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Autoregressive Conditional Skewness, Kurtosis and Jarque-bera in Lithuanian Stock Market Measurement

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Most of the statistical tools are designed to model the conditional mean of a random variable. The tools described in this article have a rather different purpose – to model the conditional variance, or volatility of a variable.

There are several reasons to model and forecast volatility. First, we need to analyze the risk of holding an asset or the value of an option. Second, forecast confidence intervals may be time-varying, so that more accurate intervals can be obtained by modeling the variance of the errors. Third, more efficient estimators can be obtained if heteroskedasticity in the errors is handled properly.

Autoregressive Conditional Heteroskedasticity (ARCH) models are specifically designed to model and forecast conditional variances. The variance of the dependent variable is modeled as a function of past values of the dependent variable and independent, or exogenous variables.

These models are widely used in various branches of econometrics, especially in financial time series analysis. This article analyses GARCH and ARCH models' two main characteristics: skewness and kurtosis.

Keywords: *asymmetry, autoregressive, conditional, distribution, Jarque-Bera, kurtosis, skewness.*

Introduction

There have been many papers studying the departures from normality of asset return distributions. It is well known that stock return distributions exhibit negative skewness and excess kurtosis (Harvey and Siddique, 1999; Peiró, 1999; and Premaratne and Bera, 2001). Specifically, excess kurtosis (the fourth moment of the distribution) makes extreme observations more likely than in the normal case, which means that the market gives higher probability to extreme observations than in normal distribution. However, the presence of negative skewness (the third moment of the distribution) has the effect of accentuating the left-hand side of the distribution. That is, the market gives higher probability to decreases than increases in asset pricing. (Leon, 2003)

The generalized autorregresive conditional heteroscedasticity (GARCH) models, introduced by Engle (1982) and Bollerslev (1986), allow for time-varying volatility (but not for time-varying skewness or kurtosis). Harvey and Siddique (1999) present a way to jointly estimate time-varying conditional variance and skewness under a non-central t distribution for the error term in the mean equation. Their methodology is applied to several series of stock index returns, and it is found that autorregresive conditional skewness is significant and that the inclusion of skewness affects the persistence in variance. It is important to point out that the paper by Harvey and Siddique (1999) allows for timevarying skewness but still assumes constant kurtosis. (Alexander, 2004)

Premaratne and Bera (2001) have suggested capturing asymmetry and excess kurtosis with the Pearson type IV distribution, which has three parameters that can be interpreted as volatility, skewness and kurtosis. This is an approximation to the non-central t distribution proposed by Pearson and Merrington (1958). However, these authors use time-varying conditional mean and variance, but maintain constant skewness and kurtosis over time. Similarly, Jondeau and Rockinger (2000) employ a conditional generalized Student-t distribution to capture conditional skewness and kurtosis by imposing a time-varying structure for the two parameters which control the probability mass in the assumed distribution1. However, these parameters do not follow a GARCH structure for either skewness or kurtosis. (Leon, 2003)

The purpose of this article is to forecast Lithuanian stock market indexes' VILSE and LITIN 10 trends with the help of skewness, kurtosis and Jarque-Bera test.

The object of this research is Lithuanian stock market.

The main methods of research are synthesis and analysis of literature, statistical and mathematical forecast methods, graphic analysis.

Autoregressive conditional skewness

Skewness, asymmetry in distribution, is found in many important economic variables such as stock index returns and exchange rate changes. Negative skewness in returns can be viewed as the phenomenon where, after the returns have been standardized by subtracting the mean, negative returns of a given magnitude have higher probabilities than positive returns of the same magnitude or vice-versa. This can be measured through the third moment about the mean. (Campbell, 2002) Skewness is a measure of asymmetry of the distribution of the series around its mean.

Skewness is a parameter that describes asymmetry in a random variable's probability distribution. Both probability density functions (PDFs) in Figure 1 have the same mean and standard deviation. The one on the left is positively skewed. The one on the right is negatively skewed. (Lambert, 2002)

The skewness of a symmetric distribution, such as the normal distribution, is zero. Positive skewness means that the distribution has a long right tail and negative skewness implies that the distribution has a long left tail. (Lanne, 2004)

Most often, the median is used as a measure of central tendency when data sets are skewed. The metric that indicates the degree of asymmetry is called, simply, skewness. Skewness often results in situations when a natural boundary is present. Typically, the skewness value will range from negative 3 to positive 3. (Huzaifah, 2002)

The Pearson mode skewness is defined by

$$\frac{[\text{mean}] - [\text{mode}]}{\sigma} \tag{1}$$

Pearson’s skewness coefficients are defined by

$$\frac{3[\text{mean}] - [\text{mode}]}{s} \tag{2}$$

The second moment of returns, variance, has been the subject of a large literature in finance. Variance of returns has been widely used as a proxy for risk in financial returns. (Campbell, 2004)

Therefore, the properties of variance by itself as well as the relation between expected return and variance have been important topics in asset pricing. Campbell (1987), Harvey (1989), Nelson (1991), Campbell and Hentschel (1992), Hentschel (1995), Glosten, Jagannathan, and Runkle (1993), and Wu (1998) have focused on the intertemporal relation between return and risk where risk is measured in the form of variance or covariance. An important concern has been the sign and magnitude of this tradeoff. (Campbell, 2002)

The generalized autoregressive conditional heteroskedasticity (GARCH) class of models, including the exponential GARCH (EGARCH) specification, have been the most widely used models in modeling time-series variation in conditional variance. Persistence and asymmetry in variance are two stylized facts that have emerged from the models of conditional volatility. Persistence refers to the tendency where high conditional variance is followed by high conditional variance. Asymmetry in variance, i.e., the observation that conditional variance depends on the sign of the innovation to the conditional mean has been documented in asymmetric variance models used in Nelson (1991), Glosten, Jagannathan, and Runkle (1993) and Engle and Ng (1993). These studies find that conditional variance and innovations have an inverse relation: conditional variance increases if the innovation in the mean is negative and decreases if the innovation is positive. (Campbell, 2004)

In contrast, skewness, the third moment, has drawn far less scrutiny in empirical asset pricing, though skewness in financial markets appears to vary through time and also appears to possess systematic relation to expected returns and variance. The time-series variation in skewness can be viewed as analogous to heteroskedasticity.

A distribution is skewed if one of its tails is longer than the other. The first distribution shown in Figure 1 has a positive skew. This means that it has a long tail in the positive direction.

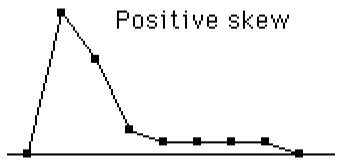

Figure 1. Positive skew

The distribution in Figure 2 has a negative skew since it has a long tail in the negative direction. Finally, the third distribution is symmetric and has no skew.

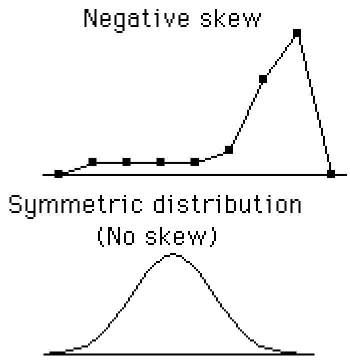

Figure 2. Negative skew and symmetric distribution

Distributions with positive skew are more common than distributions with negative skews. One example is the distribution of income. Most people make under 12000 LTL a year, but some make quite a bit more with a small number making many millions of litas per year. The positive tail therefore extends out quite a long way whereas the negative tail stops at zero.

For a more psychological example, a distribution with a positive skew typically results if the time it takes to make a response is measured. The longest response times are usually much longer than typical response times whereas the shortest response times are seldom much less than the typical response time. A histogram of the author's performance on a perceptual motor task in which the goal is to move the mouse to and click on a small target as quickly as possible is shown below. The X axis shows times in milliseconds.

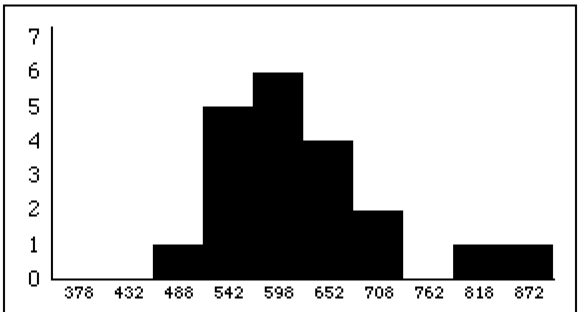

Figure 3. Negatively skewed distribution

Negatively skewed distributions do occur, however. Consider the following frequency polygon of test grades on a statistics test where most students did very well but a few did poorly. It has a large negative skew.

When the distribution has a positive skew then the mean is larger than the median.

When the distribution has a negative skew, then the mean in smaller than the median.

Autoregressive conditional kurtosis and Jarque-Bera

Kurtosis is the degree of peakedness of a distribution, defined as a normalized form of the fourth central moment μ_4 of a distribution. There are several flavors of kurtosis commonly encountered, including the kurtosis proper, denoted β_2 and defined by

$$\beta_2 \equiv \frac{\mu_4}{\mu_2^2} \tag{3}$$

where μ_i denotes the ith central moment (and in particular, μ_2 is the variance). The kurtosis "excess" is denoted γ_2 or b_2, and is defined by

$$\gamma_2 \equiv \frac{\mu_4}{\mu_2^2} - 3 \tag{4}$$

Kurtosis excess is commonly used because γ_2 of a normal distribution is equal to 0, while the kurtosis proper is equal to 3. (Rockinger, 2000)

Unfortunately, Abramowitz and Stegun (1972) confusingly refer to β_2 as the "excess or kurtosis."

A distribution with a high peak $(\gamma_2 > 0)$ is called leptokurtic, a flat-topped curve $(\gamma_2 < 0)$ is called platykurtic, and the normal distribution $(\gamma_2 = 0)$ is called mesokurtic.

An estimator $g_2 = (\gamma_2)$ for the kurtosis excess γ_2 is given by

$$g_2 = \frac{k_4}{k_2^2} \tag{5}$$

where the k_s are *k*-statistic. For a normal distribution, the variance of this estimator is

$$\mathrm{var}(\hat{g}_2) \approx \frac{24}{N} \tag{6}$$

Kurtosis measures the peakedness or flatness of the distribution of the series. Kurtosis is a parameter that describes the shape of a random variable's probability distribution. Consider the two probability density functions (PDFs) in Figure 4. (Leon, 2003)

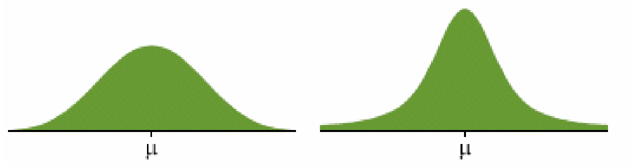

Figure 4. Kurtosis

Which would you say has the greater standard deviation? It is impossible to say. The distribution on the right is more peaked at the center, which might lead us to believe that it has a lower standard deviation. It has fatter tails, which might lead us to believe that it has a higher standard deviation. If the effect of the peakedness exactly offsets that of the fat tails, the two distributions will have the same standard deviation. The different shapes of the two distributions illustrate kurtosis. The distribution on the right has a greater kurtosis than the distribution on the left.

The kurtosis of a random variable *X* can be denoted η_2 or *kurt*(*X*). It is defined as

$$kurt(X) = \frac{E\left[(X-\mu)^4\right]}{\sigma^4} \tag{7}$$

where μ and σ are the mean and standard deviation of *X*.

Leptokurtosis is associated with distributions that are simultaneously "peaked" and have "fat tails." Platykurtosis is associated with distributions that are simultaneously less peaked and have thinner tails. In Figure 5, the distribution on the left is platykurtic. The one on the right is leptokurtic.

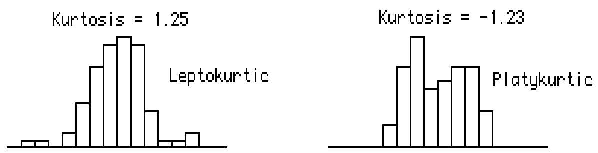

Figure 5. Kinds of kurtosis

The following two distributions have the same variance, approximately the same skew, but differ markedly in kurtosis.

Jarque-Bera is a test statistic for testing whether the series is normally distributed. The test statistic measures the difference of the skewness and kurtosis of the series with those from the normal distribution. The statistic is computed as:

$$JB = \frac{N-k}{6}\left(S^2 + \frac{1}{4}(K-3)^2\right) \tag{8}$$

where S is the skewness, K is the kurtosis, and k represents the number of estimated coefficients used to create the series. (Lawford, 2004)

Under the null hypothesis of a normal distribution, the Jarque-Bera statistic is distributed as with 2 degrees of freedom. The reported probability is the probability that a Jarque-Bera statistic exceeds (in absolute value) the observed value under the null–a small probability value leads to the rejection of the null hypothesis of a normal distribution.

Jarque-Bera test, originally devised for constant conditional variance models with no functional dependence between conditional mean and variance parameters, can be safely applied to a broad class of GARCH-M models, but not to all. (Yi-Ting, 2004)

The JB test was formally derived as a LagrangeMultiplier (LM) test of normality of the regression residuals versus the alternative that the (conditional) distribution of ξ_t belongs to the Pearson family. A closely related test was proposed by Kiefer and Salmon (1983) (KS), who developed an LMtest for normality against a Hermite polynomial expansion of the (conditional) density of ξ_t. (Yi-Ting, 2004)

Practical appliance of autoregressive conditional skewness, kurtosis and Jarque-Bera to Lithuanian stock market

For the practical analysis are taken two Lithuanian stock market's indexes VILSE and LITIN 10. The period of analysis is from 2004-01-02 to 2005-01-20.

Both indexes in the analysing period have an uptrend that's mean the situation in Lithuanian stock market is quite good.

With the help of three estimators skewness, kurtosis and Jarque-Bera we will try to estimate the stock market tendency.

In the 6 th Figure are show all statistic parametres of VILSE index. For the analysis were take 275 observations. Mean of time series is 225.978, median 216.54, maximum 321.64 and minimum 174.82, standard deviation 31.94459. Skewness of index VILSE is 1.383749 so it is positive and the mean is larger then the median, and there are right-skewed distribution. Kurtosis is 4.299084. Jarque-Bera is 107.0972. So we can forecast an uptrend.

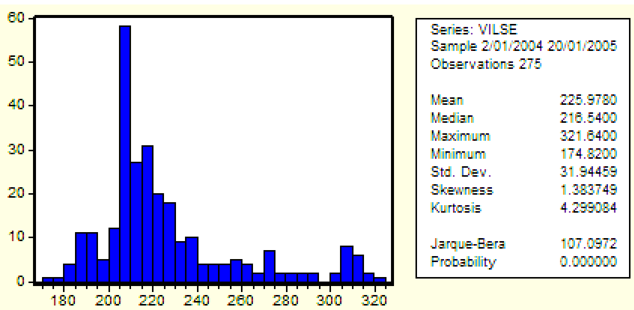

Figure 6. Descriptive statistics of VILSE

In the 7 th Figure are show all statistic parametres of LITIN 10 index. For the analysis were take also 275 observations. Mean of time series is 2765.745, median 2695.840, maximum 3510.120 and minimum 2344.78. Standard deviation 244.1560. Skewness of index LITIN 10 is 1.374076 so it is positive. The mean is larger then the median and there are right-skewed distribution. Kurtosis is 4.383886. Jarque-Bera is 108.4815. The value of index in the nearly future will rise.

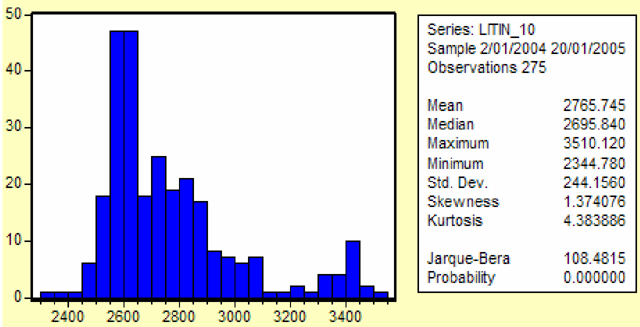

Figure 7. Descriptive statistics of LITIN 10

The most interesting thing is when the return distribution has a skewness lower than -1 and an excess kurtosis higher than 1. In this case, the probability to have sudden high negative returns increases. For a distribution with a skewness of -1 and an excess kurtosis of 5 (technology stocks, media stocks, telecom stocks or hedge funds have this kind of excess kurtosis level), a mean-variance approach will conclude that the investor will not lose more than -3.5% in the next 1 day with 99% probability. An approach, which accounts for skewness and kurtosis, gives -7.4% loss in the next 1 day with 99% probability, which is exactly what one observes on the equity market. The difference is huge: more than 100% of risk underestimation with the use of the mean-variance.

Conclusions

1. Skewness is a measure of the asymmetry of the probability distribution of a real-valued random variable. Roughly speaking, a distribution has positive skew if the positive tail is longer and negative skew if the negative tail is longer. When skewness is negative, the market will have a downtrend and when it is possitive there will be an uptrend.
2. Kurtosis is a measure of the "peakedness" of the probability distribution of a real-valued random variable. Higher kurtosis means more of the variance is due to infrequent extreme deviations, as opposed to frequent modestly-sized deviations. Kurtosis can be both positive or negative. Random variables that have a negative kurtosis are called subgaussian, and those with positive kurtosis are called supergaussian. In statistical literature, the corresponding expressions platykurtic and leptokurtic are also used. Supergaussian random variables have typically a ``spiky" PDF with heavy tails, i.e. the PDF is relatively large at zero and at large values of the variable, while being small for intermediate values. Normal distributions have a kurtosis of 3 (irrespective of their mean or standard deviation). If a distribution's kurtosis is greater than 3, it is said to be leptokurtic. If its kurtosis is less than 3, it is said to be platykurtic.
3. The Jarque-Bera Lagrange multiplier test is perhaps the most commonly used procedure for testing whether a univariate sample of t datapoints, or estimated regression residuals, are drawn from a normal distribution. It is a joint test of the null hy-

pothesis (of normality) that sample skewness equals 0 and sample kurtosis equals 3, and the null is rejected.

4. In this article three statistic characteristics were applied to Lithuanian stock market and all the results were possitive and showed uptrends to values of VILSE and LITIN 10 indexes.

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Autoregresinio sąlyginio asimetriškumo, eksceso ir Jarque-Bera testavimo modelio naudojimas vertinant Lietuvos akcijų rinką

Santrauka

Daugelis statistikos priemonių yra sukurtos atsitiktiniams kintamųjų sąlyginiams vidurkiams modeliuoti. Priemonės, aprašytos šiame straipsnyje, turi kitą paskirtį, t.y. modeliuoti sąlyginę dispersiją (kintamumą).

Yra išskiriamos dvi pagrindinės priežastys, dėl ko modeliuojamas ir prognozuojamas kintamumas. Pirmiausia – tikslinga analizuoti turimo turto riziką ar opcionų vertę. Antra – prognozuojami skirtingų laikotarpių tikimybiniai intervalai; norint sumodeliuoti tikslesnius tikimybinius intervalus, tikslinga modeliuoti paklaidų nuokrypius: šie įverčiai tikslesni tuomet, kai paklaidų modeliavimui tinkamai naudojamas heteroskedastiškumas.

Parengta daug darbų, nagrinėjančių turto grąžos normaliojo pasiskirstymo nuokrypius. Paskelbtuose straipsiuose įrodyta, kad akcijų grąžos pasiskirstymas rodo neigiamą asimetriškumą ir ekscesą (Harvey, Siddique, 1999; Peiro, 1999; Premaratne, Bera, 2001).

Apibendrinti autoregresiniai sąlyginiai heteroskedastiškumo (GARCH) modeliai, įskaitant eksponentinio GARCH (EGARCH) specifikaciją, yra plačiai naudojami modeliai, taikomi laiko eilutėms su sąlygine dispersija modeliuoti. Šiame straipsnyje aptariami keli šių modelių stilizuoti faktai, t.y. asimetrijos koeficientas ir ekscesas.

Šio straipsnio tikslas – pritaikyti asimetriškumo, eksceso ir Jarque-Bera testavimą Lietuvos akcijų rinkos LITIN 10 ir VILSE indeksams. Tyrimo objektas – Lietuvos akcijų rinka. Pagrindiniai tyrimo metodai yra literatūros sintezė ir analizė, statistiniai ir matematiniai prognozavimo modeliai, grafinė analizė.

Asimetrijos koeficientas naudojamas įvairiems svarbiems ekonomikos kintamiesiems, tokiems kaip akcijų indeksų investicijų grąžoms ir užsienio valiutos kursų pokyčiams įvertinti. Šis koeficientas įvertina skirstinio asimetriją. Normaliojo skirstinio asimetrijos koeficientas lygus 0. Teigiamas asimetrijos koeficientas reiškia, kad skirstinys turi ilgą dešinę uodegą, o neigiama koeficiento reikšmė rodo, kad skirstinys turi ilgą kairiąją uodegą. Asimetrijos koeficiento reikšmės paprastai svyruoja nuo -3 iki 3.

Skirstinys yra asimetriškas, jei vienas iš jo galų yra ilgesnis už kitą. Kai skirstinys turi teigiamą asimetrijos koeficientą, laiko eilutės vidurkis yra didesnis už medianą. Ir atvirkščiai, jei skirstinys turi neigiamą asimetrijos koeficientą, laiko eilutės vidurkis yra mažesnis nei mediana.

Dažniausiai mediana, kai duomenų rinkiniai yra asimetriški, naudojama kaip centrinės tendencijos įvertis. Kaip anksčiau minėta, asimetriškumą parodo asimetrijos koeficientas, kuris svyruoja nuo neigiamos 3 iki teigiamos 3 reikšmės (Huzaifah, 2002).

Variacijos savybės, taip pat laukiamo pelningumo ir variacijos ryšys yra pagrindiniai klausimai, nagrinėjami turto įkainojimo kontekste. Mokslininkai Campbell (1987), Harvey (1989), Nelson (1991), Campbell ir Hentschel (1992), Hentschel (1995), Glosten, Jagannathan ir Runkle (1993) bei Wu (1998) tyrimų laukas susitelkęs prie sąryšio tarp rizikos ir pelningumo matavimo variacijos ir kovariacijos formos.

Tęstinumas ir asimetriškumas variacijoje yra du stilizuoti faktai, jungiami nagrinėjant sąlyginio kintamumo modelius. Tęstinumas pasižymi tokia tendencija, kad po aukštos sąlyginės dispersijos eina vėl aukšta sąlyginė dispersija. Asimetriškumas dispersijoje, t.y. stebėjimas, kai sąlyginė dispersija priklauso nuo inovacijų lygio sąlyginiam vidurkiui, yra aprašytas asimetriškos dispersijos modeliuose, kuriuos naudoja Nelson (1991), Glosten, Jagannath ir Runkle (1993) bei Engle ir Ng (1993). Minėtų autorių tyrimai parodė, kad sąlyginė dispersija ir inovacijos turi atvirkštinį ryšį: sąlyginė dispersija didėja, kai vidutinės inovacijos yra neigiamos ir mažėja, jei inovacijos yra teigiamos (Campbell, 2004).

Praktiškai daug dažnesni skirstiniai su teigiamu asimetriškumu nei su neigiamu. Kaip pavyzdį galima pateikti uždirbamų pajamų pasiskirstymą. Daugelis Lietuvos gyventojų per metus uždirba mažiau nei 12 000 Lt, tačiau kai kurių atlyginimas daug didesnis. Pajamų sirstinio asimetriškumas turi gana ilgą teigiamą uodegą, o neigiama uodega turi apsiriboti nuliu, nes neigiamų pajamų mes neuždirbame.

Neigiamo asimetriškumo pavyzdžiu galėtų būti studentų statistikos testo rezultatai, kai daugelis studentų gauna labai gerus įvertinimus, o keletas labai blogus.

Ekscesas – tai koeficientas, kuris vertina eilučių skirstinio smailiaviršūniškumą ar plokšumą, t.y. jis apibūdina skirstinio formą. Normaliojo skirstinio ekscesas yra lygus 3. Jeigu skirstinio ekscesas didesnis už 3, teigiama, jog jis yra didesnis už normalųjį; jei mažesnis, tuomet jis yra mažesnis už normalųjį. Mažesnis už normalųjį ekscesas yra siejamas su skirstiniais, kurie tuo pat metu turi storus galus ir smailias viršūnes. Didesnis už normalųjį ekscesas būdingas tiems skirstiniams, kurie turi plonus galus ir plokščias viršūnes.

Ekscesas yra skirstinio aukščiausio taško laipsnis, apibrėžiamas kaip skirstinio ketvirto centrinio momento μ_4 normalizuota forma. Yra keletas dažniausiai pasitaikančių eksceso formų. Siaurąja prasme ekscesas žymimas β_2. Plačiąja prasme ekscesas gali būti žymimas γ_2 arba b_2.

Eksceso supratimas plačiąja prasme naudojamas dažniau, nes normaliojo skirstinio γ_2 yra lygus 0, tuo tarpu ekscesas siaurąja prasme lygus 3 (Rockinger, 2000). Tačiau literatūroje dažnai pasitaiko klaidingų eksceso interpretacijų, pavyzdžiui, Abramowitz ir Stegun (1972) klaidingai traktuoja β_2 sampratą, t.y. kaip ekscesą plačiąja prasme.

Skirstinys su aukšta viršūne $(\gamma_2 > 0)$ vadinamas normaliojo pasiskirstymo eksceso viršijimu. Skirstinys, neturintis aukštos viršūnės $(\gamma_2 < 0)$, vadinamas ekscesu, mažesniu už normalųjį. Normalusis skirstinys $(\gamma_2 = 0)$ vadinamas normaliuoju ekscesu.

Atsitiktinio X kintamojo ekscesas gali būti žymimas η_2 ar kurt(X).

Jarque-Bera yra statistinis testas, kuris tikrina, ar laiko eilutės yra pasiskirsčiusios pagal normalųjį skirstinį. Šis testavimo metodas įvertina ir normaliojo skirstinio laiko eilučių skirtumus tarp eksceso ir asimetrijos koeficientų (Lawford, 2004).

Kai normaliojo skirstinio hipotezė nulinė, Jarque-Bera statistinis testas atliekamas naudojant 2 laisvės laipsnius. Jarque-Bera testas, sukurtas pastovios sąlyginės dispersijos modeliams, kurie neturi funkcinės priklausomybės tarp sąlyginio vidurkio ir dispersijos parametrų, irgi gali būti naudojamas įvairiems GARCH-M modeliams, tačiau ne visiems (Yi-Ting, 2004).

JB testas buvo formaliai kildinamas kaip LagrangeMultiplier (LM) testas, skirtas normaliojo skirstinio regresiniams skirtumams; priešingai nei alternatyvinis (sąlyginis) skirstinys ξ_t, jis priklauso Pearson grupei. Panašus testas buvo pateiktas Kiefer ir Salmon (1983) (KS), kurie išrado LMtest normaliajam skirstiniui, kol buvo pasiūlytas Hermite polinominis skaidymas sąlyginio tankio ξ_t (Yi-Ting, 2004).

Praktinei analizei paimti du Lietuvos akcijų rinkos indeksai VILSE ir LITIN 10 (kuris nuo 2005-05-27 nebeskaičiuojamas). Analizuojamų duomenų periodas – nuo 2004-01-02 iki 2005-01-20. Straipsnyje pateikta tiek VILSE indekso, tiek LITIN 10 indekso dinamika. Indeksų statistikos kitimas rodo, kad jie abu turi kilimo tendenciją, o tai parodo, kad akcijų rinkos padėtis gana gera. Remiantis trimis statistikos įverčiais, t.y. asimetriškumu, ekscesu ir Jarque-Bera testu, bandyta nustyti tolimesnę akcijų rinkos tendenciją. Straipsnyje pateikti VILSE indeksui įvertinti dažniausiai naudojami statistiniai parametrai. Analizei paimti 275 įvykiai, laiko eilutės vidurkis 225,978, mediana 216,54, maksimali reikšmė 321,64 ir minimumas 174,82, standartinis nuokrypis 31,94459. VILSE indekso asimetriškumas yra 1,383749, t.y. teigiamas dydis, todėl vidurkis yra didesnis už medianą. Taip pat pažymėtina, kad VILSE indekso duomenys turi dešinįjį asimetriškumą. Eksceso reikšmė 4,299084, Jarque-Bera yra 107,0972. Iš išvardytų statistinių įverčių galima prognozuoti VILSE indekso kilimo tendencijas.

Tokie patys statistiniai parametrai yra apskaičiuoti ir LITIN 10 indeksui. Analizei taip pat paimti 275 įvykiai. Laiko eilutės vidurkis yra 2765,745, mediana 2695,840, maksimumas 3510,120 ir minimumas 2344,78. Standartinis nuokrypis 244,1560, indekso LITIN 10 asimetriškumas yra 1,374076, t.y. teigiamas dydis, todėl vidurkis yra didesnis už medianą ir skirstinys turi dešinįjį asimetriškumą. Ekscesas yra 4,383886, Jarque-Bera 108,4815. Taigi įvertinti dydžiai rodo indekso vertės kilimo tendenciją.

Atlikdami tyrimus ir prognozuodami finansinių instrumentų kainų kitimo tendencijas, mokslininkai pastebėjo, kad jei investicinių grąžų skirstinys turi asimetriškumą, mažesnį už -1, ir eksceso koeficientą, didesnį nei 1, tada padidėja tikimybė patirti staigią neigiamą investicinę grąžą. Skirstiniams, kurių asimetriškumo koeficientas yra -1 ir eksceso dydis 5 (pavyzdžiui, telekomunikacijų, technologijų sektorių akcijos, apsauginiai fondai turi minėtus parametrus), vidurkio-dispersijos modelis prognozuos, kad investuotojas nepatirs didesnių nuostolių kaip –3,5% per ateinančią dieną su 99% tikimybe. Tačiau modelis, kuris skaičiuoja asimetriškumo ir eksceso koeficientus, prognozuos –7,4% nuostolį per ateinančią dieną su 99% tikimybe. Skirtumas tarp modelių prognozių gana ryškus, daugiau nei 100%. Modeliai, kurie naudoja asimetriškumo ir eksceso įverčius, yra daug tikslesni.

Šiame straipsnyje pritaikyti trys statistiniai įverčiai Lietuvos vertybinių popierių rinkai signalizavo apie tolimesnį jos kilimą.

Raktažodžiai: *asimetrija, autoregresinis, sąlyginis, pasiskirstymas, Jarque-Bera, ekscesas, asimetriškumas.*

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