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Hedging Against a Price Drop Using the Inverse Vertical Ratio Put Spread Strategy Formed by Barrier Options

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This paper investigates hedging of a portfolio consisting of a risky underlying asset using the Inverse Vertical Ratio Put Spread (further IVRPS) strategy formed by single barrier options (category exotic options; subcategory path-dependent options) to study the difference between hedging using barrier and vanilla options. This strategy is useful for hedging against a price drop assuming the given underlying asset will be sold in the future. Barrier options were created to provide risk managers with cheaper means to hedge their exposures without paying for the price changes they believed unlikely to occur. They are options with second strike price, called barrier or trigger. Crossing of the barrier (in the form of frontier underlying asset price) during the life of an option implies activation or deactivation of particular barrier option. In general, they are more flexible and cheaper in comparison to plain vanilla options. In our analysis we use an interesting approach based on finding of the income functions from secured position in analytical form, which can simplify hedging process. These theoretical results are robust to different underlying assets and are useful for financial and also non-financial institutions. Many authors use this approach of analyzing for example Amaitiek, Balint and Resovsky (2010), Soltes M. (2010), Soltes V. (2001), Soltes V. and Amaitiek (2010a), Soltes V. and Amaitiek (2010b). A key difference between the previous studies is that in this paper we are concentrated on barrier options. Furthermore, we focus on the application to SPDR Gold Shares (GLD). SPDR Gold Shares offer investors an innovative, relatively cost efficient and secure way to access the gold market without being necessary to take care of delivery and safekeeping. GLD are an appropriate tool for those who want " to play" in the gold market, not for those who want to buy real gold. It is possible to use them for hedging, forming of option strategies etc. For these reasons they are very popular and SPDR Gold Trust is currently one of the largest holder of the gold in the world. We use vanilla and barrier option on these shares in our analysis. Due to the lack of real-traded barrier option data we calculate the barrier option premiums using an analytical model of Haug, who applied the Black-Scholes-Merton formula for all kinds of barrier options. We realize all calculations in the statistical program R because of simplification. The comparative comparison of proposed variants has shown the best results. We also find the best variants for the investor who speculates, and at the same time hedges against a slight or significant price drop. Our study confirms that this strategy formation using barrier options gives end-users greater flexibility to express a precise view. The results show that IVRPS strategy formation using barrier options is better than this option strategy formation using vanilla option in specific future price situations, but not in every practical situation.

Keywords: *hedging, Inverse Vertical Ratio Put Spread strategy, barrier options, vanilla option, SPDR Gold Shares.*

Introduction

Today, in the context of the globalization and liberalization process, prices of shares, commodities, interest rates and currencies in financial markets are very volatile. In consequence especially financial institutions and institutional investors have to deal with a big market risk related with their activities. Non-financial institutions are exposed to different sizes of market risk, depending on number of securities they own and to what extent commodity prices, exchange rates and interest rates affect their cash flows (Bernrud *et al.,* 2005).

The methods and instruments used to manage market risk are constantly developing. Appropriate way of efficient risk management is hedging. The analysis of available hedging strategies, their interconnection with efficient market and value theories, as well as various empirical studies are regular theme of scientific papers. For example studies of (Adam & Fernando, 2006; Brown *et al.,* 2006) analyze the corporate risk management policies of gold mining firms. (Campello *et al.,* 2011; Zou, 2010) investigate the implications of hedging for corporate financing and investment. (Korn, 2010; Loss, 2012) study a firm's optimal hedging strategies. (Hankins, 2011) investigates how firms manage risk by examining the relationship between financial and operational hedging using a sample of bank holding companies. Our theoretical analysis is useful for financial and also non-financial institutions.

According to (Zmeskal, 2004), the main idea of hedging is to add new asset or assets (usually derivatives) to risky asset in order to create new portfolio, so-called hedging portfolio, hedged against an unfavourable price movement. (Guay & Kothari, 2003) provide the empirical research documenting the importance of hedging using derivatives (Judge, 2007) analyzes why it is important to hedge.

Depending on whether we intend to buy or sell an underlying asset in the future, we have to be concerned about the anticipated increase or decrease of this underlying asset price. If we intend to buy it, we should hedge against a price increase and conversely, if we intend to sell it, we should hedge against a price decrease. In this paper we will hedge against a price drop at any price development of underlying asset to time of maturity and at time of maturity. It should be noted that our objective is not to completely avoid of a price drop but to ensure a minimum acceptable income from the sale of an underlying asset in the future. We present the most sophisticated method to manage the market risk - hedging using options strategies. Option strategies are described in popular derivative books including (Carol, 2008; Hul, 2008; Chorafas, 2008; Kolb, 1999). Generally, an option strategy involves the simultaneous combination of one or more option positions (Long Call, Short Call, Long Put, and Short Put). The paper (Lazar & Lazar, 2011) presents some of the most used option strategies on the market. (Annaert, *et al.,* 2006) elaborate a formula for determining the optimal strike price for a bond put option, used to hedge a position in a bond. (Tichy, 2009) focuses on currency hedging of non-financial institution.

Bull, bear, butterfly, condor, spreads along with straddles, strangles, combos, ladders and simple covered or protective call and put are some of the option strategies. In this paper we analyse hedging against a price drop using the IVRPS strategy formed by barrier option. To the best of our knowledge, no study has yet utilized barrier options to investigate option strategies and hedging using option strategies as well.

There are four types of barrier options. Up and knock-in (UI) call/put option is activated if an underlying price during the life of an option increases above upper barrier U or only touches it. Down and knock-in (DI) call/put option is activated if an underlying price during the life of an option decreases below lower barrier D or only touches it. Up and knock-out (UO) call/put option is deactivated if an underlying price during the life of an option increases above upper barrier. Down and knock-out (DO) call/put option is deactivated if an underlying price during the life of an option decreases below lower barrier. For example (Briys, 1998; Taleb, 1997; Tichy, 2004; Weert, 2008; Zhang, 1998) explain barrier options more detail.

We find an analytical expression of income functions from secured position. Our theoretical results are robust to different underlying assets. In this study the practical application in hedging of SPDR Gold Shares is designed. Variants for hedging of these shares are suggested and compared with unsecured position and also proposed secured positions using vanilla options.

The rest of the paper is organized as follow. First section gives the methodology and the data used in this paper. The following part deals with hedging analysis of the IVRPS strategy using barrier options. The next section contains the practical application to the SPDR Gold Shares. The last section concludes the paper.

Methodology and data

We begin, in this section, by describing the construction of the IVRPS strategy. We then go on to introduce backgrounds and specify methods used in our analysis. Finally we present data used in our analysis.

The IVRPS strategy is formed by buying a higher number n_1 of put options with a lower strike price X_1, premium p_{1B}^0 per option and at the same time by selling a smaller number n_2 of put options with a higher strike price X_2, premium p_{2S}^0 per option. It is a European type of options for the same underlying asset with the same expiration time. If we choose n_1=2 and n_2=1, we will get the well-known Long Two By One Ratio Put strategy (some authors call it Put Backspread) mentioned in paper of Jilek, 2002).

In the paper of (Soltes & Amaitiek, 2010a) authors propose usage of the IVRPS strategy using vanilla options in hedging against a price drop of an underlying asset to a future date in a way which enables hedging with zero cost, when the condition $n_2 p_{2S}^0 \geq n_1 p_{1B}^0$ is fulfilled. They use interesting method based on finding income functions in analytical form. This allows investors to express secured positions in hedging against an unfavourable price movement of an underlying asset, which simplifies the application in particular hedging.

Following the mentioned study we analyse individual ways of IVRPS strategy formation using barrier options. Furthermore, there are selected ways suitable for hedging against a price drop of an underlying asset. These ways are used in the formation of hedging possibilities. Based on analysis, conclusions are formulated. They can facilitate a selection of appropriate hedging strategy and simplify the comparison of hedging possibilities.

We use the obtained theoretical results in the application to SPDR Gold Shares. We propose hedging variants and evaluate their profitability with respect to the income of unsecured portfolio for particular intervals of underlying spot price at the time of expiration. In the end, we realize a comparative analysis of the proposed variants.

We look at vanilla and barrier European options on the SPDR Gold Shares with various strike prices and barriers. In the case of vanilla options we use real data (source: www.finance.yahoo.com and www.morningstar.com).

Due to the lack of real-traded barrier option data the barrier option premiums are calculated. We use the most popular method for option pricing - the Black-Scholes model (Black & Scholes, 1973). The classic version of this model is not designed for barrier options. By its modification (Merton, 1973) derived the first analytical formula for a down and out call European type option. (Later Rubinstein & Reiner, 1991) provided the formulas for eight types of barrier options. (Haug, 1998) gave the formulas for all types of European single barrier options. Barrier options can also be priced via lattice tree (the binomial model was first proposed by (Cox *et al.,* 1979), Monte Carlo simulation for example (Ross & Ghamami, 2010) and others.

We will consider analytical closed formulas under the Black-Scholes-Merton framework provided by Haug. To simplify the calculations of particular barrier option premiums we use the statistical programme R.

The mentioned model for shares without paying dividend is based on the following parameters:

- type of option (DI/DO/UI/UO CALL/PUT);
- actual underlying spot price;
- strike price (selected according to strike prices of vanilla options);
- expiration time (according to European standard 30E/360);
- barrier level;
- risk-free interest rate (U.S. Treasury rate (source: www.bloomberg.com) = cost of carry rate;
- Black-Scholes implied volatility.

The dataset consists of 15 vanilla put option premiums, 154 DI and DO put barrier option premiums, 210 UI and UO put barrier option premiums. Strike prices are in the range of 130 - 200, barrier levels of DI/DO options are in the range of 130 - 160 and barrier levels of UI/UO options are in the range of 170 - 200 (all in the multiples of 5). All data used in our analysis can be provided upon request.

Hedging analysis

Let us suppose that at time T in the future we will sell a portfolio consisting of n pieces of the underlying asset, but we are afraid of its price drop. Income function from unsecured position in the portfolio at time T is:

$$I\left(S_T\right) = n S_T, \tag{1}$$

where S_T is spot price of underlying asset at time T.

Let us assume that we have decided to hedge the minimum acceptable selling price using the IVRPS strategy. Based on the analysis of all possibilities of this strategy formation using barrier options we can conclude that only four possibilities ensure a minimum selling price in the case of any price movement. Other possibilities do not secure the price in every future price scenarios.

I. Let us hedge using IVRPSS strategy formed by buying a higher number n_1 of down and knock-in put options with a lower strike price X_1, premium p^0_{1BDI} per option, barrier level D and at the same time by selling a smaller number n_2 of up and knock-in put options with a higher strike price X_2, premium p^0_{2SUI} per option, barrier level U. We will select the number of options in a way which enables $n = n_1 - n_2, (n > 0)$ [1]. Down and knock-in put option will become vanilla put option, if the option is activated, i.e. the price of the underlying asset exceeds predetermined lower barrier D from above during the life of the option (the option is activated if it only touch the barrier), which the condition (2) represents.

$$\min_{0\le t\le T}\left(S_t\right) \le D. \tag{2}$$

Following Ye (2009) we assume that $D < X_1$, because otherwise DI (DO) put option is equivalent to a correspondent vanilla put. For down and knock-in/out options, we have the barrier below S_0, where S_0 is an actual spot price of an underlying asset at time of contract conclusion, i.e. 0. For up and knock-in/out, we have the barrier above S_0.

Let us denote underlying spot price at expiration time S_T. We get the profit function from buying n_1 down and knock-in put option as a payoff at time T (income) eliminated by option premium at time T (initial cost adjusted by the time value of the money[2]). The profit function is:

$$P_I\left(S_T\right) = \begin{cases} -n_1 p_{1BDI} & \text{if } \min\limits_{0\le t\le T}\left(S_t\right) > D \wedge S_T < X_1, \\ -n_1\left(S_T - X_1 + p_{1BDI}\right) & \text{if } \min\limits_{0\le t\le T}\left(S_t\right) \le D \wedge S_T < X_1, \\ -n_1 p_{1BDI} & \text{if } S_T \ge X_1. \end{cases} \tag{3}$$

Up and knock-in put option will become vanilla put option if the option is activated, which is expressed by the condition.

$$\max_{0\le t\le T}\left(S_t\right) \ge U. \tag{4}$$

Following Ye (2009) we assume for UI (UO) put options $U < X_2 \vee U = X_2 \vee U > X_2$. The profit function from selling n_2 UI put options has the form:

$$P_{II}\left(S_T\right) = \begin{cases} n_2 p_{2SUI} & \text{if } \max\limits_{0\le t\le T}\left(S_t\right) < U \wedge S_T < X_2, \\ n_2\left(S_T - X_2 + p_{2SUI}\right) & \text{if } \max\limits_{0\le t\le T}\left(S_t\right) \ge U \wedge S_T < X_2, \\ n_2 p_{2SUI} & \text{if } S_T \ge X_2. \end{cases} \tag{5}$$

If it is valid $D < X_1 \le U \le X_2$ or $D < X_1 \le X_2 \le U$ in the context of the previous assumptions, we get the income function from secured position using the IVRPS strategy (6) as a sum of the functions (1), (3) and (5) from the individual operations.

$$SI_I\left(S_T\right) = \begin{cases} nS_T + n_2 p_{2SUI} - n_1 p_{1BDI} & \text{if } \min\limits_{0\le t\le T}\left(S_t\right) > D \wedge \max\limits_{0\le t\le T}\left(S_t\right) < U \wedge S_T < X_1, \\ n_1 S_T - n_2 X_2 + n_2 p_{2SUI} - n_1 p_{1BDI} & \text{if } \min\limits_{0\le t\le T}\left(S_t\right) > D \wedge \max\limits_{0\le t\le T}\left(S_t\right) \ge U \wedge S_T < X_1, \\ -n_2 S_T + n_1 X_1 + n_2 p_{2SUI} - n_1 p_{1BDI} & \text{if } \min\limits_{0\le t\le T}\left(S_t\right) \le D \wedge \max\limits_{0\le t\le T}\left(S_t\right) < U \wedge S_T < X_1, \\ n_1 X_1 - n_2 X_2 + n_2 p_{2SUI} - n_1 p_{1BDI} & \text{if } \min\limits_{0\le t\le T}\left(S_t\right) \le D \wedge \max\limits_{0\le t\le T}\left(S_t\right) \ge U \wedge S_T < X_1, \\ nS_T + n_2 p_{2SUI} - n_1 p_{1BDI} & \text{if } \max\limits_{0\le t\le T}\left(S_t\right) < U \wedge X_1 \le S_T < X_2, \\ n_1 S_T - n_2 X_2 + n_2 p_{2SUI} - n_1 p_{1BDI} & \text{if } \max\limits_{0\le t\le T}\left(S_t\right) \ge U \wedge X_1 \le S_T < X_2, \\ nS_T + n_2 p_{2SUI} - n_1 p_{1BDI} & \text{if } S_T \ge X_2. \end{cases} \tag{6}$$

By comparing the functions (1) and (6) in meeting the condition $n_2 p^0_{2S} \ge n_1 p^0_{1B}$ we have formulated the following conclusions:

[1] Every followed function in this work is built on this assumption.

[2] The premium is calculating using the formula for simple or compound interest, i.e. $p = p^0\left(1 + rt\right)$ or $p = p^0\left(1 + r\right)^t$, where r is nominal interest rate. In other functions of this work we will be using the option premium adjusted by the time value of the money according to given formulas.

- If the price moves between barriers during the option life and at time T it is $S_T < X_1$, then by hedging we have ensured a constant profit of $n_2 p_{2SUI} - n_1 p_{1BDI}$.
- If the price does not fall below D and it grows above U during the option life and at time T it is $S_T < X_1$, then we have hedged the minimum income of $n_1 D - n_2 X_2 + n_2 p_{2SUI} - n_1 p_{1BDI}$.
- If the price falls below D and it does not grow above U during the option life and at time T it is $S_T < X_1$, then our profit will not be lower than it would be without hedging.
- If the price falls below D and it grows above U during the option life and at time T it is $S_T < X_1$, then we have hedged a constant income of $n_1 X_1 - n_2 X_2 + n_2 p_{2SUI} - n_1 p_{1BDI}$.
- If the asset price does not grow above U and at time T it is $X_1 \le S_T < X_2$, then we have ensured a constant profit of $n_2 p_{2SUI} - n_1 p_{1BDI}$ .If the asset price grows above U and at time T it is $X_1 \le S_T < X_2$, then we will get a loss or we will make a profit, but the maximum of profit can be $n_2 p_{2SUI} - n_1 p_{1BDI}$.
- If at time T the asset price is $S_T \ge X_2$, then our profit will not be lower than it would be without hedging.

If $D < U \le X_1 < X_2$, the income function from secured position is:

$$SI_I(S_T)=\begin{cases} nS_T+n_2p_{2SUI}-n_1p_{1BDI} & \text{if} \min\limits_{0\le t\le T}(S_t)>D\wedge\max\limits_{0\le t\le T}(S_t)<U\wedge S_T<X_1, \\ n_1S_T-n_2X_2+n_2p_{2SUI}-n_1p_{1BDI} & \text{if} \min\limits_{0\le t\le T}(S_t)>D\wedge\max\limits_{0\le t\le T}(S_t)\ge U\wedge S_T<X_1, \\ -n_2S_T+n_1X_1+n_2p_{2SUI}-n_1p_{1BDI} & \text{if} \min\limits_{0\le t\le T}(S_t)\le D\wedge\max\limits_{0\le t\le T}(S_t)<U\wedge S_T<X_1, \\ n_1X_1-n_2X_2+n_2p_{2SUI}-n_1p_{1BDI} & \text{if} \min\limits_{0\le t\le T}(S_t)\le D\wedge\max\limits_{0\le t\le T}(S_t)\ge U\wedge S_T<X_1, \\ n_1S_T-n_2X_2+n_2p_{2SUI}-n_1p_{1BDI} & \text{if} \max\limits_{0\le t\le T}(S_t)\ge U\wedge X_1\le S_T<X_2, \\ nS_T+n_2p_{2SUI}-n_1p_{1BDI} & \text{if } S_T\ge X_2. \end{cases} \quad (7)$$

II. Let us create this option strategy by buying a higher number n_1 of down and knock-in put options with a lower strike price X_1, premium p^0_{1BDI} per option, barrier level D and at the same time by selling a smaller number n_2 of up and knock-out put options with a higher strike price X_2, premium p^0_{2SUO} per option, barrier level *U*.

Up and knock-out put option will behave like vanilla put option if the price of an underlying asset does not grow above barrier level *U*. The profit function is:

$$P_{III}(S_T)\begin{cases} n_2 p_{2SUO} & \text{if} \max\limits_{0\le t\le T}(S_t)\ge U\wedge S_T<X_2, \\ n_2(S_T-X_2+p_{2SUO}) & \text{if} \max\limits_{0\le t\le T}(S_t)<U\wedge S_T<X_2, \\ n_2 p_{2SUO} & \text{if } S_T\ge X_2. \end{cases} \quad (8)$$

If $D < X_1 \le U \le X_2$ or $D < X_1 \le X_2 \le U$, the income function from secured position (9) can be obtain as a sum of the functions (1), (3) and (8).

$$SI_{II}(S_T)=\begin{cases} nS_T+n_2p_{2SUO}-n_1p_{1BDI} & \text{if} \min\limits_{0\le t\le T}(S_t)>D\wedge\max\limits_{0\le t\le T}(S_t)\ge U\wedge S_T<X_1, \\ n_1S_T-n_2X_2+n_2p_{2SUO}-n_1p_{1BDI} & \text{if} \min\limits_{0\le t\le T}(S_t)>D\wedge\max\limits_{0\le t\le T}(S_t)<U\wedge S_T<X_1, \\ -n_2S_T+n_1X_1+n_2p_{2SUO}-n_1p_{1BDI} & \text{if} \min\limits_{0\le t\le T}(S_t)\le D\wedge\max\limits_{0\le t\le T}(S_t)\ge U\wedge S_T<X_1, \\ n_1X_1-n_2X_2+n_2p_{2SUO}-n_1p_{1BDI} & \text{if} \min\limits_{0\le t\le T}(S_t)\le D\wedge\max\limits_{0\le t\le T}(S_t)<U\wedge S_T<X_1, \\ nS_T+n_2p_{2SUO}-n_1p_{1BDI} & \text{if} \max\limits_{0\le t\le T}(S_t)\ge U\wedge X_1\le S_T<X_2, \\ n_1S_T-n_2X_2+n_2p_{2SUO}-n_1p_{1BDI} & \text{if} \max\limits_{0\le t\le T}(S_t)<U\wedge X_1\le S_T<X_2, \\ nS_T+n_2p_{2SUO}-n_1p_{1BDI} & \text{if } S_T\ge X_2. \end{cases} \quad (9)$$

If $D < U \le X_1 < X_2$, the income function from secured position is:

$$SI_{II}(S_T)=\begin{cases} nS_T+n_2p_{2SUO}-n_1p_{1BDI} & \text{if} \min\limits_{0\le t\le T}(S_t)>D\wedge\max\limits_{0\le t\le T}(S_t)\ge U\wedge S_T<X_1, \\ n_1S_T-n_2X_2+n_2p_{2SUO}-n_1p_{1BDI} & \text{if} \min\limits_{0\le t\le T}(S_t)>D\wedge\max\limits_{0\le t\le T}(S_t)<U\wedge S_T<X_1, \\ -n_2S_T+n_1X_1+n_2p_{2SUO}-n_1p_{1BDI} & \text{if} \min\limits_{0\le t\le T}(S_t)\le D\wedge\max\limits_{0\le t\le T}(S_t)\ge U\wedge S_T<X_1, \\ n_1X_1-n_2X_2+n_2p_{2SUO}-n_1p_{1BDI} & \text{if} \min\limits_{0\le t\le T}(S_t)\le D\wedge\max\limits_{0\le t\le T}(S_t)<U\wedge S_T<X_1, \\ nS_T+n_2p_{2SUO}-n_1p_{1BDI} & \text{if} \max\limits_{0\le t\le T}(S_t)\ge U\wedge X_1\le S_T<X_2, \\ nS_T+n_2p_{2SUO}-n_1p_{1BDI} & \text{if } S_T\ge X_2. \end{cases} \quad (10)$$

III. Let us form the IVRPS strategy by buying a higher number n_1 of DI put options with a lower strike price X_1, premium p^0_{1BDI} per option, barrier level *D* and at the same time by selling a smaller number n_2 of DI put options with a higher strike price X_2, premium p^0_{2SDI} per option and the same barrier level *D*. The income function from secured position in this case is expressed by (11).

$$SI_{III}(S_T)=\begin{cases} nS_T+n_2p_{2SDI}-n_1p_{1BDI} & \text{if} \min\limits_{0\le t\le T}(S_t)>D\wedge S_T<X_1, \\ n_1X_1-n_2X_2+n_2p_{2SDI}-n_1p_{1BDI} & \text{if} \min\limits_{0\le t\le T}(S_t)\le D\wedge S_T<X_1, \\ nS_T+n_2p_{2SDI}-n_1p_{1BDI} & \text{if} \min\limits_{0\le t\le T}(S_t)>D\wedge X_1\le S_T<X_2, \\ n_1S_T-n_2X_2+n_2p_{2SDI}-n_1p_{1BDI} & \text{if} \min\limits_{0\le t\le T}(S_t)\le D\wedge X_1\le S_T<X_2, \\ nS_T+n_2p_{2SDI}-n_1p_{1BDI} & \text{if } S_T\ge X_2. \end{cases} \quad (11)$$

By analyzing the functions (1) and (11) in meeting the condition $n_2 p^0_{2S} \ge n_1 p^0_{1B}$ we conclude the following:

- If the price does not fall below D during the option life and at time *T,* then by hedging we have ensured a constant profit of $n_2 p_{2SDI} - n_1 p_{1BDI}$.
- If the price falls below D during the option life and at time T it is $S_T < X_1$, then we have hedged an income of $n_1 X_1 - n_2 X_2 + n_2 p_{2SDI} - n_1 p_{1BDI}$.
- If the asset price falls below D and at time T it is $X_1 \le S_T < X_2$, then we will get a loss or we will make a profit, but the maximum of profit can be $n_2 p_{2SDI} - n_1 p_{1BDI}$.
- If at time T the asset price is $S_T \ge X_2$, then our profit will not be lower than it would be without hedging.

IV. Finally, we will buy a higher number n_1 of DI put options with a lower strike price X_1, premium p^0_{1BDI} per option, barrier level D and at the same time sell a smaller number n_2 of DO put options with a higher strike price X_2, premium p^0_{2SDO} per option and the same barrier level.

The income function from secured position using IVRPS strategy is expressed by the equation (12).

$$SI_{IV}(S_T)=\begin{cases} n_1S_T-n_2X_2+n_2p_{2SDO}-n_1p_{1BDI} & if \min_{0\le t\le T}(S_t)>D\wedge S_T<X_1,\\ -n_2S_T+n_1X_1+n_2p_{2SDO}-n_1p_{1BDI} & if \min_{0\le t\le T}(S_t)\le D\wedge S_T<X_1,\\ nS_T+n_2p_{2SDO}-n_1p_{1BDI} & if \min_{0\le t\le T}(S_t)\le D\wedge X_1\le S_T<X_2,\\ n_1S_T-n_2X_2+n_2p_{2SDO}-n_1p_{1BDI} & if \min_{0\le t\le T}(S_t)>D\wedge X_1\le S_T<X_2,\\ nS_T+n_2p_{2SDO}-n_1p_{1BDI} & if\ S_T\ge X_2. \end{cases} \quad (12)$$

It is possible to suppose two various lower barriers in the case of III. and IV. hedging variants and by analogy we can derive income functions from secured position for all possible relations between strike prices and lower barriers.

Application of hedging results to SPDR Gold Shares

Let us suppose that we own a portfolio consisting of 100 SPDR Gold Shares. On 22 November 2011, the shares were traded at the New York Stock Exchange at approximately USD 165 per share. At the same time, we are going to apply the above mentioned IVRPS hedging strategy using barrier and vanilla options to hedge a minimum price of the given shares to the certain future date (March 2013). We will select the number of traded options in a way that enables condition $n_2p_{2S}^0\ge n_1p_{1B}^0$ to be met, i.e. zero-cost strategy to be formed. We will propose some hedging variants, which meet the above stated requirements.

1. We will buy n_1 =150 DI put options with the strike price X_1 =140, the premium p_{1BDI}^0 =10.41 per option, the barrier level *D*=130 and at the same time, we will sell n_2 =50 UI put options with the strike price X_2 =195, the premium p_{2SUI}^0 =34.78 per option, the barrier level *U*=170. The income function from the sale of 100 shares hedged for this variant is the function (13).

$$SI_{1A}(S_T)\begin{cases} 100S_T+177.8 & if \min_{0\le t\le T}(S_t)>130\wedge\max_{0\le t\le T}(S_t)<170\wedge S_T<140,\\ 150S_T-9572.2 & if \min_{0\le t\le T}(S_t)>130\wedge\max_{0\le t\le T}(S_t)\ge170\wedge S_T<140,\\ -50S_T+21177.8 & if \min_{0\le t\le T}(S_t)\le130\wedge\max_{0\le t\le T}(S_t)<170\wedge S_T<140,\\ 11427.8 & if \min_{0\le t\le T}(S_t)\le130\wedge\max_{0\le t\le T}(S_t)\ge170\wedge S_T<140,\\ 100S_T+177.8 & if \max_{0\le t\le T}(S_t)<170\wedge140\le S_T<195,\\ 150S_T-9572.2 & if \max_{0\le t\le T}(S_t)\ge170\wedge140\le S_T<195,\\ 100S_T+177.8 & if\ S_T\ge195. \end{cases} \quad (13)$$

Let us change the numbers of options, i.e. n_1 and n_2, other parameters remain the same. The results are noted into Table 1.

The comparison of hedging variants 1A, 1B and 1C at various development of share price during time to maturity and at time to maturity of options is shown in Figure 1. It can be seen, but it also can be calculated exactly using data from Table 1, that:

- hedging variant 1C does not enable the condition $n_2p_{2S}^0\ge n_1p_{1B}^0$ $\left(n_2p_{2S}^0=95.6<n_1p_{1B}^0=1249.2\right)$ to be met,
- from the remaining two variants, i.e. 1A and 1B, the best results will be obtained through:
  - the hedging variant 1A, if the spot price of shares during time to maturity grows above upper barrier *U=170* and it is lower than 170.6 at time to maturity,
  - the hedging variant 1B, if the spot price of shares during time to maturity does not grow above upper barrier or if the spot price of shares during time to maturity grows above upper barrier *U* and it is higher than 170.6 at time to maturity.

As we mentioned earlier, hedging against a price drop is ensuring a minimum acceptable underlying asset price. The actual spot price is USD 165. It follows that a price drop below 165 is expected. Zero-cost hedging strategy is also preferred. We can see that if our assumptions are fulfilled, then the hedging variant 1B ensures the highest income at expected intervals of spot price at time *T*. Therefore we recommend this particular case to use in hedging. If the investor speculates on a slight increase (above 170) during time to maturity and he expects the spot price lower than 170.6 at time to maturity, but he limits the loss at a price drop, then the hedging variant 1A can be used.

Table 1

Comparison of particular hedging variants

Spot price scenarios during time to maturity t and at maturity time T	Hedging variant 1A n_1 =150; n_2 =50	Hedging variant 1B n_1 =200; n_2 =100	Hedging variant 1C n_1 =120; n_2 =20
$\min_{0\le t\le T}(S_t)>130\wedge\max_{0\le t\le T}(S_t)<170\wedge S_T<140$	$100S_T+177{,}8$	$100S_T+1398$	$100S_T-554.4$
$\min_{0\le t\le T}(S_t)>130\wedge\max_{0\le t\le T}(S_t)\ge170\wedge S_T<140$	$150S_T-9572.2$	$200S_T-18102$	$120S_T-4454.4$
$\min_{0\le t\le T}(S_t)\le130\wedge\max_{0\le t\le T}(S_t)<170\wedge S_T<140$	$-50S_T+21177.8$	$-100S_T+29398$	$-20S_T+16245.6$
$\min_{0\le t\le T}(S_t)\le130\wedge\max_{0\le t\le T}(S_t)\ge170\wedge S_T<140$	11427.8	9898	12345.6
$\max_{0\le t\le T}(S_t)<170\wedge140\le S_T<195$	$100S_T+177{,}8$	$100S_T+1398$	$100S_T-554.4$
$\max_{0\le t\le T}(S_t)\ge170\wedge140\le S_T<195$	$150S_T-9572.2$	$200S_T-18102$	$120S_T-4454.4$
$S_T\ge195$	$100S_T+177{,}8$	$100S_T+1398$	$100S_T-554.4$

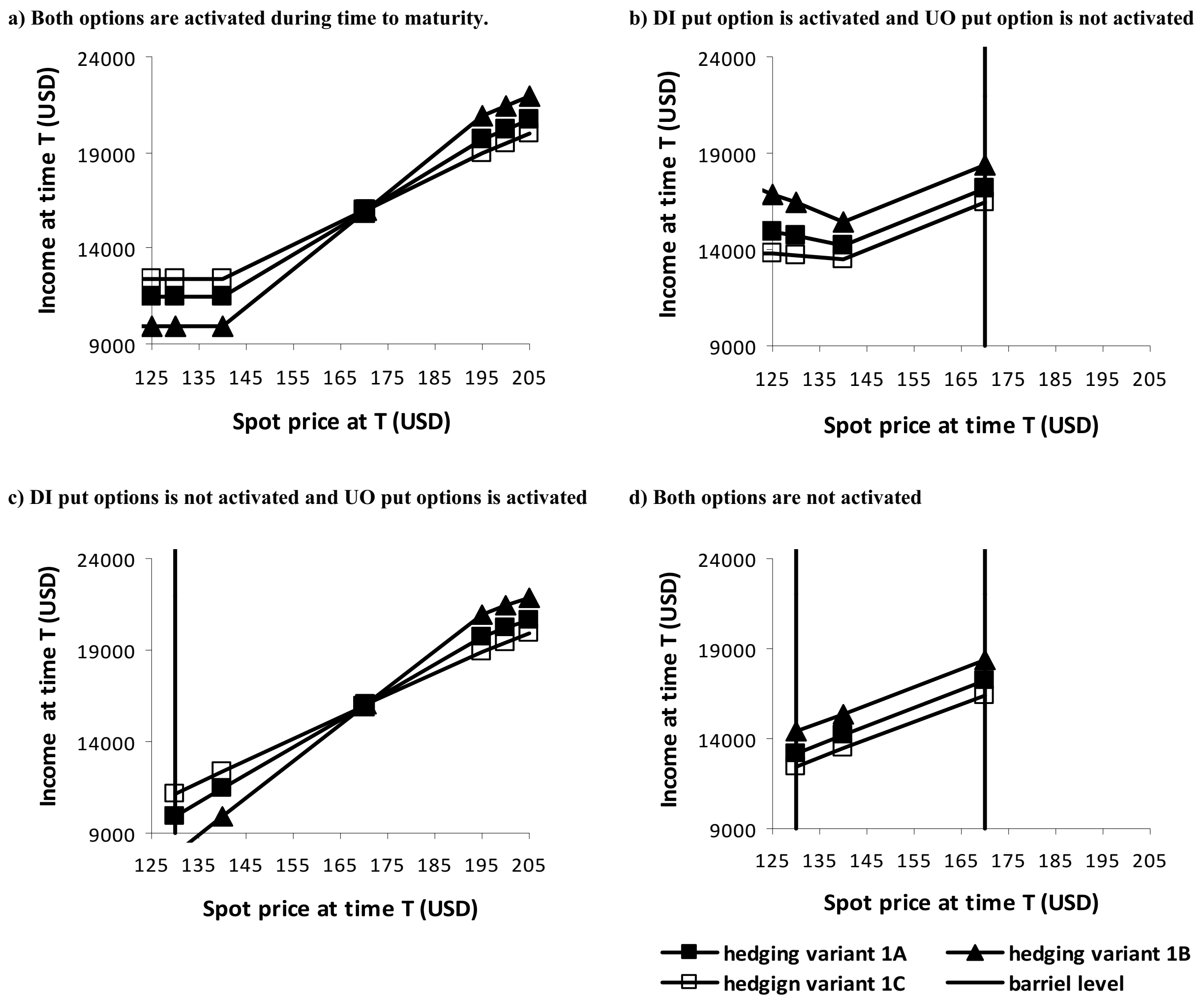

Figure 1. Comparison of the income functions at time T of particular hedging variants

2. We will buy n_1=200 DI put options with the strike price X_1=140, the premium p_{1BDI}^0=10.41 per option, the barrier level D=130 and at the same time, we will sell n_2=100 UI put options with the strike price X_2=195, the premium p_{2SUI}^0=29.83 per option, the barrier level U=195. The income function from secured position is:

$$SI_2(S_T)\begin{cases} 100S_T+902.3 & if \min\limits_{0\le t\le T}(S_t)>130 \wedge \max\limits_{0\le t\le T}(S_t)\ge 195 \wedge S_T<140, \\ 200S_T-18597.7 & if \min\limits_{0\le t\le T}(S_t)>130 \wedge \max\limits_{0\le t\le T}(S_t)<195 \wedge S_T<140, \\ -100S_T+28902.3 & if \min\limits_{0\le t\le T}(S_t)\le 130 \wedge \max\limits_{0\le t\le T}(S_t)\ge 195 \wedge S_T<140, \\ 9402.3 & if \min\limits_{0\le t\le T}(S_t)\le 130 \wedge \max\limits_{0\le t\le T}(S_t)<195 \wedge S_T<140, \\ 100S_T+902.3 & if \max\limits_{0\le t\le T}(S_t)\ge 195 \wedge 140\le S_T<195, \\ 200S_T-18597.7 & if \max\limits_{0\le t\le T}(S_t)<195 \wedge 140\le S_T<195, \\ 100S_T+902.3 & if\ S_T\ge 195. \end{cases} \quad (14)$$

Results of the comparative analysis of hedging variants 1B and 2:

- if the spot price of shares during time to maturity grows above upper barrier *U=195* and it is lower than 190 at time to maturity, then the hedging variant 2 is better;
- otherwise, the hedging variant 1B.

It can be concluded from the above statements and assumptions mentioned earlier that the hedging variant 1B is the best variant from till now analyzed variants.

3. In this case we will buy n_1 DI put options with the strike price X_1=140, the premium p_{1BDI}^0=10.41 per option, the barrier level D=130 and at the same time, we will sell n_2 DI put options with the strike price X_2=195, the premium p_{2SUI}^0=35.52 per option, the same barrier level D=130.

A n_1 = 150 and n_2 = 50,

B n_1 = 200 and n_2 = 100.

The income functions are:

$$SI_{3A}(S_T)=\begin{cases} 100S_T+214.8 & if \min\limits_{0\le t\le T}(S_t)>130 \wedge S_T<140, \\ 11464.8 & if \min\limits_{0\le t\le T}(S_t)\le 130 \wedge S_T<140, \\ 100S_T+214.8 & if \min\limits_{0\le t\le T}(S_t)>130 \wedge 140\le S_T<195, \\ 150S_T-9535.2 & if \min\limits_{0\le t\le T}(S_t)\le 130 \wedge 140\le S_T<195, \\ 100S_T+214.8 & if\ S_T\ge 195, \end{cases} \quad (15)$$

$$SI_{3B}(S_T)=\begin{cases}100S_T+1472.1 & if \min\limits_{0\le t\le T}(S_t)>130 \wedge S_T<140,\\ 9972.1 & if \min\limits_{0\le t\le T}(S_t)\le 130 \wedge S_T<140,\\ 100S_T+1472.1 & if \min\limits_{0\le t\le T}(S_t)>130 \wedge 140\le S_T<195,\\ 200S_T-18027.9 & if \min\limits_{0\le t\le T}(S_t)\le 130 \wedge 140\le S_T<195,\\ 100S_T+1472.1 & if\ S_T\ge 195.\end{cases} \quad (16)$$

Based on comparison of the proposed hedging variants 3A and 3B we have the following conclusions:

- if the spot price of shares during time to maturity drops below lower barrier *D=130* and it is lower than 169.9 at time to maturity, then the hedging variant 3A is better,
- otherwise, the hedging variant 3B.

We can see that the selection of appropriate variant must be made by the investor depending on his expectations. If the investor expects a significant price drop then the hedging variant 3A ensures the highest income. If the investor expects a slight price drop (above 130) then the hedging variant 3B ensures better results.

The comparison of hedging variants 1A and 1B or 3A and 3B is in Table 2. Hedging variants 1B, 3A and 3B ensure the best results for expected price scenarios.

Table 2

Comparison of particular hedging variants

Spot price scenarios during time to maturity t	Spot price scenarios at maturity time T	Better hedging variant	Spot price scenarios during time to maturity t	Spot price scenarios at maturity time T	Better hedging variant
$\max\limits_{0\le t\le T}(S_t)\ge 170$	$S_T<170.6$	1A	$\min\limits_{0\le t\le T}(S_t)\le 130$	$S_T<169.9$	**3A**
$\max\limits_{0\le t\le T}(S_t)\ge 170$	$S_T\ge 170.6$	1B	$\min\limits_{0\le t\le T}(S_t)\le 130$	$S_T\ge 169.9$	3B
$\max\limits_{0\le t\le T}(S_t)<170$		**1B**	$\min\limits_{0\le t\le T}(S_t)>130$		**3B**

4. In the last case let us form the IVRPS strategy using vanilla options (see Šoltés V and Amaitiek, 2010a), i.e. by buying n_1 put options with the strike price X_1=140 and the premium p_{1B}^0= 10.45 per option and at the same time by selling n_2 put options with the strike price X_2=195, the premium p_{2S}^0 = 41.85 per option.

A n_1 = 150 and n_2 = 50,

B n_1 = 200 and n_2 = 100.

The income functions from secured position have the following form:

$$SI_{4A}(S_T)=\begin{cases}11775.8 & if\ S_T<140,\\ 150S_T-9224.2 & if\ 140\le S_T<195,\\ 100S_T+525.8 & if\ S_T\ge 195,\end{cases} \quad (17)$$

$$SI_{4B}(S_T)=\begin{cases}10598 & if\ S_T<140,\\ 200S_T-17402 & if\ 140\le S_T<195,\\ 100S_T+2098 & if\ S_T\ge 195.\end{cases} \quad (18)$$

Results of the analysis:

- if the spot price of shares is lower than 163.6 at time to maturity, then the hedging variant 4A is better,
- if the spot price of shares is higher than 163.6 at time to maturity, then the hedging variant 4B ensures higher income.

Now we will compare secured positions 1B, 3B, 4B, 3A and 4A with unsecured position $100S_T$. We will calculate a minimum (min) and maximum (max) profit for selected spot price intervals at time *T* as a difference between the particular secured position and unsecured position. If the profit will be higher than 0 then secured position ensures higher income than unsecured position. The results are in Table 3. From this data we can deduce following conclusions:

- if it is expected the significant price drop (below 130), then the hedging variant 1B is the best,
- if it is expected the slight price drop (under 130), then the highest income is in the hedging variant 3B,
- both variants ensure higher income in comparison to the income from unsecured position for these particular price scenarios.

It should be noted that, if the price does not meet investor expectations, it could be lossy. It is a fee of hedging.

If the investor speculates:

- on a significant price increase during time to maturity (above 170) and a significant price drop during time to maturity (below 130) and a price drop at time to maturity (below 165), then the variant 4A is the best;
- on a price increase at time to maturity (above 170) but he limits the loss in a significant price drop during time to maturity (below 130), then the variant 4B is the best;
- on a price increase at time to maturity (above 170) but he limits the loss in a slight price drop during time to maturity (above 130), then the variant 3B is the best for $S_T\in\langle 170,189\rangle$ and the variant 4B for $S_T\ge 189$;
- on a slight price increase at time to maturity , i.e. $S_T\in\langle 165,170\rangle$ but he limits the loss in a slight price drop during time to maturity (above 130), then the variant 3B is the best;
- on a slight price increase at time to maturity, i.e. $S_T\in\langle 165,170\rangle$ but he limits the loss in a slight price drop during time to maturity (above 130), then the variant 3B is the best;

- on a slight price increase at time to maturity, i.e. $S_T \in \langle 165, 170 \rangle$ and he does not expect a price increase above 170 during time to maturity but he limits the loss in a significant price drop during time to maturity (below 130), then the variant 1B is the best;
- on a slight price increase at time to maturity, i.e. $S_T \in \langle 165, 170 \rangle$ and he expects a price increase above 170 during time to maturity but he limits the loss in a significant price drop during time to maturity (below 130), then hedging variant 4B is the best.

These future price scenarios are the combination of trading and hedging.

Table 3

Comparison of selected hedging variants

4A	**Profit**	**max**	**11776**	11776	**-1046**	-1046	-1046	-1046	-724	-724	-724	-724	226	226	526	526	526
		min	**-1224**	-1224	**-1224**	-1224	-1224	-1224	-1046	-1046	-1046	-1046	-724	-724	226	226	526
3A	**Profit**	**max**	11465	11465	-1357	-1357	215	215	-1035	-1035	215	215	-85	215	215	215	215
		min	-1535	-1535	-1535	-1535	215	215	-1357	-1357	215	215	-1035	215	-85	215	215
4B	**Profit**	**max**	10598	10598	-1046	-1046	-1046	-1046	**-402**	-402	-402	-402	**1472**	1472	**2098**	**2098**	**2098**
		min	-2402	-2402	-2402	-2402	-2402	-2402	**-1046**	-1046	-1046	-1046	**-402**	-402	**1472**	**1472**	**2098**
3B	**Profit**	**max**	9972	9972	-1672	-1672	**1472**	**1472**	-1028	-1028	**1472**	**1472**	872	**1472**	1472	1472	1472
		min	-3028	-3028	-3028	-3028	**1472**	**1472**	-1672	-1672	**1472**	**1472**	-1028	**1472**	872	1472	1472
1B	**Profit**	**max**	9898	**29398**	-1746	**1398**	-1746	1398	-1102	**1398**	-1102	1398	798	798	1398	1398	1398
		min	-3102	**3398**	-3102	**1398**	-5102	1398	-1746	**1398**	-1746	1398	-1102	-1102	798	798	1398
Barrier conditions (0≤t≤T)			Max (St)≥ 170	Max (St)≤ 170	Max (St)≥ 170	Max (St)≤ 170	Max (St)≥ 170	Max (St)≤ 170	Max (St)≥ 170	Max (St)≤ 170	Max (St)≥ 170	Max (St)≤ 170					
					Min (St)≤ 130	Min (St)≤ 130	Min (St)≥ 130	Min (St)≥ 130	Min (St)≤ 130	Min (St)≤ 130	Min (St)≥ 130	Min (St)≥ 130	Min (St)≤ 130	Min (St)≥ 130	Min (St)≤ 130	Min (St)≥ 130	
Spot price intervals at time T			$S_T \leq 130$		$130 \leq S_T \leq 164$				$164 \leq S_T \leq 170$				$170 \leq S_T \leq 189$		$189 \leq S_T \leq 195$		$S_T \geq 195$

Conclusions

This paper presents the hedging analysis using barrier options. By these options we formed the well-known Inverse Vertical Ratio Put Spread strategy and have given formulas of secured positions at the future date. To the best of our knowledge, no study has yet provided hedging analysis using option strategies formed by barrier options. This work therefore contributes to the literature by filling this gap, including practical application to SPDR Gold Shares.

We analyze the Inverse Vertical Ratio Put Spread strategy using barrier options and propose its utilization in hedging against the price drop of an underlying asset. The interesting approach based on the analytical expression of the income functions from secured position is presented, which can also be used in practice as a priceless aid in deciding which hedging variant is the most suitable.

In the practical section of this paper we show how to apply this strategy in hedging of SPDR Gold Shares. We demonstrate its usage in hedging against a price drop in some model variants. The comparison with hedging using vanilla options is presented as well.

The selection of traded options enables zero-cost Inverse Vertical Ratio Put Spread strategy formation. Opponents of hedging often argue that hedging is expensive. We demonstrated that there is more sophisticated hedging which does not have to be expensive at all.

The results showed differences between proposed hedging variants formed by the Inverse Vertical Ratio Put Spread strategy. The selection of appropriate hedging variant must be made by the investor depending on his expectations.

We found the best variants for the investor who hedges against a slight or significant price drop. If it is expected the significant price drop (below the lower barrier) and it does not expected the price increase above the higher barrier then the hedging variant formed by buying DI put options and selling UI put options is the best. Otherwise, if it is expected the slight price drop (upper the lower barrier), then the highest income is in the hedging variant formed by buying DI put options and selling DI put options. In both cases, it is preferable to select the number of options in a way which minimizes the ratio n_1 / n_2.

We also identified the best variants for the investor who hedges against a slight or significant price drop and at the same time he speculates on a price increase. If the investor speculates on the slight price increase and he hedges against a significant price drop then the highest income is in the hedging variant formed by buying DI put options and selling DI put options. If he speculates on the slight price increase and he hedges against a slight price drop then the hedging variant formed by buying DI put options and selling UI put options or buying put vanilla options and at the same time selling put vanilla options is the best (it depends on more specific expectations). Further, it can be argued that if the investor speculates on the significant price increase then the hedging variant formed by vanilla put options is the best.

The selection of strike prices, lower and upper barrier is extremely significant for the profit profile. The findings also indicate that this strategy formation using barrier options gives end-users greater flexibility to express a precise view and produces better results in the hedging away the risk involved with the price drop.

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Vincent Šoltés, Martina Rusnáková

Apsidraudimas nuo kainos kritimo naudojant *atvirkštinio vertikalaus santykio pirkimo Spredo strategiją*, suformuotą iš barjero pasirenkamų sandorių

Santrumpa

Vykstant globalizacijos ir liberalizacijos procesui, akcijų kainos, prekės, palūkanų normos ir valiutos finansų rinkose nuolat kinta. Metodai ir priemonės, naudojami norint suvaldyti riziką rinkose, nuolat tobulinami. Vienas iš efektyvių rizikos valdymo metodų yra hedžingas. Galimų hedžingo strategijų analizė, jų tarpusavio ryšiai su našios rinkos ir vertės teorijomis, taip pat ir įvairūs empiriniai tyrimai, dabartiniu metu yra pagrindinė mokslinių darbų tema.

Zmeškal (2004) teigia, jog pagrindinė hedžingo idėja, yra pridėti naują turtą arba lėšas (dažniausiai vertybinius popierius) prie rizikingo turto, norint sukurti naują portfelį, (kitaip vadinamą hedžingo portfeliu), apsaugotą nuo nepageidaujamų kainos svyravimų. Mes pristatome patį sudėtingiausią metodą rinkos rizikai valdyti – hedžingą, tam naudodami pasirinkimo sandorio strategijas. Yra įvairių pasirinkimo sandorių strategijų. Šis darbas tyrinėja portfelio, kurį sudaro rizikingas bazinis turtas hedžingą, panaudojant *atvirkštinio vertikalaus santykio pirkimo Spredo* (*plg. angl.* Inverse Vertical Ratio Put Spread) strategiją, suformuotą iš atskirų *barjero* pasirenkamų sandorių. Tai atliekama norint ištirti skirtumą tarp hedžingo, naudojant *barjero* pasirinkimo sandorius ir hedžingo, naudojant *vanilla* pasirinkimo sandorius (analizavo Šoltés V. ir Amaitiek, 2010a). Ši strategija yra naudinga, kai apsidraudžiama nuo kainos kritimo žinant, kad duotasis bazinis turtas bus parduotas ateityje. Kiek mums žinoma, dar joks tyrimas nepanaudojo *barjero* pasirinkimo sandorių, tiriant pasirinkimo sandorių strategijas, taip pat ir hedžingo, naudojant pasirinkimo sandorių strategijas. *Barjero* pasirinkimo sandoriai buvo sukurti norint aprūpinti rizikos valdytojus pigesnėmis priemonėmis, siekiant apdrausti savo padėtį ir nemokant už kainos pokyčius, kurių jų nuomone neturėtų atsirasti. Šie pasirinkimo sandoriai vadinami *barjeru* arba *priežastimi. Barjero* kritimas pasirenkamo sandorio egzistavimo metu, reiškia, tam tikrų barjero pasirinkimo sandorių aktyvavimą arba deaktyvavimą. Apskritai tariant, palyginti su paprastais *vanilla* pasirinkimo sandoriais, jie dažniausiai yra daug lankstesni ir pigesni.

Savo analizėje mes naudojame įdomų metodą, pagrįstą duomenimis apie apdraustų pozicijų pajamų funkcijas, kurios gali palengvinti tam tikrų hedžingo strategijų pasirinkimą ir supaprastinti hedžingo galimybių palyginimą. Šie teoriniai rezultatai yra svarbūs kalbant apie skirtingas bazines lėšas ir yra naudingi finansinėms ir ne finansinėms institucijos. Praktinėje šio darbo dalyje parodyta kaip taikyti šią strategiją SPDR *Gold Shares* hedžinge. Atskleistas jo panaudojimas, prieš tai apsidraudžiant nuo kainos kritimo kai kuriuose modelio variantuose. Dar daugiau, apdraustos hedžingo variantų pozicijos suformuotos iš *barjero* ar *vanilla* pasirenkamų sandorių, yra lyginamos su neapdraustomis pozicijomis.

SPDR *Gold Shares* siūlo investuotojams naujovišką, iš dalies efektyvų ir saugų būdą patekti į aukso rinką, nesirūpinant pristatymu ir saugiu laikymu. Analizuojant šias akcijas, naudotasi *vanilla* ir *barjero* pasirinkimo sandoriais. Mes pateikėme *vanilla* ir *barjero* europietiškus pasirinkimo sandorius dėl šių akcijų įvairesnių vykdymo kainų ir barjerų. *Vanilla* pasirenkamų sandorių atveju mes naudojame tikrus duomenis. Kadangi trūksta duomenų apie *barjero* pasirinkimo sandorių tikrą prekybą, mes skaičiuojame *barjero* pasirenkamų sandorių premijas, naudodami Haug analitinį modelį, kuris pritaikė Black-Scholes-Merton formulę visoms *barjero* pasirenkamų sandorių rūšims. Visi skaičiavimai atlikti statistine programa R.

Pasiūlytų variantų palyginimas leido nustatyti geriausius rezultatus. Jei tikimasi žymaus kainos kritimo (žemiau 130), tai geriausias hedžingo variantas yra 1B. Kitu atveju, kai tikimasi nežymaus kainos kritimo (iki 130), tada didžiausios pajamos yra hedžingo variante 3B. Tam tikrą hedžingo variantą turi pasirinkti investuotojas pagal savo lūkesčius. Mes taip pat galime pasakyti geriausią variantą ir investuotojui, kuris lošia biržoje ir tuo pat metu apsidraudžia nuo nežymaus arba žymaus kainos kritimo. Mūsų tyrimas patvirtina, kad šis strategijos formavimas, naudojant barjero pasirinkimo sandorį, suteikia galutiniam vartotojui daugiau galimybių susidaryti tikslesnį vaizdą. Rezultatai parodė, kad *atvirkštinio vertikalaus santykio pirkimo Spredo* strategijos formavimas, naudojant *barjero* pasirinkimo sandorį, yra geresnis, negu šio pasirinkimo sandorio strategijos formavimas, naudojant *vanilla* pasirinkimo sandorį ir numatant tam tikras kainų situacijas ateityje (išskyrus praktines situacijas).

Raktažodžiai: *hedžingas, atvirkštinio vertikalaus santykio pirkimo Spredo strategija, barjero pasirinkimo sandoris, vanilla pasirinkimo sandoris, SPDR Gold Shares.*

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