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Development, Implementation and Evaluation of Multistage Investment Strategies

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Construction of optimal investment portfolio is very complicated task due to many diverse factors which might affect risk and return of the portfolio in the future. Values and impact level of unique factors on the portfolio are changing over time; therefore every investor should take into account the fact that there always will be a certain level of risk associated with any portfolio involving stocks.

There is a number of ways to form a collection of most appropriate stocks and bonds for investment portfolio as well as to allocate weights of assets based on various criteria. All of these methods, dedicated for selection and allocation of assets, have their specific features and some disadvantages. In order to be able to conclude which of the asset selection methods have least disadvantages, four popular techniques were analysed and compared. These techniques were based on different variables: correlation coefficients between asset returns, maximisation of the utility function (diverse values of risk aversion coefficients were analysed), selection of assets with highest historical returns, and employment of modified price-to-earnings ratio.

The article deals with multistage extension to the mean-variance and expected utility maximisation portfolio choice.

Multistage investing consists of several essential stages, where each stage forms a basis for the next stage by providing useful input data, derived by stage-specific analysis. For construction of optimal portfolio the following stages are used: asset allocation, security selection, investment strategy development, construction of the model and its evaluation.

After asset allocation was made and stocks for the portfolios had been selected, different theoretical asset allocation models (equal weight asset allocation, Markowitz model, Capital Asset Pricing Model (CAPM), model where risk free asset is incorporated when constructing a portfolio) have been modified and adapted in order to become suitable for real market situation. Such prerequisites as normal distribution of stock returns were not satisfied by most Lithuanian companies' stocks when different interims were investigated, therefore authors set a presupposition that distribution properties of the stocks can be disregarded when Markowitz and CAPM models were applied to real market. Some other changes for the prerequisites of models were made; otherwise these theoretical models could not have been applied to Lithuanian market.

After all models had been applied in Lithuanian equity market, back testing was carried out and certain characteristics of outcomes of different investment strategies were compared. Results were judged against characteristics of popular stock exchange indices of Baltic States in order to obtain conclusions.

Most models were developed for broad financials markets (global markets). In the paper we analyse financial market of Lithuania. Since this market does not fit assumed conditions of general models, the models were slightly modified to apply for Lithuania market. The results of portfolio were compared with Baltic States index.

It was concluded that the highest return rate is achieved by constructing the investment portfolio with employing modified Capital Asset Pricing Model. The best technique for selecting stocks proved to be the maximisation of utility function when risk aversion coefficient A=3. In addition to this, after comparison of different asset selection methods, it was noted that the highest value of the Sharpe ratio was achieved by utilising the same technique. After investigation it was noted that investors should add a risk free asset into portfolio of stocks because it usually improves the results of most portfolios, irrespective of their contents. Constructing portfolios based on asset allocation according to indices analysed in the paper is not recommended because characteristics of indices were worse than the ones of constructed portfolios.

Stocks of every company quoted in NASDAQ OMX Baltic (2011) Stock Exchange in Vilnius Official list for more than 10 years (2001 beginning – 2010 end) were investigated. Stocks of 14 companies satisfied preset 10 year interim criterion.

Keywords*: stocks, asset selection, Markowitz model, asset allocation, asset pricing, Sharpe ratio, utility function, rate of return.*

Introduction

Quantitative models can be found in modern investment theory. There are models that quantify the relationship between the expected returns among a set of assets and their relative risk levels, such as the Capital Asset Pricing Model (CAPM) (Sharpe, 1964; Jagannathan, & Wang, 1996) or the Arbitrage Pricing Theory (APT) (Ross, 1976). There are models used to price financial products, including stocks, bonds, options, et cetera. And of particular interest for this article, there are models used to guide asset allocation decisions, such as the Markowitz Mean-Variance criterion or the set of expected utility-theoretic paradigms.

Analysis of historical stock returns had shown that investing in stock market is the best way to protect money from inflation and that variability of short-term investments is directly proportional to probability of exceeding the rate of inflation during certain interim (Valakevicius, 2007). There are various methods to construct a portfolio (Ang & Chen, 2002), and each of these methods have specific flaws. Therefore, the best combination of methods used in various stages of forming, forecasting and managing the investment portfolio should be devised so that investors could obtain maximum benefit from their money. Methods of forming the portfolio differ in their complexity, time and financial resources needed to be established, reliability, etc. E.g., the models involving copulas initially were regarded successful (Junker, Szimayer & Wagner, 2006); later these models, according to Salmon (2009), were called one of the Wall Street crash reasons. Variables (stock prices, interest rates, currency rates) in financial markets often cannot be described properly by using normal distribution (Glasserman, Heidelberger & Shahabuddin, 2002) and not always it is possible to find appropriate copula function for financial variables (Malevergne & Sornette, 2003), sometimes the combination of several copula functions has to be created (Rachev et al, 2009). When misinterpreted and unprofessionally applied these functions can cause a lot of losses. One of the time-consuming models is the model based on factors (Grinblatt & Titman, 1983). Valakevicius & Zolyte (2003) used this method in their research and extracted 8 factors – consumer price index, export and import of goods, unemployment level, etc. Roll (1980) states that for this type of model 3 to 4 factors should be enough if arbitrage pricing theory is adapted. Mansor (2011) had identified connections between stock market development and GDP, ratio of market capitalisation and investment, and aggregate price level. In this article we will concentrate on the models which do not need such high amount of external information about country's economy. Input information for all the stages of forming the portfolio requires only one external macro economical variable return norm of riskless asset. All other input information is calculated using price data of the companies in the Lithuanian market. Figure 1 shows the change of prices of Lithuanian companies stocks, selected for the research.

Bonds are regarded riskless assets. They can be issued by Lithuanian government, public limited companies and private limited companies (LVPK, 2008).

Let S_i denote the price of a single stock at time i and let R_i denote the corresponding return over period from $(t_{i-1}, t_i]$, defined as:

$$R_i = \frac{S_i - S_{i-1}}{S_{i-1}},$$

which represents the percentage change in value of the stock. If inequality $R_i < 0$ becomes valid for some particular case, then investment is loss-making. If short selling is not prohibited, then it is possible to sell assets without owning them. According to Luenberger (1998), short selling is very risky. This is because amount of loss is not undetermined in this case. Therefore, in our research we did not incorporate transactions involving short selling.

Risk can be treated variously. In this publication we regarded risk as square root of variance σ^2 of the rate of return. Let us have two assets, A ir B, and their average rates of return $E(R_A)$ and $E(R_B)$. Then sum of asset weights must be equal to one: $\omega_A + \omega_B = 1$. Then average rate of return is calculated using this formula:

$$E(R) = \omega_A E(R_A) + \omega_B E(R_B)$$

Risk is calculated using such formula:

$$\sigma_p = \sqrt{\omega_A^2 \sigma_A^2 + 2\omega_A \omega_B \mathrm{cov}(R_A, R_B) + \omega_B^2 \sigma_B^2}.$$

Risk is proportional to volatility of prices of the stocks.
Let us have investment portfolio which contains n assets. Then average rates of return of stocks will be denoted as $E(R_1), E(R_2), \ldots, E(R_i), \ldots, E(R_n)$, $i = \overline{1,n}$. Weight ω_i is amount of the asset i invested in the portfolio. Therefore, equalities $1 = \sum_{i=1}^{n} \omega_i$ and $E(R_p) = \sum_{i=1}^{n} \omega_i E(R_i)$ must be satisfied. Following this logic, variance for portfolio with many assets can be calculated by the following formula:

$$\sigma_p^2 = E\left(R_p - E(R_p)\right)^2 = E\left(\sum_{i=1}^{n} \omega_i R_i - \sum_{i=1}^{n} \omega_i E(R_i)\right)^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} \omega_i \omega_j \sigma_{ij},$$

where $i = \overline{1,n}$, $j = \overline{1,n}$. Then risk is expressed using this

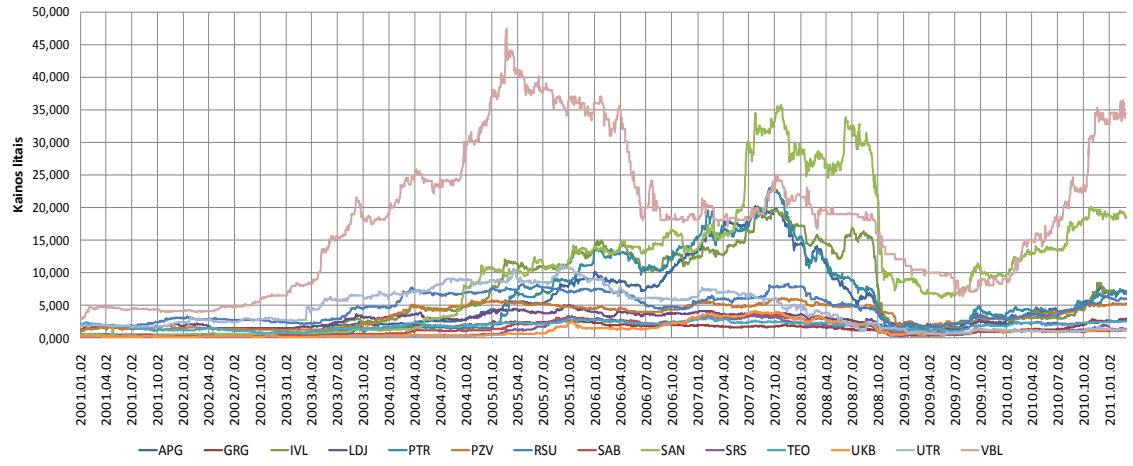

Figure 1. Stock prices of Lithuanian companies, quoted in NASDAQ OMX Baltic, and selected for research

formula: $\sigma_p = \sqrt{\sigma_p^2} = \sqrt{E(R_p - E(R_p))^2}$.

The aim of the research is to develop and evaluate multistage extension to the mean-variance and expected utility maximization portfolio choice and implement modified models for Lithuania's equity market.

Objectives of the research: (1) determine optimal duration of the interim which has to be chosen in order know how often contents of investment portfolios have to be updated so that best results can be achieved; (2) after research to conclude which of the investigated asset selection method is the best taking into account different preferences; (3) determine which of the asset allocation techniques is optimal; (4) carry out comparative analysis of aforementioned portfolio models and stock exchange indices so that unbiased conclusions about quality of formed portfolios can be made.

Object of the research – stocks of Lithuanian companies, return rate of riskless assets and stock exchange indices of Baltic States.

Methods of the research – methods used in statistics and financial mathematics.

A premise was made that transactions with assets do not cost anything. Such elimination enables separate stages of portfolio construction to be compared representatively and research results are not limited to particular situation with specific amounts of fees imposed on returns. This gives the research more applicability to further investigations. If transaction costs were included, real rate or return would be lower than stated in the article.

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Table 1

List of Lithuanian companies

No.	Ticker symbol	Name of the company
1	APG	Apranga
2	GRG	Grigiškės
3	IVL	Invalda
4	LDJ	Lietuvos dujos
5	PTR	Panevėžio statybos trestas
6	PZV	Pieno žvaigždės
7	RSU	Rokiškio sūris
8	SAB	Šiaulių bankas
9	SAN	Sanitas
10	SRS	Snoras
11	TEO	TEO LT
12	UKB	Ūkio bankas
13	UTR	Utenos trikotažas
14	VBL	Vilniaus baldai

The list of Lithuanian companies (in alphabetical order) which were analysed is provided in Table 1. This table shows all the companies which were quoted in Main List of NASDAQ OMX Baltic Vilnius stock exchange for at least ten years from 2001 to 2010.

Riskless interest rates

For many calculations in this research, riskless interest rate was used. Average annual interest rates (obtained for year 2001-2009 from World Bank, 2011) of deposits in Lithuanian banks were used as riskless interest rate. Similar information for year 2010 was unavailable; therefore for the last year return rate of government bonds, obtained from Central Bank of the Republic of Lithuania (Lietuvos Respublikos centrinis bankas, 2011), was used. Table 2 shows annual riskless rates for ten years and their corresponding daily riskless interest rates.

Table 2

Riskless interest rates (years 2001-2010)

Year	Annual riskless interest rate, %	Daily interest rate
2001	3.00	0.0118%
2002	1.70	0.0067%
2003	1.27	0.0050%
2004	1.22	0.0048%
2005	2.40	0.0094%
2006	2.97	0.0117%
2007	5.40	0.0210%
2008	7.65	0.0294%
2009	4.81	0.0187%
2010	3.17	0.0124%

Asset selection techniques

Before asset allocation process the asset selection has to be conducted. This helps investor obtain a manageable number of different assets which have been selected according to the criteria which are important to investor and find appropriate compromise among potential return and risk of the portfolio (Campbell, Koedijk & Kofman, 2002). Asset selection techniques are described in the following paragraphs.

One of the ways to select appropriate stocks for the portfolio is employment of price-to-earnings ratio, or P/E ratio (Basu, 1977). It defines the ratio between stock price and net profit. P/E ratio describes demand for particular company's stocks because it shows how much investor is prepared to pay in order to get one Litas of company's profit, therefore high value of this ratio can mean that prompt rise in profit of particular company is expected (Auditum, 2011). There are several techniques to assess the value of P/E ratio.

Another technique to select assets for portfolio is application utility function, expressed in such formula:

$$U = E(R) - 0{,}005A\sigma^2,$$

where number 0,005 denotes coefficient of calibration. There are three main groups of investors based on their risk aversion: risk-averse, risk-neutral and risk-seeking. Most of the investors are risk-averse (Valakevicius, 2008) and their utility functions are convex. Higher value of risk aversion coefficient means that investor will be inclined to invest in less risky assets and the majority of investors have risk aversion coefficient *A* ranging from 2 to 4 (Bodie

et al., 2004).

The third technique is based on calculation of highest average values of correlation coefficients stock prices companies (Longin, & Solnik, 2001). Theoretically, investment portfolios composed of negatively correlated stocks are more advantageous than those of positively correlated stocks. This is due to the fact that companies, whose stocks and whose prices are not rising or falling simultaneously, decrease the risk of the portfolio. Therefore, the stocks of appropriate companies can be selected by calculating averages of correlation coefficients ρ_{ij} among all the pairs of companies with numbers i and j (where $i = \overline{1,n},\ j = \overline{1,n}$). A set of companies which have lowest average coefficients of correlation will become constituents of the investment portfolio. Comparative analysis will show whether this type of selection leads to better portfolio characteristics. In order to calculate correlation, first of all, covariance has to be determined between two assets (A, B) and two states of economy with probabilities P_1 and P_2:

$$\begin{aligned}\operatorname{cov}(R_A,R_B)&=E((R_A-E(R_A))\times(R_B-E(R_B)))=\\&=P_1((R_{A_1}-E(R_A))\times(R_{B_1}-E(R_B)))+\\&+P_2((R_{A_2}-E(R_A))\times(R_{B_2}-E(R_B))).\end{aligned}$$

If value of $\operatorname{cov}(R_A,R_B)$ is zero, then variables R_A and R_B are uncorrelated. Correlation coefficient is expressed using this formula:

$$\rho_{AB}=\frac{\operatorname{cov}(R_A,R_B)}{\sigma_A\sigma_B}=\frac{\sigma_{AB}}{\sigma_A\sigma_B}.$$

Let us denote covariance between any different assets as $\operatorname{cov}(R_i,R_j)=\sigma_{ij}$, $i\neq j$ (if $i=j$, then we have variance of asset i or asset j). Covariance, when P_i is i state of economy and $i = \overline{1,n},\ j = \overline{1,n}$, is of this form:

$$\sigma_{ij}=\operatorname{cov}(R_i,R_j)=E((R_i-E(R_i))\times(R_j-E(R_j))).$$

According to Markowitz (1952), specific risks are "cancelled out" less if returns of different assets are changing in the same direction more, and this lead to higher volatility of entire portfolio return.

Another technique is selection of stocks which have highest average rates of return at specific time period. If asset A has higher average rate of return than asset B ($E(R_A)\geq E(R_B)$), then asset A will be prioritised during asset selection process.

The last technique, selection of assets having normal distribution of the returns, could not be implemented due to the nature of the data. Although in modern portfolio theory models (i.e. Markowitz and CAPM) it is implied that asset returns are normally distributed, in paper published by Kitt and Kalda (2005) it was stated that normal distribution does not represent appropriately real market stocks. Having implemented the analysis of returns distribution of Lithuanian companies' stocks, it was discovered that there are only few cases of normal distribution and only at few periods investigated, therefore there was not possible to form the portfolio with assets having normal distribution because there were no assets having such distribution for several periods.

Markowitz Model

This asset allocation model was developed by economist Harry Markowitz. In his publication "Portfolio Selection" (Markowitz, 1952) described construction of optimal portfolio by diversifying it, i.e. forming portfolio by incorporating as many assets as possible and thus minimising variance σ_p^2(and risk) for particular level of average rate of return $E(R_p)$. In this article the results of research of applying Markowitz model in Lithuanian equity market will be presented and compared with the results of other ways of investing.

Let us have a portfolio composed from n assets with average rates of return $E(R_1), E(R_2), \dots, E(R_n)$ and covariance σ_{ij}^2, where $i = \overline{1,n}$ and $j = \overline{1,n}$. If to each of n assets we have invested part ω_i of the entire initial portfolio, then $1 = \sum_{i=1}^{n} \omega_i$. If condition $\omega_i \geq 0$ is added, then short selling is prohibited. Otherwise short selling is allowed and weights ω_i can obtain negative values.

The aim of creation of such portfolio is to find minimum risk portfolio for specific average rate of return and this problem can be solved by using linear programming. We have to find minimum of $\sum_{i=1}^{n}\sum_{j=1}^{n}\omega_i\omega_j\sigma_{ij}$, $i = \overline{1,n}$ and $j = \overline{1,n}$, with conditions $\sum_{i=1}^{n}\omega_i E(R_i) = E(R_p)$ and $\sum_{i=1}^{n}\omega_i = 1$ (Luenberger, 1998). If we need to solve the problem where short selling is prohibited, then we have to add one more condition: $\omega_i \geq 0$. In this case we have formed the problem of non-linear programming which can be solved using various mathematical software packages. In this case MATLAB from MathWorks had been employed.

Capital Asset Pricing Model

Acronym for this asset allocation model is CAPM. CAPM expands theoretical model of portfolio diversification, proposed by H. Markowitz. CAPM theory was developed by William Sharpe, John Lintner and Jan Mossin (Sharpe, 1964; Lintner, 1965; Mossin, 1966).When model is developed according to CAPM, investor has opportunity to incorporate riskless asset to the investment portfolio. The results of the research showed that sometimes this feature is very advantageous, especially during crisis, because it helps form the portfolio which has positive rate of return and has no risk. The research showed that during crisis efficient frontier quite often consists only of the one portfolio which has the only asset and that asset is riskless while portfolios formed according to Markowitz theory often bring losses during crisis. If we compare CAPM, compared to Markowitz models at the same fixed level of rate of return, CAPM has the feature of providing lower risk.

Several of the major CAPM prerequisites which have to be ignored so that this model could be applied in the market were: all the assets are infinitely divisible, equity market is in balanced condition, all investors have the same level of access to needed financial information and act rationally according to that financial knowledge. These conditions do not reflect real market situation (they

simplify real market behaviour too much), so for the sake of the research modified CAPM without these prerequisites was applied to Lithuanian market. The results of the modified CAPM showed that formal prerequisites do not have to be satisfied for construction of the portfolio which brings profit. Main mathematical formula of CAPM is called Capital Allocation Line (CAL):

$$E(R_c) = R_f + \frac{E(R_m) - R_f}{\sigma_m}\sigma_c,$$

where $E(R_c)$ is average rate of return of complete effective portfolio, containing risky and riskless assets, σ_c – risk of complete portfolio; R_f – return norm of risk-free portfolio; $E(R_m)$ – average rate of return of market portfolio; σ_m – risk of market portfolio; expression $E(R_m) - R_f$ denotes risk premium.

In CAPM theory it is stated that expected return of entire portfolio should exceed return of riskless asset due to proportionality risk premium and beta coefficient. However, (Fleuret, 2003) does not agree with this statement and says that empirical examinations do not reaffirm this theory. According to (Hagen, 1993), if market portfolio is effective, then there exists direct proportionality between beta coefficient of any asset and expected return of that asset.

The results of Black, Jensen and Scholes test, conducted in 1972 with the stocks quoted in NYSE 1926-1965, have shown that one can apply CAPM successfully to lend out, but not to borrow. Roll (1977) did not agree with the conclusions about validity of CAPM stating that the theory is not testable because exact composition of the market portfolio is unknown. Shanken (1987) after analysis of CAPM theory stated that unambiguous inference regarding validity of CAPM is probably unattainable. Connor & Korajczyk (1993) also stated that CAPM is not ideal model. However, CAPM is acknowledged model, used by rating agencies and other organisations.

Stock exchange indices

Stock exchange indices are useful benchmarks for evaluating stock market in general and its sectors (e.g., energy, manufacture, health care, financial institutions, commodities, information technologies, etc. (Standard & Poor's, 2011), individual companies in different sectors, mutual funds, groups of companies whose stocks are most actively bought and sold in the market, and other entities. Therefore characteristics of two popular indices (OMX Vilnius and OMX Baltic Benchmark) quoted in NASDAQ OMX Baltic Stock exchange were compared with those of constructed portfolios; results of such comparison will show quantitatively how much parameters of models and indices differ.

OMX Vilnius: according to NASDAQ OMX Baltic (2009), index OMX Vilnius (OMXV), established on 1999-12-31, represents the situation in Lithuanian stock market and is composed of the stocks of all the companies which are quoted in the exchange's Main and Secondary lists, except for those whose 90% or more of stocks are owned by one shareholder.

OMX Baltic Benchmark: this index is abbreviated as OMXBB. According to NASDAQ OMX Baltic (2009), OMXBB, established on 1999-12-31 and updated biannually (in order to ensure optimal investment strategy with minimal costs), it is composed from stocks of the companies having highest capitalisation, highest liquidity and belonging to all the sectors in the Baltic market; amount of the stocks of the company in the index depends on each company's value of the stocks and the amount of the stocks in the market. This index should be helpful in creating personal effective investment portfolio inexpensively and is very useful for managers of investment products and for investors as a benchmark (NASDAQ OMX Baltic, 2009). Therefore, we used this index in this article as representative benchmark.

Sharpe ratio

Sharpe ratio is also referred to as Sharpe measure and Sharpe index. It is popular, simple and representative characteristic to evaluate investments. Higher value of Sharpe ratio implies higher quality of investment portfolio or an individual stock (Sharpe, 1964). Sharpe ratio describes what premium $(E(R_A) - R_f)$ the investor will receive for each addition unit of risk. This ratio is written in such expression:

$$S_A = \frac{E(R_A) - R_f}{\sigma_A},$$

where $E(R_A)$ is average rate of return of a stock or portfolio, σ_A – its risk; R_f – rate of return of a riskless asset.

Beta coefficient

This coefficient describes the level of sensitivity of an asset with number i asset to the entire market volatility (Shanken, 1992). According to Fleuriet (2003), beta coefficient represents systemic risk of the asset comparing asset's change of the price with financial market's fluctuation. The coefficient has such mathematical expression:

$$\beta_i = \frac{\sigma_{im}}{\sigma_m^2}, \quad i = \overline{1,n},$$

where σ_{im} represents covariance between the rate of return of the asset with a number i and market portfolio, σ_m^2 – variance of rate of return of the market portfolio. Asset which as beta coefficient is equal to 1, has a risk equal to the risk of the market. Numerical value lower than 1 means that asset is less risky than entire market. Beta coefficient of entire portfolio is denoted this way:

$$\beta_p = \sum_{i=1}^{n} \omega_i \beta_i.$$

Equality below represents connection between average return of a single asset and its beta coefficient:

$$E(R_i) = R_f + \beta_i (E(R_m) - R_f).$$

Practical implementation of models

There were several stages of the research which helped to isolate certain methods, models or techniques and compare them properly. Before beginning of practical implementation of models in Lithuanian equity market, it was assumed that year has 252 trading days, because such number of days is considered a standard (Borodin et al., 2004). Therefore, when models were implemented with interims of 6 months, 126 days data were used, and 63 days data was applied for interims of 3 months. Length of the interim determines how often contents (which companies' stock will be included and what weigh each asset will have) of the investment portfolio will be updated. In addition to this, chosen interim length is essential factor which is used to regulate forecasts of results of the portfolios.

Another stage is the implementation of technique of asset selection. All techniques, which were described earlier, were applied in Lithuanian equity market.

Models which were differing only in one factor – inclusion/exclusion of riskless asset were realised. Markowitz and CAPM models were implemented and comparative analysis was carried out.

Due to large amount of the output data all results will not be presented in this article. Several examples of output are presented in Figure 2 and Figure 3.

Figure 2 shows that the rate of return of the Markowitz model, when 14 different companies' stocks are incorporated into the portfolio in both cases (when the model is compared with OMXV and OMBB), is higher, when value of the risk is similar.

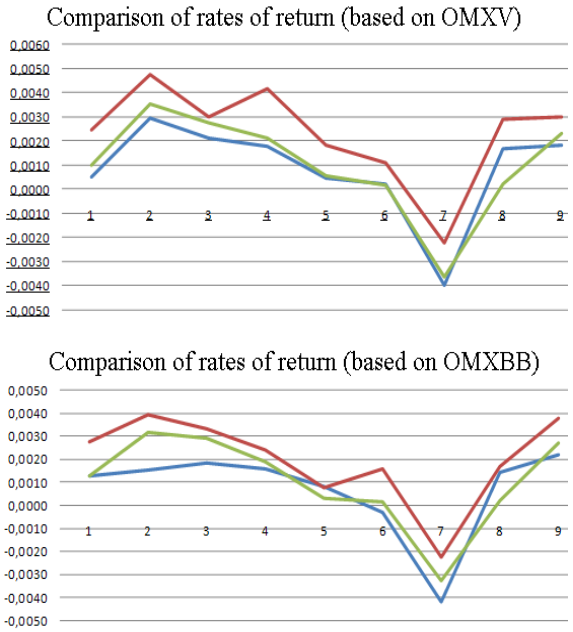

Figure 2. Comparison of rates of returns (y axis) through periods (x axis) among index (blue line), efficient frontier (red line) and Markowitz model (including 14 different assets) being tested (green line)

Upper part of Figure 3 shows graphical representation of comparative analysis dependence of values of Sharpe ratio on portfolio contents. Portfolios having higher numbers are located higher on the efficient frontier, this mean that they have both higher risk and higher rate of return. Lower part of Figure 3 depicts how values of Sharpe ratio depend on interim number. The latter dependency is much more representative, values of Sharpe ratio depend more on wellness of economic condition of the market rather than on of the composition of the portfolio.

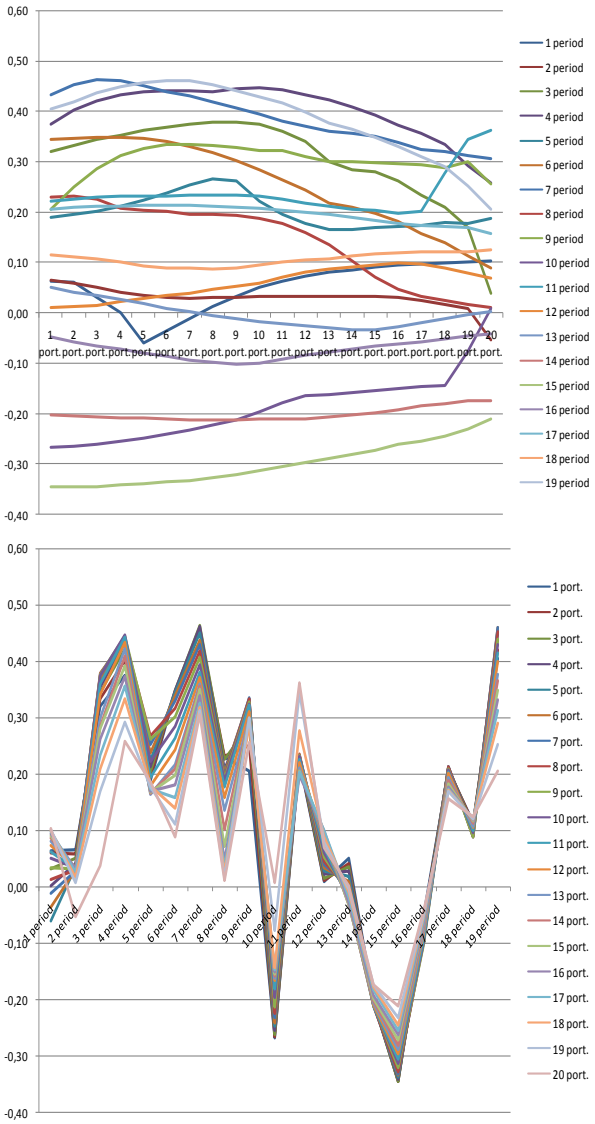

Figure 3. Dependence of Sharpe ratio values(depicted on y axis) on portfolio contents (upper part of the figure) and on interim number (lower part of the image), when efficient frontier portfolios are formed using 14 different assets and updated biannually

Results of the research

The portfolios which were formed using equal asset weights method generally have tendency to provide lower rate of return and higher risk, compared to portfolios, formed according Markowitz and CAPM models. Therefore it is not recommended to construct the portfolio using equal asset weights method. However, if portfolio

using this method has already been constructed, then adding riskless asset is recommended.

Most substantial disadvantage of CAPM and Markowitz models is that formal prerequisites have to be satisfied before the theoretical models are applied into practice. However, both literature review and this empirical research has shown that real market acts very unpredictably and aforementioned models have to be modified by disregarding various prerequisites and only then applied to market.

Application of CAPM and Markowitz models bring most benefits when market situation is stable or improving. This is because of the fact that in these cases future is successfully predicted using past historical data.

After investigation of the portfolios constructed according to Markowitz model, using the same amount of different companies' stocks, with and without incorporation of riskless asset, it was concluded that portfolios containing riskless asset generally convey several better characteristics simultaneously such as higher value of Sharpe's ratio, smaller value of beta coefficient and similar or higher value of rate of return. Therefore it can be generally recommended to add riskless asset into the portfolio.

If the portfolios are constructed only according to some indices, it is better to choose index OMV Vilnius over OMX Baltic Benchmark, because OMX Vilnius conveys higher average value of Sharpe's ratio.

When stocks are selected using correlation coefficients as well, there is no tendency for the portfolio's characteristics to improve when more stocks are added into portfolio.

When stocks are selected using their price-to-earnings coefficients, higher amount of stocks in the portfolio conveys better characteristics, compared to the portfolio constructed from lower amount of companies' stocks.

During recession it is recommended to invest only in riskless assets because stocks' rates of return tend to be negative during crisis.

Beta coefficient is not always appropriate characteristic to evaluate investments impartially. One of the reasons – lowest (i.e. best) values of beta coefficients were conveyed by portfolios which had lowest rate of returns. These portfolios were constructed using price-to-earnings coefficient and updated quarterly.

Conclusions

When the portfolios constructed using different duration (3 months, 6 months and one year) of interims (ceteris paribus) were compared, it was noted that highest returns are conveyed by portfolios updated annually. The second best option is half of the year.

If investor wants to get the highest premium for every additional unit of risk obtained, in this case it is recommended to form the investment portfolio using maximisation of utility function method when value of risk aversion coefficient A=3.

After comparative analysis of OMX Baltic Benchmark index and portfolios, formed using Markowitz model, which have similar risk, it was determined that portfolios, constructed using Markowitz model, having even low amount of different assets, have better values of various characteristics (rate of return, Sharpe's ratio and beta coefficient) than those of the index. Aforementioned statement is not valid when very short interims (i.e. 3 months) are chosen; such duration is too short to form the portfolio having better characteristics than OMX Baltic Benchmark.

Comparative analysis has shown that portfolios, constructed according to CAPM and Markowitz models, and located on the effective frontier, generally have better characteristics (higher rate or return, higher value of Sharpe's ratio and lower value of beta coefficient) when they are formed using more different companies' assets, compared to portfolios formed similarly, but using less different assets. Also, CAPM yielded better results compared to Markowitz model.

Optimal portfolio, which showed better characteristics than any other portfolio, was constructed out of 14 companies' stocks using capital asset pricing model and updated annually, had 0.0013/0.0008=1,625 times higher rate of return than OMX Vilnius, 0.0013/0.0007=1,86 times higher than OMX Baltic Benchmark return rate and 0.0027/0.0013=2,08 times lower than ideal portfolio (efficient frontier portfolio which gives maximum rate of return at corresponding interim) rate of return. Sharpe's ratio of optimal portfolio is 0.1399/0.0976=2,43 times higher than the one of OMX Vilnius and 0.1341/0.0951=1,41 times higher than OMX Baltic Benchmark Sharpe's ratio. Values of beta coefficients of optimal portfolio are also better than the ones of OMX Vilnius and OMX Baltic Benchmark.

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Daugiapakopių investavimo strategijų kūrimas, įgyvendinimas ir vertinimas

Santrauka

Investicinių portfelių vidutinė grąžos norma ir rizika priklauso nuo įvairių veiksnių, pavyzdžiui, bendros ekonominės ir finansinės pasaulio, šalies ar regiono padėties, verslo šakos, kuriai priklauso kompanija būklės, kompanijos pelningumo ir kitų rodiklių (politinių, socialinių, ekologinių faktorių). Visų veiksnių ir jų įtakos, daromos akcijų kainoms, tiksliai nustatyti neįmanoma. Todėl kuriami, pritaikomi praktiškai ir vertinami bei tobulinami įvairūs modeliai, skirti atrinkti ir paskirstyti aktyvams investiciniuose portfeliuose. Du svarbūs dalykai, apie kuriuos reikia pagalvoti prieš investuojant į vertybinius popierius – tai investavimo trukmė ir tinkamų aktyvų pasirinkimas. Investicinius portfelius galima sudaryti iš rizikingų ir nerizikingų vertybinių popierių. Akcijų pirkėjai tampa bendrovių nuosavybės dalies turėtojais. šiuo darbu siekta patikrinti, kaip akcijų atrinkimo ir svorių portfeliuose optimizavimo būdai gali būti pritaikyti Lietuvos vertybinių popierių rinkoje (taip pat į portfelius buvo įtraukiami nerizikingi aktyvai ir tiriamas jų poveikis portfelio charakteristikoms).

Tyrimo tikslas: sukurti daugiapakopes investicinių portfelių formavimo strategijas, jas praktiškai pritaikyti ir nustatyti, kuri strategija yra tinkamiausia Lietuvos akcijų rinkai.

Uždaviniai: (1) nustatyti optimalią periodo trukmę, kurią pasirinkus bendru atveju būtų gaunami geriausias charakteristikų reikšmes turintys investiciniai portfeliai; (2) po tyrimo padaryti išvadą apie geriausią akcijų atrinkimo portfeliams modelį atsižvelgiant į skirtingus prioritetus; (3) nustatyti, kuris iš aktyvų paskirstymo portfelyje būdų yra geriausias; (4) atlikus iš kelių pakopų sudarytų modelių ir populiarių akcijų biržos indeksų lyginamąją analizę padaryti išvadas apie sumodeliuotų portfelių kokybę.

Tyrimo objektai – akcijos, egzistuojančios Lietuvos vertybinių popierių rinkoje, nerizikingoji grąžos norma ir Baltijos šalių akcijų indeksai.

Šiame tyrime buvo ištirti tokie akcijų atrinkimo būdai: mažiausiai koreliuojančių su kitomis akcijų suradimas, didžiausią naudingumo funkcijos reikšmę turinčių akcijų atrinkimas, kai rizikos vengimo koeficientas A=3, ir kai A=10, didžiausią grąžą turinčių akcijų atrinkimas, ir didžiausias P/E koeficiento reikšmes turinčių akcijų atrinkimas. Atrenkant akcijas pagal koreliacijos koeficientus remiamasi faktu, kad teoriškai investiciniai portfeliai, sudaryti iš neigiamai koreliuotų akcijų, yra geresni nei iš teigiamai koreliuotų. Taip yra todėl, kad akcijos, kurių grąžų normos kinta priešingomis kryptimis, sumažina portfelio riziką. Atrinkus tų įmonių akcijas, kurių koreliacijų koeficientų vidurkiai yra mažiausi, buvo patikrinta, ar šis akcijų atrinkimo metodas yra naudingesnis praktikoje nei kiti metodai. Aktyvus atrenkant pagal naudingumo funkcijos reikšmes yra atsižvelgiama į individualaus investuotojo rizikos vengimo koeficiento reikšmę A. Atrenkant akcijas pagal didžiausių akcijų grąžų normų metodą yra neatsižvelgiama į riziką. Šis metodas – atskiras naudingumo funkcijos metodo atvejis. Kitas būdas tinkamų įmonių akcijoms parinkti yra P/E koeficiento (*plg. angl.* price-to-earnings ratio), nusakančio akcijos rinkos kainos ir grynojo pelno santykį, suradimas. Šis koeficientas nusako tam tikros įmonės akcijų paklausą rinkoje – juo parodoma, kiek litų investuotojas sutiktų mokėti už vieną litą įmonės pelno.

Investiciniam portfeliui sudaryti buvo pasirinkti tokie modeliai: Markovičiaus modelis, kai tam tikrai grąžai reikia rasti mažiausią galimą riziką, finansinių aktyvų įkainojimo modelis, portfelių sudarymas į investicinį portfelį prie rizikingų aktyvų aibės prijungiant nerizikingą aktyvą ir portfelio sudarymas iš vienodus svorius turinčių aktyvų. Pagrindiniai Markovičiaus ir CAPM modelių trūkumai – prielaidų, neatitinkančių tikrovės, darymas. Todėl šie modeliai buvo modifikuojami atmetant realybės neatitinkančias prielaidas ir tada pritaikomi Lietuvos vertybinių popierių rinkoje.

Tyrimui paimtos „NASDAQ OMX Baltic“ Vilniaus biržos Oficialiajame sąraše dešimt arba daugiau metų (2001-01-01 – 2010-12-31) egzistuojančių įmonių akcijos. Iš dabar kotiruojamų 18 įmonių akcijų šią sąlygą tenkina 14 įmonių. Buvo sumodeliuoti ir testuoti portfeliai, kurių sudėtis atnaujinama taikant tris skirtingos trukmės periodus – metų, pusės metų ir trijų mėnesių. Vykdant skaičiavimus buvo laikoma, kad metuose yra 252 prekybos akcijomis dienos. Atitinkamai, pasirinkus pusmečio trukmės periodą skaičiavimai buvo atliekami su 126 dienų įmonių akcijų duomenų matrica, o pasirinkus ketvirčio metų ilgio periodą – su 63 dienų. Nuo pasirinktos periodo trukmės priklauso, kaip dažnai bus atnaujinama investicinio portfelio sudėtis ir kokios trukmės periodui bus sudaromos prognozės apie būsimą portfelio grąžą.

Sukurtų ir rinkoje pritaikytų daugiapakopių modelių charakteristikos buvo palygintos ne tik vienos su kitomis, bet ir su OMXV ir OMXBB indeksais. Buvo padarytos išvados, kurios galėtų tapti pagrindu tolesniems tyrimams.

Nustatyta, jog portfeliai, sudaryti iš aktyvų su vienodais svoriais, bendruoju atveju turi mažesnę grąžą ir didesnę riziką negu analogiški portfeliai, sudaryti pagal Markovičiaus modelį arba CAPM. Todėl nereikėtų sudaryti portfelių paskirstant aktyvus portfelyje vienodais svoriais. Jei vis dėlto toks portfelis sudaromas, norint gauti kuo didesnę grąžą reikėtų įtraukti į portfelį nerizikingą aktyvą.

Ant efektyviojo portfelių krašto esančių portfelių, sudarytų iš didesnio kiekio aktyvų, (palyginus tuo pačiu metodu sudarytais portfeliais iš mažesnio skaičiaus aktyvų), charakteristikos bendruoju atveju yra geresnės (didesnę grąžos norma, Šarpo rodiklio ir mažesnė beta koeficiento reikšmė).

Palyginus iš to paties skirtingų akcijų kiekio sudarytus portfelius pagal Markovičiaus modelį su analogiškais portfeliais, sudarytais įtraukus nerizikingą aktyvą (t.y. pagal CAPM), padaryta išvada, kad antruoju atveju yra gaunama ne mažesnė grąža nei pirmu atveju, taip pat antru atveju gaunama aukštesnė Šarpo rodiklio ir mažesnė beta koeficiento reikšmė. Todėl ir šiuo atveju rekomenduojama įtraukti nerizikingą aktyvą į investicinį portfelį.

Parinkus investiciniams portfeliams skirtingos trukmės periodus (12 mėn., 6 mėn. ir 3 mėn.) ir palyginus suformuotų portfelių charakteristikas padaryta išvada, kad didžiausios grąžos gaunamos pasirinkus 12 mėn. trukmės periodus. Mažiausios grąžos gautos suformavus portfelius taikant 3 mėn. trukmės istorinius duomenis.

Geriausias charakteristikas turinčio, optimalaus portfelio, t.y., portfelio, sudaryto taikant modifikuotą CAPM (iš 14 skirtingų akcijų ir nerizikingo aktyvo), kurio sudėtis atnaujinama kas 12 mėn., grąžos norma yra 0.0013/0.0008=1,625 karto aukštesnė nei OMXV ir 0.0013/0.0007=1,86 karto aukštesnė nei OMXBB grąžos norma, bei vidutiniškai 0.0027/0.0013=2,08 karto mažesnė nei grąžos norma, kuri gali būti gauta idealiu atveju sudarius efektyvųjį portfelių kraštą. Optimalaus realizuoto modelio Šarpo rodiklis yra 0.1399/0.0976=2,43 karto aukštesnis nei OMXV ir 0.1341/0.0951=1,41 karto aukštesnis nei OMXBB Šarpo rodiklio reikšmė. Optimalaus realizuoto modelio beta koeficientų reikšmės yra mažesnės nei OMXV ir OMXBB indeksų atitinkamos reikšmės.

Atrenkant akcijas pagal P/E koeficientų reikšmes, didesnis akcijų kiekis portfelyje yra proporcingas didesnei grąžai (esant tai pačiai rizikai).

Įvertinus akcijų atrinkimo portfeliams būdus, nustatyta, kad didžiausią Šarpo rodiklio bei grąžos normos reikšmę turintys portfeliai buvo gauti taikant naudingumo funkcijos metodą, kai rizikos vengimo koeficientas A=3, todėl norint gauti didžiausią premiją už papildomą prisiimtą rizikos vienetą verta taikyti šį metodą.

Palyginus indeksą OMX Baltic Benchmark ir portfelius, sumodeliuotus pagal istorinius duomenis taikant Markovičiaus modelį ir turinčius riziką, analogišką OMXBB indekso rizikai, nustatyta, kad net iš nedidelio kiekio skirtingų akcijų, pvz., penkių įmonių, suformuoti investiciniai portfeliai turi geresnes charakteristikų (grąžos normos, Šarpo indekso ir beta koeficiento) reikšmes. Šis teiginys nėra teisingas, jei portfeliams kurti taikomi labai trumpi – tik trijų mėn. trukmės – duomenys, tada indeksas OMXBB turi geresnes charakteristikų reikšmes.

Nustatyta, kad kai akcijos atrenkamos pagal naudingumo funkciją, kai A=3 ir A=10, ir pagal koreliacijos koeficientus, didinat aktyvų kiekį portfelyje, bendru atveju (esant tai pačiai rizikai) nėra suformuojami geresnes charakteristikas turintys portfeliai.

Markovičiaus ir CAPM modelių esminis trūkumas – būtinybė atmesti kai kurias teorinių modelių prielaidas ir modelius modifikuoti prieš pradedant juos taikyti praktiškai.

Nustatyta, kad mažiausios grąžos (esant tai pačiai rizikai, palyginus su kitais pagal moderniąją portfelio teoriją sudarytais portfeliais) buvo gautos taikant P/E koeficientų metodą, kai portfelis yra atnaujinamas kas 3 mėnesius. Šiuo atveju gautos mažiausios beta koeficientų reikšmės, todėl galima daryti išvadą, kad beta koeficientas yra nereprezentatyvi investicinio portfelio charakteristika.

Empirinio tyrimo metu pastebėta, kad krizės metu verta investuoti tik į nerizikingus aktyvus. Apibendrinus tyrimo rezultatus bendruoju atveju gautos tokios išvados:

- Investiciniai portfeliai duoda didesnę grąžą, kai naudojami ilgesnės trukmės istoriniai duomenys
- Geriausius rezultatus duoda portfeliai, į kuriuos akcijos buvo atrinktos pagal naudingumo funkciją, o blogiausius- pagal P/E koeficientą
- Didžiausią grąžą sugeneravo optimalus portfelis, naudojant modifikuotą CAPM modelį
- Nustatyta, kad optimalusis portfelis iš keturiolikos OMX Vilnius akcijų duoda didesnię grąžą, negu OMX Baltic Benchmark indeksas.

Raktažodžiai: *akcijos, aktyvų parinkimas, Markovičiaus modelis, finansinių aktyvų įkainojimo modelis, Šarpo rodiklis, naudingumo funkcija, grąžos norma.*

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