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Deep Learning Model for Estimation Market Risk in Insurance Sector

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The paper developed a model for market risk assessment based on deep learning in combination with non-parametric, parametric and semi-parametric VaR and ES models. We presented the ANN-GRU model more precisely. It is intended for insurance companies operating in emerging markets because the model was developed to cover all the characteristics of emerging markets. The research was conducted on the example of 8 optimal investment portfolios for insurance companies operating in the Balkan countries. The portfolios were calibrated at the daily level and calculated for the period from 1 January 2020 to 31 December 2023. The first 500 data were used to estimate the calibration of the VaR model, the other 250 to estimate the validity of the VaR model, and the last 250 to test the validity of the VaR/ES-GRU-DL market risk estimates in accordance with Directive II. Conditional and unconditional coverage tests were used to test the validity of VaR estimates, while Berkowitz's ES test was used to test the validity of ES estimates. Due to the limitations of these tests, the validity of the backtesting VaR estimate was performed using Dufour Monte Carlo simulations, while the validity of the backtesting ES estimate used the Bootstrap procedure. The backtesting results, as well as the results of the validity of the backtesting results, show that the model generates reliable estimates of VaR and ES in accordance with the Solvency II directive as well as produces better estimates compared to the popular and widely used VaR and ES models.

Keywords: *Insurance Companies; Market Risk; Value at Risk; Expected Shortfall; Emerging Market; Deep Learning.*

Introduction

From the earliest days, people have employed various methods and techniques to protect themselves from risks. The application of different risk management techniques and principles has provided them with security about future outcomes. In modern business conditions, insurance, as a form of organized risk protection based on solidarity and reciprocity, represents the most significant form of risk protection. Traditional insurance companies manage risk by retaining it, forming technical reserves as a buffer. The inability to predict payments based on obligations, both in terms of amount and timing, necessitates the formation of technical reserves as direct support for ensuring the solvency and liquidity of the insurance company. However, since technical reserves, from the perspective of the dynamics of cash flows, represent immobilized assets of the company, directly limiting the earnings power of the company, the formation of optimal technical reserves means making a compromise between unconditional solvency and liquidity, on one hand, and the shareholders' desire to maximize returns on investment, on the other.

From the perspective of the national economy, insurance companies play a crucial role in developing financial markets, creating and trading financial instruments, and efficiently allocating capital. This highlights the need for regulatory institutions to enforce strong regulations on the formation of technical reserves and their investment options. This is the reason why investment of insurance funds in all countries is subject to strict legal regulation. The goal of such frames is to ensure organized, fair, and controlled trading of financial assets, to prevent possible irregularities and crises in financial markets (Stancic & Radivojevic, 2021).

Considering the significance of insurance and the functions of insurance companies, insurance supervisors have undertaken a series of significant activities to ensure adequate protection for participants in the financial markets. This has been accomplished through defining capital requirements for cushioning market risks and ensuring their adequate control. However, the complexity of regulating the operations of insurance companies stems from the fact that the ideal synchronization of cash inflows from insurance premiums and outflows from the payment of incurred obligations is not entirely feasible in reality (Stancic & Radivojevic, 2021). Prescribing regulatory standards as measure control of exposure to market risk, often does not reflect the real risks to which portfolios of insurance companies, formed from a portion of technical reserves, are exposed. Moreover, by prescribing equal standards for different insurance companies, the investment capabilities of insurance company management are not considered. Additionally, the effects of diversification, i.e., the degree of correlation between different assets and markets, are not taken into account.

Following the paradigm adopted from banking, where a single standard cannot be defined for measuring market risk that will suit all portfolios, the level of required capital to cover market risk, which refers to the risk of negative deviations of the value of an open trading position, should be determined by the actual exposure of the portfolio to market risk, consistent with the philosophy of insurance company operations (Rea *et al.,* 2018). Therefore, supervisors have adopted the Solvency II directive, which is built around the principles of market consistency and embeds strong risk management and governance within insurance companies. The implementation of the Solvency II Directive represents the most significant change in the field of solvency regulation at the European Union level in recent decades. The Directive allows insurance companies, alongside the Standard Approach, to determine the Solvency Capital Requirement (SCR) for market risk coverage using internal models for risk assessment (Value at Risk – VaR). The Directive sets requirements for the application of internal VaR models instead of defining a percentage of required capital for different levels of exposure to risks.

In this manner, insurance companies have gained the ability to use a new tool for managing market risks. This new tool enables them to more realistically assess the riskiness of their portfolios and capital adequacy (SCR), while minimizing capital costs, as the SCR is consistent with the actual degree of risk exposure. The Directive promotes incentives for weaker companies to hold more capital and/or reduce their risk exposure without significantly distorting the decisions of financially sound insurers. Furthermore, since the level of SCR that insurance companies are required to set aside for market risk coverage depends on the validity of VaR models, insurance companies are motivated to continually improve them. The development of new, more efficient models not only increases the validity of risk assessment and reduces the risk of insolvency but also reduces capital costs, resulting in a decreased agency problem due to the alignment of management and supervisory objectives and incentives (Doff, 2008).

The directive requires insurance companies to calibrate VaR models according to a set of precisely defined quantitative and qualitative rules and to integrate these models into the risk management process. The Directive does not prescribe the type of models, but it requires that the models be continuously reviewed in terms of their ability to accurately predict maximum loss. However, the proper use of VaR models demands the fulfillment of certain highly restrictive assumptions (Radivojevic *et al.,* 2016, 2019). The choice of a theoretical distribution that best describes the distribution of empirical data is crucial in determining the correctness of the model in providing conditional or unconditional risk coverage. The directive assumes that changes in portfolio values are stochastic and generally follow a normal distribution. Popular and widely used models in developed markets are generally developed based on the assumption that changes in portfolio values can be described by a fixed parameter and a random variable, which follows an independently and identically distributed (IID) distribution. However, numerous empirical studies show that financial asset return series do not follow IID and normal distribution (Al Janabi *et al.*, 2017; BenSaida *et al.*, 2018; Eling & Jung, 2018; Arreola Hernandez, & Al Janabi, 2020). Furthermore, empirical research indicates that fluctuations in the return series of financial assets do not follow stochastic processes that can be easily represented by a random walk model, using ARCH/GARCH models, as well as linear ARMA or ARIMA models (Rossignolo *et al.*, 2012, 2013; Zikovic & Filer, 2103; Louzis *et al.,* 2014; Radivojevic *et al.,* 2019, 2020).

Given the complexities surrounding traditional risk modeling measures such as VaR and the limitations of traditional techniques, there is a growing need for new approaches. Specifically, the application of data mining, machine learning, and artificial neural networks (ANN) has gained significance, particularly in the development of new VaR models and risk measures such as Expected Shortfall (ES). This is primarily driven by the desire to overcome the limitations of traditional techniques for predicting asset behavior in financial markets. Research results by Hiransha *et al.* (2018), Fischer & Krauss (2018), Rundo *et al.* (2019), Nti et al. (2019), Shah *et al.* (2019), and Sezer *et al*. (2020) highlighted the advantages of using neural networks relative to traditional statistical techniques in modeling time series.

However, as VaR heavily relies on modeling the tail of a probability distribution, accurate tail modeling is essential. It's important to note that ANN models aren't capable of capturing extreme returns, as the tail distributions typically involve a small number of returns. Consequently, the application of classic ANN models based on historical data and values of insurance company portfolios in the context of the Solvency II directive requires special attention. Additionally, ANN models are based on the assumption of independence between observations, affecting their ability to incorporate heteroskedasticity in return series. Given this, the application of ANN in emerging markets, such as the financial markets of the Balkan countries, is quite debatable, as they feature relatively short financial market histories and exhibit significant correlations among return variables.

Despite these challenges, the ability to capture non-linear dependencies among risk factors in the portfolios of insurance companies, arising due to national regulations restricting investment structures within the technical reserves of insurance companies, justifies the effort to develop new VaR and ES models for estimating market risk in the context of the Solvency II directive.

Therefore, the aim of this work is to develop a market risk evaluation model based on deep learning (DL) for emerging markets, addressing the limitations of traditional risk modeling techniques. Recent works by Sirignano and Cont (2019), Choi *et al.* (2023) indicate the validity of this approach. With the precise goal of improving market risk evaluation models according to the Solvency II directive, the intent is to leverage the advantages of parametric and non-parametric models for market risk assessment and the application of machine learning and artificial intelligence. To this end, data mining models have been developed, incorporating parametric or non-parametric VaR and ES models, while successfully integrating the characteristics of emerging markets. Importantly, the significance of this research also lies in the fact that most studies focusing on the application of machine learning and deep learning for studying market risk behavior predominantly center around the United States.

Literature Review

In recent years, there has been a notable surge in literature dedicated to exploring the potential applications of advancements in IT within the domain of financial risk management. As a result, there has been a proliferation of new models for assessing market risk, primarily based on data mining, deep learning, and the utilization of artificial neural networks (ANN).

Among the early authors who attempted to exploit the benefits of ANN in risk assessment were Donaldson and Kamstra (1996). However, the random weighting in the hidden layer led to unstable risk estimates, rendering their solution to have little practical application. Other authors such as Lahmiri (2017), and Bijelic and Ouijjane (2019) attempted to combine ANN with volatility models. Ding *et al.* (2008) combined ANN with three volatility models from the ARCH family: GARCH, EGARCH, and TGARCH, while Kristjanpoller *et al.*, (2014) included only the GARCH model in ANN. Lahmiri (2017) combined ANN with GARCH and EGARCH with different assumptions of innovation distribution of the volatility model's residuals, while Hajizadeh *et al.*, (2012) combined it only with an EGARCH model to capture nonlinear dependencies. The successful combination of ANN with volatility models that capture asymmetry in volatility data was also demonstrated by Bildirici and Ersin (2009), who combined ANN with the APGARCH model. Chen *et al.* (2009) demonstrated that ANN models can generate better risk estimates than ARMA-GARCH models.

However, while all these studies show improvements compared to ARCH models in market risk assessment, it is important to note that the use of volatility estimates obtained solely from GARCH models may not necessarily be the best approach for predicting historical instability, as they rely on certain assumptions regarding the distribution of variables and errors (residuals). Mostafa *et al.* (2017) also caution against the limitations of these models in capturing all stylized features of financial markets, especially heteroskedasticity. For this reason, they recommend the use of Mixture Density Networks (MDN), as they are effective in estimating conditional densities with non-constant variance. They have presented satisfactory outcomes in capturing the dynamics of portfolio returns series and a narrow tail distribution; however, the main drawback of the model lies in its complexity and inability to be used for ES estimation. Miazhynskaia *et al.* (2003) hold a similar stance, using both linear and non-linear MDN for risk estimation.

Among the first authors to use ANN for VaR estimation were Locarek-Junge and Prinzler (1998). They developed a model based on MDN and presented findings suggesting better performance of this model compared to the RiskMetrics model. However, the primary drawback of this model is its incompatibility with time series characteristics. For market risk assessment using VaR models, Bartlmae and Rauscher (2000) used Neural Network Volatility Mixture (NNVM), while Dunis and Chen (2005) utilized regression models of neural networks for VaR estimation for market risk. Similar models were used by Wu et al. (2005), specifically employing quantile regression neural networks and support vector regression (SVR). The results of applying all these models indicate certain improvements compared to stochastic volatility models.

The Deep Belief Network Ensemble-based approach for VaR estimation was utilized by He *et al.* (2018). They presented results indicating that the application of this network type leads to improved VaR estimates compared to a Fully Connected Neural Network. With the demands of using recurrent neural networks (RNNs) and their advantages in dealing with time series, Bijelic and Ouijjane (2019) combined RNN with the standard GARCH model to predict VaR/ES, and they obtained satisfactory results.

A common aspect of all these studies is the estimation of volatility, from which VaR was calculated, or the estimation of the distribution of portfolio value changes, used for market risk assessment. Unlike the aforementioned studies, Doncic *et al.* (2023) took VaR/ES estimates obtained from the hybrid VaR model based on Extreme Value Theory as inputs for their network, showing significant improvements. A similar model was also used by Musah et al. (2018). However, the models are based on the standard multilayer perceptron (MLP) model, which is less efficient compared to the GRU model for working with time series. Furthermore, they take VaR/ES estimates from only a single VaR model as inputs. Therefore, our idea is to develop a ANN model based on the GRU, with inputs being VaR/ES estimates obtained from different VaR/ES estimation models. A similar approach regarding input data is also seen in Zhang *et al.* (2022). The justification for this approach in risk assessment can be found in the work of Sirignano and Cont (2019), indicating the universal characteristics of asset formation value.

Methodological Framework for Modeling Market Risk Using Deep Learning

Considering the advantages and disadvantages of using ANN in predicting financial phenomena, as well as the strengths and weaknesses of existing market risk assessment models based on neural networks, a new model for VaR and ES assessment has been developed in this work. The model is built on the foundations of the so-called GRU-ANN model, using VaR and ES estimates obtained from eight commonly used parametric, non-parametric and semi-paramtric VaR models as inputs: Standard Historical Simulation model (HS), Bootstrap (BHS) and Mirrored Historical Simulation (MHS), RiskMetrics (RM) based on a normal GARCH(p,q), and Student's t GARCH(p,q) volatility model, and RiskMetrics based on a Threshold GARCH (TARCH) volatility model, Filtered Historical simulation model (FHS) and VaR/ES model based on Extreme Value Theory (VaR/ES-EVT500).

The GRU-ANN has been selected because it falls into the category of so-called DL models, which enable high-level abstraction for data modeling and whose key advantages lie in automatically extracting good features from input data using a general-purpose learning procedure (Sezer *et al.*, 2020). These models demonstrate better performance when working with financial market data series compared to traditional machine learning (ML) models, as evidenced by Ozbayoglu *et al.*, (2020). Additionally, GRU-ANN has been chosen because it belongs to the group of recurrent neural networks (RNN) which have been proven to work well with time series data. However, unlike traditional RNN models, GRU-ANN represents an enhancement of these models in many aspects, especially in dealing with challenges of long-term dependencies and gradient problems. It solves the gradient problem by using gate and reset gates that help control the flow of information through sequential data. This helps reduce issues with vanishing and exploding gradients that are common in traditional RNNs. The internal architecture of GRU allows for better handling of long-term dependencies compared to classical RNNs, making them more suitable for working with time series. Additionally, they typically require fewer parameters, which implies faster training and lower susceptibility to overfitting.

The new model, VaR/ES-GRU-DL, comprises 8 input layers with one neuron each, five hidden layers with varying numbers of neurons, and one output layer with one neuron (Figure 1). In each hidden layer, the activation function of the Hyperbolic Tangent was used, while Linear was used in Output Layer. In order to attain an optimal network structure, 500 VaR/ES estimations were generated using: HS500, BHS500, MHS500; RM500-GARCH based on a normal GARCH(p,q) volatility model, RM500-GARCHt based on a Student's t GARCH(p,q) volatility model, RM500-TARCH based on a Threshold GARCH (TARCH) volatility model, FHS500 and VaR/ES-EVT500.

The HS500 model was chosen considering the inherent characteristics of RNN. Given that RNN is model-free, it seems logical to perform VaR/ES estimations using the standard HS model. Considering that the HS method encounters difficulties in generating VaR for extremely high confidence levels due to the limited number of extreme values falling into the tail distribution after applying the Directive-defined sample size, VaR/ES estimations were generated using two additional non-parametric models that directly extend the HS model and mitigate the limitation of a small number of data points in the tail distribution: BHS500 and MHS500. The VaR/ES estimates using BHS500 are obtained as the average values of VaR estimations for the insurance portfolio returns, derived from (M) simulated return series using the Bootstrap method, following the procedure outlined by Radivojevic et al. (2017).

Given the numerous empirical research studies related to the validity of applying non-parametric models in emerging markets (Radivojevic *et al.,* 2019, 2020) the other hand, and the theoretical advantages of parametric models in capturing specific characteristics of these markets such as heteroskedasticity and leptokurtosis (volatility of clusters and leptokurtosis in the data), VaR/ES estimates have been generated as previously highlighted by employing three parametric VaR/ES models. RM500-GARCH was chosen because empirical studies by Radivojevic *et al.* (2017) demonstrate that incorporating a normal GARCH(p,q) model into the RM methodology contributes to improving the applicability of this model in emerging markets.

The authors Rossignola *et al.* (2013) emphasize that the assumption about the distribution of returns is more important than the specification of the volatility model itself. Therefore, for markets characterized by volatility clusters and heavy tails, it is considered more appropriate to use a volatility model based on the Student's t distribution rather than assuming normal distribution. For this reason, the Student's t GARCH(p,q) model was used, and the VaR/ES estimates were made using the RM500-GARCHt model. Furthermore, the findings from various studies (Radivojevic *et al.*, 2020), Rossignolo et al. (2013), etc., indicate that in most emerging markets, negative innovations have a greater impact on market volatility than positive innovations. Therefore, the Threshold GARCH (TARCH) model was used in this study.

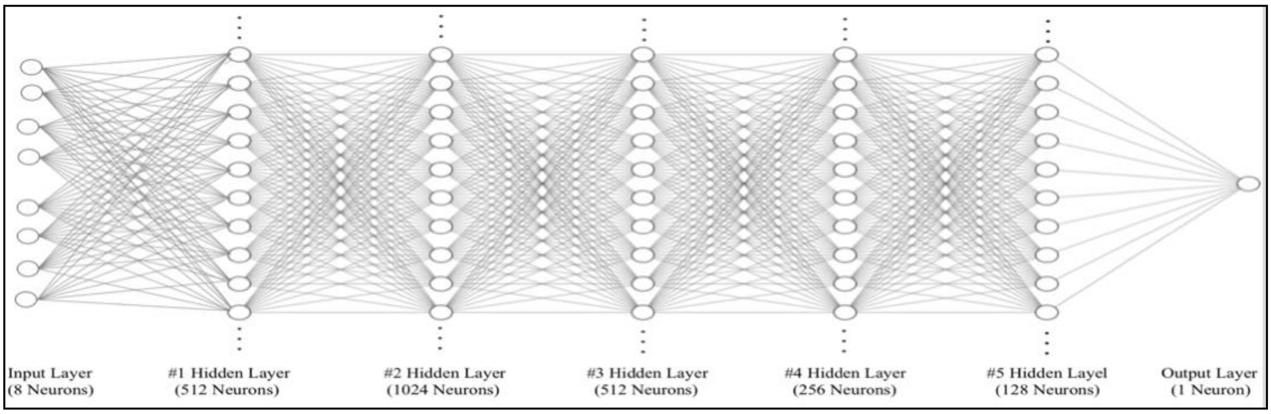

Figure 1. VaR/ES-GRU-DL Architecture

Since semi-parametric models represent a compromise between the advantages and disadvantages of the two previous groups of models, their use in developing the VaR/ES-GRU-DL model seems justified, particularly FHS model, which has shown remarkable performance when applied in emerging markets (Zikovic & Randall, 2013). For this reason, the FH500 model was employed in this study. The semi-parametric model that has also demonstrated excellent performance in emerging markets is VaR/ES-EVT500. The model is capable of addressing leptokurtosis, asymmetry, autocorrelation, and heteroscedasticity, and was developed by Radivojevic *et al.* (2016). For this reason, the VaR/ES-EVT500 model was utilized in this study.

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It is important to note that in the case of both semi/parametric models, the conditional volatility estimates used in the VaR calculation were obtained by applying different conditional volatility models and that models with the ability to capture asymmetries, such as Taylor/Schwert Generalized ARCH (GARCH), Exponential GARCH (EGARCH), The Glosten-Jagannathan-Runkle GARCH (GJR-GARCH) and Asymmetric Power GARCH model (APARCH). The selection of the most likely volatility model was performed using the Log-likelihood information criterion. Also, in choosing the model, we followed the findings of Radivojevic et al. (2019, 2020) and Rossignolo *et al.* (2013), stating that the assumption of innovation distribution is more important for emerging markets than the specification of volatility models. Thus, volatility models are based on assumptions that best match the characteristics of the individual markets from the sample. Four types of distributions are considered: normal, student t, GED, Skewed T, and Skewed GED.

VaR/ES estimates were made for the period from May 2020 to May 2022. 70% of the collected data is allocated to the training set, which is used for the purposes of model selection, while 20% is allocated to the validation set, and 10% to the testing set. The validation set was used to avoid network overfitting. The order of the data is preserved due to the presence of temporal dependencies between observations, and they are chronologically entered into the model to further avoid any bias during the training process. Data were included in the validation process only after the training was completed. This was intended to prevent any leakage of information into the network and to keep the validation process unbiased. To minimize the problem of RNNs' excessive sensitivity to the issue of exploding gradients, where there is a significant difference in the magnitude of weight adjustments between different time steps, the data were normalized.

The networks were trained and tested on 8 insurance companies' optimal investment portfolios, which operate in the countries of the Western Balkans, and that one optimal structure was obtained and later used to predict the market. Backpropagation Through Time (BPTT) has been selected as the learning algorithm because BPTT helps in adjusting the network's weights based on the error that is propagated backwards through time steps. Hence, in combination with the GRU architecture, it enables efficient learning and solving the problem of vanishing gradients. Adam was chosen because it dynamically adjusts the learning rate for each individual network weight, meaning the speed of learning during training (Brownlee, 2017) with a learning rate of 0.1. A batch size of 32 data points was used during training. The model considered historical data up to 90 days using a look-back function. The training process involved 500 epochs to iteratively improve the model's performance. The loss function used in network training and testing is Mean Squared Error (MSE), which values for neural network training and testing for each portfolio are shown in Table 1.

Table 1

Performance Results of Artificial Neural Network

	MSE			**MSE**	
Portfolio	**Training**	**Testing**	**Portfolio**	**Training**	**Testing**
Serbia	0.00038	0.00044	Bosnia and Hercegovina	0.00412	0.00509
Croatia	0.00016	0.00018	North Macedonia	0.00013	0.00013
Slovenia	0.00024	0.00025	Turkey	0.00020	0.00021
Montenegro	0.00029	0.00032	Bulgaria	0.00229	0.00242

Note: To sake of identity protection, the companies are marked with the country of origin.

Source: Authors' own calculation

Discussion

The research was conducted on the example of 8 optimal investment portfolios for insurance companies operating in the Balkans countries: Republic of Serbia, Croatia, Slovenia, Montenegro, Bosnia and Hercegovina (BiH), North Macedonia, Turkey and Bulgaria. These markets were selected because they show a high trend of growth of the insurance market. Namely, although local insurance markets have not yet reached the level of development, in terms of insurance density, in relation to the EU, the growth of insurance premiums has made these markets very popular investment alternatives. The growth rate, the lack of correlation of these markets with the financial markets of developed Western European countries, and the possibility of generating extreme profits, the need to service their multinational clients, attracted foreign insurers to the insurance markets in the Balkans.

The portfolios were formed considering the legal restrictions related to the investment of part of the technical reserves. They were obtained by applying the Markowitz optimization model, with the note that the yield on treasury bills was used as a proxy for the minimum (risk-free) rate of return, since there are no legal restrictions on the investment of technical reserves in government securities. The portfolios were calibrated at the daily level and calculated for the period from 1 January 2020 to 31 December 2023. The first 500 data were used to estimate the calibration of the VaR model, the other 250 to estimate the validity of the VaR model, and the last 250 to test the validity of the VaR/ES-GRU-DL market risk estimates in accordance with Directive II.

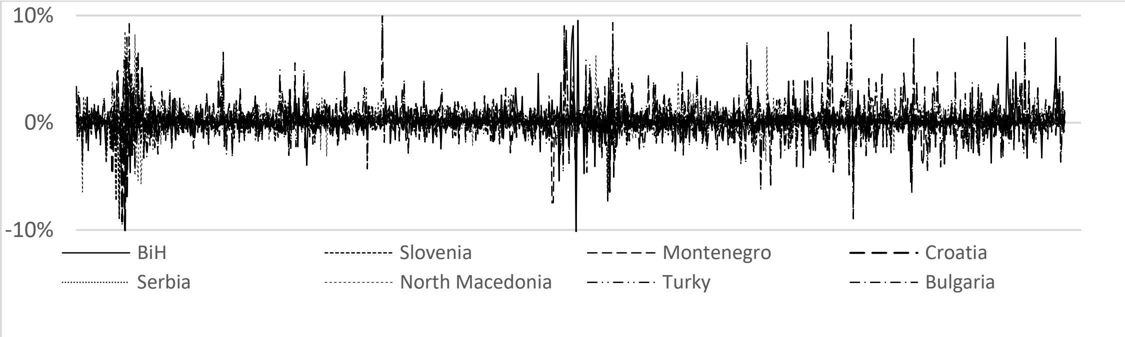

Figure 2. The Movements of the Returns of Optimal Oortfolios

The movements of optimal portfolio returns, shown in Figure 2, display periods of both high and low volatility. This suggests the presence of autocorrelation and heteroscedasticity. The examination of autocorrelation and conditional heteroscedasticity (ARCH effect) in returns is conducted through the Ljung-Box Q' statistic and the Lagrange Multiplier test. The findings are detailed in Table A1 in the appendix, confirming the presence of these characteristics in portfolios within emerging markets. An exception is observed in the Republic of Serbia, where the LM test indicates the absence of the ARCH effect.

Table 2 shows the volatility model parameters used to estimate VaR when applying parametric VaR models, while Table 3 displays the estimates of the volatility model parameters for semi-parametric VaR models. The estimates of the GDP parameters are presented in Table 4. The threshold was set by applying Christoffersen's (2011) rule of thumb for determining the threshold. The appropriate volatility model (structures and assumptions of innovations distribution) was selected based on the Log-likelihood criterion.

Table 2

The Estimates of the Parameters of the Volatility Model for Parametric VaR Models

	Serbia			**Croatia**		
Type of ARCH model	**GARCH(1,1) normal dis.**	**GARCH(1,1) Student's t**	**TARCH(1,1)**	**GARCH(1,1) normal dis.**	**GARCH(1,1) Student's t**	**TARCH(1,1) Skewed T Dis.**
α	0.065	1.999	0.641	0.191	0.157	0.189
β	0.705	0.882	0.374	0.735	0.779	0.771
ω	6.5e-09	0.000	8.4e-09	0.000	3.6e-06	5.9e-06
γ			1.476			0.206
λ						-0.021
η		2.000			3.631	3.487
Log-likelihood	7276.477	8004.76	7463.25	3585.70	3722.77	3718.64
	Slovenia			**Montenegro**		
Type of ARCH model	**GARCH(1,1) normal dis.**	**GARCH(1,1) Student's t**	**GJR GARCH(1,1) GED dis.**	**GARCH(1,2) normal dis.**	**GARCH(1,1) GED**	**TARCH(1,1) Skewed GED dis.**
α	0.234	0.188	0.146	0.300	0.259	0.385
β	0.645	0.726	0.816	0.813	0.538	0.430
ω	9.7e-06	6.8e-06	6.0e-06	3.8e-06 -	2.1e-05	1.8e-05
γ			0.355			-0.291
λ						0.037
η		5.123	1.231		0.577	0.571
Log-likelihood	3397.92	3442.26	3436.23	3588.68	3854.52	3861.27
	BiH			**North Macedonia**		
Type of ARCH model	**GARCH(1,1) normal dis.**	**GARCH(1,1) Student's t**	**TARCH(1,1) normal dist.**	**GARCH(1,1) normal dis.**	**GARCH(1,1) Student's t**	**TARCH(1,1) normal dis.**
α	0.077	-	0.187	0.138	0.180	0.165
β	0.893	-	0.797	0.806	0.695	0.822
ω	3.9e-06	-	1.1e-05	6.4e-06	1.2e-05	6.6e-06
γ		-	-0.714			0.204
η		-			4.005	
Log-likelihood	3245.54		3278.19	3241.14	3331.07	3240.82

	Turkey			Bulgaria		
Type of ARCH model	**GARCH(1,1) normal dis.**	**GARCH(1,1) Student's t dis.**	**TARCH(1,1) Student's t dis.**	**GARCH(1,1) normal dis.**	**GARCH(1,1) Student's t**	**TARCH(1,1) Skewed GED dis.**
α	0.230	0.170	0.171	0.141	0.170	0.178
β	0.601	0.730	0.720	0.762	0.729	0.716
ω	6.8e-05	4.3e-05	5.4e-05	4.1e-05	4.3e-05	5.25036e-05
γ			-0.540			-0.43
λ						0.123
η		4.114	4.169		4.114	1.179
Log-likelihood	2586.21	2648.10	2658.54	2523.23	2648.10	2657.99

Source: Authors' own calculation

Table 3

The Estimates of the Parameters of the Volatility Model for Semi-Parametric VaR Models

	Serbia	Croatia	Slovenia	Montenegro
Type of ARCH model	**Taylor/Schwert's GARCH(1,1) normal dist.**	**APARCH(1,1) normal dis.**	**Taylor/Schwert's GARCH(1,1) Skewed T**	**APARCH(1,1) GED dis.**
α	0.0399	0.095	0.187	0.365
β	0.898	0.759	0.772	0.472
ω	0.000	3.3E0-06	7.40E-06	1.60E-05
γ		0.212		-0.273
λ			-0.097	
δ		2.911		1.014
η			5.026	0.578
Log- likelihood	7290.71	3595.07	3437.04	3858.95
	BiH	**North Macedonia**	**Turkey**	**Bulgaria**
Type of ARCH model	**GJR GARCH(1,1) GED**	**APARCH(1,1) normal dis.**	**EGARCH(1,1) Student's t dis.**	**GJR(1,1) Skewed GED dis.**
α	0.149	0.157	0.281	0.2
β	0.838	0.811	0.858	0.607
ω	5.10E-06	6.63E-06	-1.341	6.40E-05
γ	-0.242	0.148	0.129	-0.271
λ				0.123
δ	5.1e-06	1.555		6.493
η	0.428		4.161	1.157
Log- likelihood	3815.9		2656.56	2653.24

Source: Authors' own calculation

Table 4

The Estimates of the GPD Parameters

Parametars	**Serbia**	**Croatia**	**Slovenia**	**Montenegro**
u	-1.757	-0.132	-0.174	-0.119
ξ	0.168	0.621	0.407	0.431
sigma	4.795	0.238	0.17	0.157
Parametars	**BiH**	**North Macedonia**	**Turkey**	**Bulgaria**
u	-0.123	-0.135	-0.163	-0.181
ξ	0.554	0.504	0.336	0.375
sigma	0.164	0.119	0.076	0.113

Source: Authors' own calculation

Backtesting Result

The obtained VaR/ES estimates were compared with the estimates obtained using the VaR/ES models used in this study to answer the question of whether the VaR/ES-GRU-DL model produces better estimates compared to traditionally widely used models. For this purpose, the ASMF (average squared magnitude function) was used. Before this comparison was conducted, testing of the model's validity itself was performed. For this purpose, the Kupiec unconditional model (LR_{uc}) and Christoffersen conditional coverage model (LR_{cc}) were used. The results of both tests are displayed in Table 5.

Table 5

The Validity test's Results

		Serbia	Croatia	Slovenia	Montenegro	BiH	North Macedonia	Turkey	Bulgaria
No. VaR breaks		1	4	4	1	3	3	1	2
Cluster VaR breaks		0	0	0	0	0	0	0	0
LR_u	Cr. value	0.048	3.836	3.836	0.054	1.765	1.765	0.054	0.382
	p-value	0.826	0.050	0.050	0.816	0.184	0.184	0.816	0.536
LR_{cc}	Cr. value	0.050	3.836	3.836	0.055	1.765	1.765	0.055	0.382
	p-value	0.976	0.147	0.147	0.973	0.414	0.414	0.973	0.826
$MC\text{-}LR_{uc}$		0.451	0.106	0.116	0.209	0.371	0.212	0.414	0.378
$MC\text{-}LR_{cc}$		0.388	0.183	0.098	0.127	0.322	0.264	0.401	0.276
No. ES breaks		0	3	2	0	1	0	0	2
Cluster ES breaks		0	0	0	0	0	0	0	0
LR_{BT}		n/a	0.084	0.129	0.251	0.096	n/a	n/a	0.101
Bootstrap-LR_{BT}		0.332	0.096	0.210	0.173	0.106	0.274	0.377	0.142
VaR estimations									
$ASMF_{VaR\text{-}GRU\text{-}DL}$		0.052	0.366	0.206	0.421	0.531	0.670	0.819	0.732
$ASMF_{HS}$		0.100	0.560	0.9043	0.453	0.699	0.712	0.900	0.802
$ASMF_{BHS}$		0.181	0.445	0.976	0.491	0.605	0.801	0.832	0.811
$ASMF_{MHS}$		0.190	0.479	0.915	0.498	0.712	0.722	0.882	0.890
$ASMF_{RM\text{-}Garch}$		0.105	0.662	0.842	0.472	0.672	0.715	0.915	0.911
$ASMF_{RM\text{-}S(t)Garch}$		0.103	0.324	0.720	0.066	-	0.829	0.899	0.929
$ASMF_{RM\text{-}Tarch}$		0.097	0.448	0.619	0.438	0.698	0.788	0.901	0.898
$ASMF_{FHS}$		0.057	0.354	0.500	0.449	0.593	0.694	0.994	0.794
$ASMF_{EVT}$		0.059	0.377	0.234	0.279	0.601	0.700	0.892	0.773
ES estimations									
$ASMF_{ES\text{-}GRU\text{-}DL}$		0.278	0.417	0.431	0.333	0.271	0.098	0.187	0.223
$ASMF_{HS}$		0.715	0.548	0.962	0.680	0.664	0.579	0.821	0.548
$ASMF_{BHS}$		0.695	0.665	0.783	0.591	0.417	0.582	0.491	0.541
$ASMF_{MHS}$		0.751	0.713	0.866	0.668	0.420	0.720	0.827	0.633
$ASMF_{RM\text{-}Garch}$		0.530	0.782	0.745	0.371	0.424	0.458	0.703	0.695
$ASMF_{RM\text{-}S(t)Garch}$		0.544	0.706	0.766	0.382	-	0.465	0.614	0.455
$ASMF_{RM\text{-}Tarch}$		0.597	0.590	0.863	0.431	0.504	0.477	0.591	0.363
$ASMF_{FHS}$		0.368	0.495	0.591	0.419	0.368	0.171	0.287	0.326
$ASMF_{EVT}$		0.464	0.571	0.454	0.454	0.276	0.177	0.196	0.248

Source: Authors' own calculation

As seen in Table 5, no cluster VaR exceedances were observed in any instance. The highest number of exceedances, at four, was noted for the portfolio in Croatia and Slovenia, while the lowest number of exceedances, at one each, was observed for the portfolios in Serbia, Montenegro, and Turkey. The results of the LR_{uc} and LR_{cc} tests indicate that the model successfully passed both validity tests. However, both tests' validity is questioned when there are few exceedances or with a limited sample size as outlined in the Directive. Namely, the LR_{uc} test is asymptotically distributed as $\chi 2$ with one degree of freedom under the null hypothesis that the tail probability *(p)* is the true probability. The LR_{cc} test is asymptotically distributed as $\chi 2$ with two degrees of freedom under the null hypothesis that the hit sequence is IID Bernoulli with the mean equal to the VaR coverage rate. Asymptotically, that is as the number of observations, T, goes to infinity, the LR_{uc} test will be distributed as a $\chi 2$ with one degree of freedom. It is the same with the LR_{cc} test. In large enough samples, the LR_{cc} test will be distributed as a $\chi 2$ with two degree of freedom. Radivojevic *et al.* (2016) have shown that when the number of VaR breaks is small, there are substantial differences between asymptotic probability distributions of the considered tests and their finite sample analogues. Therefore they point out that in case of a small sample size, (as in sample size defined by Directive), i.e. in case of a small number of VaR breaks (T_1), which are the informative observations, it is better to rely on Monte Carlo simulated p-values rather than on those from the $\chi 2$ distribution. To address this, the model's VaR estimation validity was further examined in the study through Monte Carlo simulations. This involved generating 9999 samples of random IID Bernoulli (p) variables, matching the actual sample size. Following this, 9999 simulated LR_{uc} and LR_{cc} tests were computed based on these artificial samples. Finally, simulated p-values were calculated as test values greater than the actual test value using the following expression:

$$p - value = \frac{1}{10000}\left\{1 + \sum_{i=1}^{9999} I(LR\widetilde{(\iota) > LR}\right\} \quad (1)$$

where I(.) takes the value of 1 if the argument is true and 0 otherwise.

Based on the results presented in Table 5 cannot dispute the validity of the model, in terms of the backtesting rules. Particularly good results are gained in meeting LR_{uc}. The explanation lies in the fact that the approach is designed in such a way that it can perfectly capture the dynamics in the series of return. VaR is not a coherent risk measure. For this reason, banking supervisors have suggested using ES to assess market risk. Although this measure is not yet mandatory in the insurance sector, its advantages in relation to VaR make it a good choice also when it comes to the insurance sector. As a result, the entire model development process has been repeated, noting that the ES estimates were obtained using the model proposed by Artzner *et al.* (1999):

$$ES_{cl} = -E[r|r \leq -VaR_{cl}] \quad (2).$$

Although Acerbi and Tasche (2002) presented a more rigorous measure of risk with Expected Shortfall (ES) compared to the measure obtained using expression (2), it has been chosen to use Artzner's version. This decision is based on the fact that when applied to continuous distributions, which is a valid distribution in the case of assessing the market risk of an investment portfolio, both measures yield the same risk estimates

Just like in the case of VaR, no cluster of ES breaks were recorded. The model demonstrates somewhat weaker results for the investment portfolio of the insurance company operating in Croatia. Unlike testing the validity of VaR, testing the validity of ES is much more complex, and there is no consensus on the best test. Since Berkowitz's ES test (LR_{BT}) is widely used, it was utilized in the study. Like the case with VaR coverage tests, this test is also based on asymptotic assumptions, and therefore, it is necessary to validate the results of this test. In the research, Berkowitz's ES validation is done using a method outlined by Radivojevic *et al.* (2019). This method involves using simulations to estimate the unknown distribution F of the estimator θˆ. The density of ES estimates from F is approximated by running simulations of the model multiple times. The number of bootstrap repetitions is determined based on the Andrews and Buchinsky procedure (1997). The determination of the number of the bootstrap repetitions is particularly important in this case because the sample of the breaks utilized in obtaining a single ES estimate is a small fraction of the number of draws. The procedure for calculating the p-value is then continued by analogy, as previously described. The results of this test, as well as Berkowitz's test, are also presented in Table 5. The test results indicate that the model can be reliably used for assessing ES. It shows slightly weaker results for the investment portfolio of the company operating in Croatia.

The analysis of the ASMF results shows that the model generated better VaR estimates than all other models. The only exception is in the case of the insurance company operating in Croatia and Montenegro. In the first scenario, the FHS model generated a lower ASMF value compared to the VaR-GRU-DL model, while in the second scenario, the EVT model generated a lower ASMF value compared to the VaR-GRU-DL model. According to the ASMF criterion, after the VaR-GRU-DL model, the FHS and EVT models follow, which, in the case of the same number of portfolios, generated the lowest and second lowest ASMF value. Furthermore, analysis of the ASMF values implies that parametric models produce better estimates compared to non-parametric models, and BHS generates the best estimates compared to the other two non-parametric models. In the case of ES, the analysis of the ASMF results significantly favors the ES-GRU-DL model, as this model consistently generated the lowest ASMF values across all scenarios. The findings for the other models are the same as in the VaR estimation case.

Conclusion

The paper developed a new model for evaluating the market risk of optimal portfolios of insurance companies in accordance with the Solvency II Directive. The model is based on Deep Learning. More precisely, the model is based on the GRU-ANN model using VaR and ES estimates derived from eight widely utilized VaR models. These models include the standard Historical Simulation, Bootstrap, Mirrored Historical Simulation, RiskMetrics using normal GARCH(p,q) and Student's t GARCH(p,q) volatility models, RiskMetrics with a Threshold GARCH (TARCH) volatility model, Filtered Historical Simulation, and the VaR/ES model based on Extreme Value Theory (VaR/ES-EVT500).

The validity of the model was tested for VaR and ES estimates. For this purpose, both unconditional and conditional coverage models were used, as well as the Berkowitz test. Since these tests were developed based on asymptotic assumptions that predict infinite samples, the results of these tests were subjected to validation. For this purpose, the DuFou Monte Carlo procedure was used, as well as the bootstrap procedure proposed by Radivojevic and colleagues. The test results indicate that the model can reliably be used for market risk management of optimal investment portfolios of insurance companies operating in emerging markets, such as the markets of the Balkan countries.

The research was conducted using the example of 8 optimal portfolios of insurance companies operating in the Republic of Serbia, Croatia, Slovenia, Montenegro, Bosnia and Herzegovina, North Macedonia, Turkey, and Bulgaria. The research covers the period from January 2020 to December 2023. The optimal portfolios were formed taking into account the regulations in these countries.

To determine whether the model generates better VaR and ES estimates compared to widely used and popular VaR models, a comparison of the model with the abovementioned models was carried out using the average squared magnitude function. The analysis results of the average squared magnitude function show that the model generates the smallest values of the average squared magnitude function for the estimation of both VaR and ES in all portfolios, except for two cases where smaller values were generated by the FHS and EVT models in the estimation of VaR, specifically in the case of the optimal portfolio of an insurance company operating in Croatia, and in Montenegro.

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Annexes

Table A1

Results of the Ljung-Box Q' Statistics and of the Presence of ARCH Effects

Name	Q' statistics	p-value	LM	p-value
Serbia	1187.8	3.1e-239	0.002	0.963
Croatia	155.51	5.4e-23	100.60	1.1e-23
Slovenia	71.415	1.1e-07	44.658	2.3e-11
Montenegro	70.181	1.7e-07	162.079	3.9e-37
Bosnia and Hercegovina	69.432	2.2e-07	22.468	2.1e-06
North Macedonia	37.753	0.009	40.516	1.9e-10
Turkey	31.109	0.054	5.563	0.018
Bulgaria	28.419	0.099	5.899	0.015

Source: Authors' own calculation

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