	-0.2882	4.2947	-0.5180	2.2797	-0.2160	3.2377
Kurtosis	12.3888	35.6376	9.4860	34.8569	21.5953	32.394	4.7751	7.6897	5.1044	21.9910
Maximum	0.1633	0.2035	0.1384	0.2245	0.2523	0.2754	0.0903	0.1064	0.0906	0.1523
Minimum	-0.1390	0.0023	-0.1625	0.0035	-0.2066	0.0000	-0.0926	0.0025	-0.1045	0.0000
ACF (1)	1.0009	1.0001	1.0001	1.0001	1.0001	1.0001	1.0001	1.0001	1.0001	1.0001
ACF(15)	0.0050	0.3950	0.0000	0.3820	0.0230	0.4610	0.0360	0.3940	0.0280	0.4100
Q(15)	65.474 **	16135**	37.318**	17255**	46.969**	19170**	63.287**	14122**	44.421**	15733**

Panel B: Descriptive Statistics for Developed Indices for the Period 2003-2021

Statistics	SPX close-to-close returns	SPX Range based returns	DAX close-to-close returns	DAX Range based returns	UXX close-to-close returns	UXX Range based returns
Mean	0.0003	0.0122	0.0003	0.0152	0.0001	0.0133
Std Deviation	0.0120	0.0099	0.0106	0.0106	0.0112	0.0096
Skewness	-0.5529	3.5411	-0.1957	2.6241	-0.3652	3.3171
Kurtosis	14.1951	20.2225	7.8786	11.4089	10.4494	18.612
Maximum	0.1096	0.1090	0.1080	0.1114	0.0938	0.1151
Minimum	-0.1277	0.0015	-0.1305	0.0014	-0.1151	0.0023
ACF (1)	1.0001	1.0001	1.0001	1.0001	1.0001	1.0001
ACF(15)	-0.0710	0.4810	-0.0190	0.4670	-0.0260	0.4460
Q(15)	171.31**	24791**	15.2180	21969**	59.326**	21600**

Source: Prepared by authors
*Note: Q(15) is the Ljung Box Q statistics for autocorrelation ** denotes significance at 5%*

[2] *To be inserted within Text*

Table 2

Return and Range-Based Volatility Model Estimates for the Nifty50 Index for the In-Sample Period

Parameters	GARCH(1,1)	TARCH(1,1)	RGARCH(1,1,1)	RTARCH(1,1,1)
Ω	2.40E-06**	2.87E-04**	2.88E-12	7.44E-16
A	0.1006**	1.01E-01**	0.0005	0.0004
B	0.8906**	0.8997**	0.0002	0.0002
Y		0.4810**		0.0242
Ө			0.0075**	0.02419**
LLF	11016.47	11111	11553.31	11551.37
AIC	-5.9904	-6.0418	-6.2830	-6.2809
BIC	-5.9803	-6.0317	-6.2746	-6.2691

Source: Prepared by authors
*Note: ** and * represents significance at 5 % and 10 %, respectively*
L is the log-likelihood function value, and the AIC and BIC denote the Akaike and Schwarz information criteria, respectively.

Table 3

Return and Range-Based Volatility Model Estimates for the Shanghai Index for the In-Sample Period

Parameters	GARCH(1,1)	TARCH(1,1)	RGARCH(1,1,1)	RTARCH(1,1,1)
Ω	7.6944E-07	0.0001**	2.8518E-16	9.4133E-07
A	0.0547**	0.0664**	0.0278*	0.0569**
B	0.9439**	0.9467**	0.0167	0.9423**
Y		0.0372		0.0008
Ө			0.0088**	1.1561E-08
LLF	10161.02	10269.54	10673.46	10266.82
AIC	-5.6923	-5.7531	-5.9801	-5.7510
BIC	-5.6819	-5.7427	-5.9714	-5.7389

Source: Prepared by authors
*Note: ** and * represents significance at 5 % and 10 %, respectively*

Table 4

Return and Range-Based Volatility Model Estimates for the IBOVESPA Index for the In-Sample Period

Parameters	GARCH(1,1)	TARCH(1,1)	RGARCH(1,1,1)	RTARCH(1,1,1)
Ω	5.3863E-06**	0.0004**	1.09862E-12	3.18776E-16
A	0.0666**	0.6816**	0.0001	0.0001
B	0.9118**	0.9232**	0.0001	0.0001
Y		0.5296**		0.0058
Ө			0.0089**	0.0088**
Parameters	**GARCH(1,1)**	**TARCH(1,1)**	**RGARCH(1,1,1)**	**RTARCH(1,1,1)**
LLF	10159.5	10211.9	10735.84	10732.13
AIC	-5.5574	-5.5862	-5.8735	-5.8703
BIC	-5.5473	-5.5759	-5.8649	-5.8584

Source: Prepared by authors
*Note: ** and * represents significance at 5 % and 10 %, respectively*
L is the log-likelihood function value, and the AIC and BIC denote the Akaike and Schwarz information criteria, respectively.

Table 5

Return and Range-Based Volatility Model Estimates for the MOEX index for the In-Sample Period

Parameters	GARCH(1,1)	TARCH(1,1)	RGARCH(1,1,1)	RTARCH(1,1,1)
Ω	6.1234E-06**	0.0002**	2.9511E-07	2.9598E-07
A	0.1077**	0.1089**	1.6999E-05	2.8713E-05*
B	0.8766**	0.9043**	0.0003**	0.0003
Y		0.9043**		-1.1544E-05**
Ɵ			0.0095**	0.0095**
LLF	10141.93	10281.53	10759.74	10765.02
AIC	-5.4626	-5.5379	-5.7961	-5.7979
BIC	5.4526	-5.5278	-5.7878	-5.7863

Source: Prepared by authors
*Note: ** and * represents significance at 5 % and 10 %, respectively*

Table 6

Return and Range-Based Volatility Model Estimates for the South Africa Index for the In-Sample Period

Parameters	GARCH(1,1)	TARCH(1,1)	RGARCH(1,1,1)	RTARCH(1,1,1)
Ω	2.0055E-06	0.0002	1.3383E-15	1.7208E-06
A	0.0879**	0.0624**	0.0021	0.0020
B	0.8999**	0.9365**	0.0555**	0.9210
Y		0.9014**		0.1289**
Ɵ			0.0066**	1.1901E-08
LLF	11383.58	11448.19	11803.16	11438.34
AIC	-6.1417	-6.1766	-6.3688	-6.1708
BIC	-6.1317	-6.1666	-6.3604	-6.1590

Source: Prepared by authors
*Note: ** and * represents significance at 5 % and 10 %, respectively*
L is the log-likelihood function value, and the AIC and BIC denote the Akaike and Schwarz information criteria, respectively.

Table 7

Return and Range-Based Volatility Model Estimates for the SPX Index for the In-Sample Period

Parameters	GARCH(1,1)	TARCH(1,1)	RGARCH(1,1,1)	RTARCH(1,1,1)
Ω	1.81E-06**	0.0002**	1.67E-16	2.24E-16
A	0.0968**	0.0878**	0.0010	3.61E-07
B	0.8843**	0.9144**	0.0007	0.0013
Y		0.9999**		0.0276
Ɵ			0.0059**	0.0058**
LLF	12372	12518.44	12902.69	12909.8
AIC	-6.6199	-6.6983	-6.9045	-6.9073
BIC	-6.6099	-6.6883	-6.8962	-6.8956

Source: Prepared by authors
*Note: ** and * represents significance at 5 % and 10 %, respectively*

L is the log-likelihood function value, and the AIC and BIC denote the Akaike and Schwarz information criteria, respectively.

Table 8

Return and Range-Based Volatility Model Estimates for the UXX Index for the In-Sample Period

Parameters	GARCH(1,1)	TARCH(1,1)	RGARCH(1,1,1)	RTARCH(1,1,1)
Ω	1.56E-05	0.0002**	4.18E-14	1.32E-15
A	0.1037**	0.0774**	0.0089	1.31E-05
B	0.8825**	0.9221**	0.0017	0.0004
Y		0.9999**		0.0546
Ө			0.0053**	0.0051**
Parameters	**GARCH(1,1)**	**TARCH(1,1)**	**RGARCH(1,1,1)**	**RTARCH(1,1,1)**
LLF	12883.96	13006.96	13436.77	13443.26
AIC	-6.5905	-6.6535	-6.8739	-6.8762
BIC	-6.5809	-6.6438	-6.8659	-6.8650

Source: Prepared by authors
*Note: ** and * represents significance at 5 % and 10 %, respectively*
L is the log-likelihood function value, and the AIC and BIC denote the Akaike and Schwarz information criteria, respectively.

Table 9

Return and Range-Based Volatility Model Estimates for the DAX Index for the In-Sample Period

Parameters	GARCH(1,1)	TARCH(1,1)	RGARCH(1,1,1)	RTARCH(1,1,1)
Ω	2.25E-06**	0.0002**	3.44E-14	2.47E-16
A	0.0855**	0.0747**	0.0007	4.05E-06
B	0.9013**	0.9247**	0.0003	0.0021
Y		0.9999**		0.0330
Ө			0.0078**	0.0079**
LLF	11901.71	12042	12382.17	12390.53
AIC	-6.0878	-6.1596	-6.3343	-6.3375
BIC	-6.0782	-6.1500	-6.3263	-6.326

Source: Prepared by authors
*Note: ** and * represents significance at 5 % and 10 %, respectively*
L is the log-likelihood function value, and the AIC and BIC denote the Akaike and Schwarz information criteria, respectively.

Table 10

Performance of Volatility Forecasting Models for the BRICS Indices Based on the MSE and QLIKE Criteria for the Out-Of-Sample Period

Models	Nifty50	
	MSE	**QLIKE**
GARCH	0.000209	-8.45866
TARCH	0.000203	-9.21218
RGARCH	0.000125	-8.45108
RTARCH	0.000126	-8.4511
Models	**Shanghai**	
	MSE	**QLIKE**
GARCH	0.000266	-8.22335
TARCH	0.000273	-8.97847
RGARCH	0.000169	-8.22189
RTARCH	0.000274	-8.22182
Models	**IBOVESPA**	
	MSE	**QLIKE**
GARCH	0.000267	-8.14897
TARCH	0.000263	-8.81951
RGARCH	0.000176	-8.14848
RTARCH	0.000177	-8.14849

Models	**MOEX**	
	MSE	**QLIKE**
GARCH	0.000384	-7.79508
TARCH	0.000374	-8.68431
RGARCH	0.000199	-7.79432
RTARCH	0.000199	-7.79431
Models	**South Africa**	
	MSE	**QLIKE**
GARCH	0.000163	-8.68673
TARCH	0.000162	-9.32788
RGARCH	0.000109	-8.68688
RTARCH	0.000163	-8.68712

Source: Prepared by authors

Table 11

Performance of Volatility Forecasting Models for the Developed Indices Based on the MSE and QLIKE Criteria for the Out-Of-Sample Period

Models	**SPX**	
	MSE	**QLIKE**
GARCH	0.000122	-8.92234
TARCH	0.00013	-9.79782
RGARCH	0.000069	-8.91187
RTARCH	0.000070	-8.91190
Models	**UXX**	
	MSE	**QLIKE**
GARCH	0.000119	-8.99147
TARCH	0.000118	-9.79222
RGARCH	0.000069	-8.98877
RTARCH	0.000070	-8.98895
Models	**DAX**	
	MSE	**QLIKE**
GARCH	0.000178	-8.45418
TARCH	0.000185	-9.06146
RGARCH	0.000118	-8.45434
RTARCH	0.000123	-8.45455

Source: Prepared by authors

Table 12

Diebold-Mariano Test Statistics for Volatility Forecasting of BRICS Indices for the Out-Of-Sample Period

		NIFTY50	
Models	TARCH	RGARCH	RTARCH
GARCH	-1.3826	-1.7263*	-1.7219*
TARCH		-2.539**	-2.5315**
RGARCH			
RTARCH		-3.2583**	
		Shanghai	
Models	TARCH	RGARCH	RTARCH
GARCH	-1.0656	-0.9220	-1.0271
TARCH		1.5436	1.6583*
RGARCH			
RTARCH		1.504	

		MOEX	
Models	TARCH	RGARCH	RTARCH
GARCH	-0.0092	-0.5546	-0.56239
TARCH		-2.3611**	-2.3662**
RGARCH			
RTARCH		2.7715**	
		IBOVESPA	
Models	TARCH	RGARCH	RTARCH
GARCH	-0.0091	-0.9898	-0.97278
Models	TARCH	RGARCH	RTARCH
TARCH		-1.895**	-1.888**
RGARCH			
RTARCH		-2.2984**	
		South Africa	
Models	TARCH	RGARCH	RTARCH
GARCH	0.8534	0.2298	0.8626
TARCH		-1.0321	-0.5085
RGARCH			
RTARCH		-1.2155	

Source: Prepared by authors
*Note: ** and * represents significance at 5 % and 10 %, respectively*

Table 13

Diebold-Mariano Test Statistics for Volatility Forecasting of Developing Indices for the Out-Of-Sample Period

		UXX Index	
Models	TARCH	RGARCH	RTARCH
GARCH	0.2097	0.0248	0.0624
TARCH		-1.6436	-1.5367
RGARCH			
RTARCH		-2.3365**	
		DAX Index	
Models	TARCH	RGARCH	RTARCH
GARCH	1.0367	0.4783	0.55461
TARCH		-2.6975**	-2.6127**
RGARCH			
RTARCH		-3.5904**	
		SPX	
Models	TARCH	RGARCH	RTARCH
GARCH	-1.0206	-1.2594	-1.2456
TARCH		-1.9948**	-1.9757**
RGARCH			
RTARCH		-2.5836**	

Source: Prepared by authors
*Note: ** and * represents significance at 5 % and 10 %, respectively*

Table 14

VaR Back-Testing for BRICS Economies

. One-step-ahead VaR back-testing at a 95% confidence level for the NIFTY50 index for the out-of-sample period

Models	LR(UC)	LR(CC)	Expected exceedances @5%	Actual Exceedances	Actual exceedances (%)
GARCH	0	2.20	50	50	5%
TARCH	0.021	0.09	50	51	5.10%

Models	LR(UC)	LR(CC)	Expected exceedances @5%	Actual Exceedances	Actual exceedances (%)
RGARCH	2.253	3.33	50	40	4%
RTARCH	1.812	1.87	50	41	4%

.One-step-ahead VaR back-testing at a 95% confidence level for the Shanghai index for the out-of-sample period

Models	LR(UC)	LR(CC)	Expected exceedances @5%	Actual Exceedances	Actual exceedances (%)
GARCH	1.421	2.211	50	42	4.2%
TARCH	0.085	0.297	50	48	4.8%
RGARCH	5.268**	5.714	50	32	3.5%
RTARCH	1.421	1.454	50	42	4.2%

.One-step-ahead VaR back-testing at a 95% confidence level for the MOEX index for the out-of-sample period

Models	LR(UC)	LR(CC)	Expected exceedances @5%	Actual Exceedances	Actual exceedances (%)
GARCH	4.553**	4.908	50	36	3.6%
TARCH	0.544	0.544	50	45	4.5%
RGARCH	17.475**	19.782**	50	24	2.4%
RTARCH	17.475**	19.782**	50	24	

.One-step-ahead VaR back-testing at a 95% confidence level for the IBOVESPA index for the out-of-sample period

Models	LR(UC)	LR(CC)	Expected exceedances @5%	Actual Exceedances	Actual exceedances (%)
GARCH	0.083	0.117	50	52	5.2%
TARCH	0.021	0.747	50	51	5.1%
RGARCH	0.544	0.996	50	45	4.5%
RTARCH	0.788	1.342	50	44	4.4%

.One-step-ahead VaR back-testing at a 95% confidence level for the South Africa index for the out-of-sample period

Models	LR(UC)	LR(CC)	Expected exceedances @5%	Actual Exceedances	Actual exceedances (%)
GARCH	9.022**	9.694**	50	72	7.2%
TARCH	4.918**	5.561	50	66	6.6%
RGARCH	0.544	1.235	50	45	4.5%
RTARCH	1.783	1.811	50	43	4.3%

Source: Prepared by the authors

*Note: * indicates a 5% significance level, and LRuc and LRcc are the statistics of unconditional and conditional coverage tests, respectively. The critical values for the unconditional and conditional coverage tests are 3.841 and 5.991, respectively. The null hypothesis of the UC test is Correct exceedances, and the Null hypothesis of the CC test is correct exceedances and independence of failures.*

Table 15

VaR Back-Testing for Developed Economies

One-step-ahead VaR back-testing at a 95% confidence level for the SPX index for the out-of-sample period

Models	LR(UC)	LR(CC)	Expected exceedances @5%	Actual Exceedances	Actual exceedances (%)
GARCH	2.836	6.86**	50	62	6.2%
TARCH	4.918**	7.803**	50	66	6.6%
RGARCH	4.345**	9.193**	50	65	6.5%
RTARCH	1.616	6.661**	50	59	5.9%

.One-step-ahead VaR back-testing at a 95% confidence level for the DAX index for the out-of-sample period

Models	LR(UC)	LR(CC)	Expected exceedances @5%	Actual Exceedances	Actual exceedances (%)
GARCH	9.813**	10.372**	50	73	7.3%
TARCH	5.524**	5.588	50	67	6.7%
RGARCH	3.299	3.581	50	63	6.3%
RTARCH	0.228	1.002	50	47	4.7%

.One-step-ahead VaR back-testing at a 95% confidence level for the UXX index for the out-of-sample period

Models	LR(UC)	LR(CC)	Expected exceedances @5%	Actual Exceedances	Actual exceedances (%)
GARCH	1.616	4.82	50	59	5.9%
TARCH	2.826	5.259	50	62	6.2%
RGARCH	0	6.308**	50	50	5%
RTARCH	3.776	7.144**	50	41	4.1%

Source: Prepared by the authors

*Note: * indicates a 5 % significance level, and LRuc and LRcc are the statistics of unconditional and conditional coverage tests, respectively. The critical values for the unconditional and conditional coverage tests are 3.841 and 5.991, respectively. The null hypothesis of the UC test is Correct exceedances, and the Null hypothesis of the CC test is correct exceedances and independence of failures.*

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