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Project Delivery System Decision Making using Pythagorean Fuzzy TOPSIS

Limin Su[1], Huimin Li[2,3,4*], Yongchao Cao[2,3], Lelin Lv[2,3]

[1]*School of Management and Economics, North China University of Water Resources and Electric Power*
No. 136, Jinshui Road, 450046, Zhengzhou, China
E-mail. suliminlove2010@163.com

[2*]*Department of Construction Engineering and Management, North China University of Water Resources and Electric Power*
No. 136, Jinshui Road, 450046, Zhengzhou, China
E-mail. lihuimin3646@163.com
**Corresponding author: lihuimin3646@163.com*

[3]*Henan Key Laboratory of Water Environment Simulation and Treatment*
No. 136, Jinshui Road, 450046, Zhengzhou, China
E-mail. 18638188626@163.com

[4]*Collaborative Innovation Center of Water Resources Efficient Utilization and Protection Engineering*
No. 136, Jinshui Road, 450046, Zhengzhou, China
E-mail. 13253671135@163.com

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The decision making of project delivery systems is a complex process, which is also a critical task for owners. The complexity problem arises from the uncertainty of decision making environment and construction project itself. Pythagorean fuzzy set (PFS), as an extension from intuitionistic fuzzy set (IFS) to deal with uncertainty information, has attracted more scholars' attention in the decision making area. This paper aims to develop a project delivery systems decision making approach under Pythagorean fuzzy environment. The main contributions of this paper are as follows: (1) Three similarity measures (i.e., 1-type PFSs similarity measure, 2-type PFSs weighted similarity measure, 3-type PFSs weighted similarity measure) are developed, and their properties are also investigated. (2) An improved TOPSIS decision making framework is further established with PFSs information, in which the proposed similarity measures are employed to measure the similarity degree between each alternative and positive ideal solution and negative ideal solution. (3) A project delivery system decision making method using Pythagorean fuzzy TOPSIS decision making framework. Finally, a case study of the selection of project delivery systems is presented to prove the performance of the proposed decision making method.

Keywords: *Project Delivery System; Pythagorean Fuzzy Sets; Similarity Measure; Pythagorean Fuzzy TOPSIS Method.*

Introduction

A project delivery system (PDS) defines the relationship between project participants, and how a proposed project is delivered from the contractor to the owner (ASCE, 1988; Chen, *et al.*, 2011). And PDS totally affects the construction performance including schedule, cost and quality (Blayse & Manley, 2004; Chan, *et al.*, 2001; Khalil, 2002; Mollaoglukorkmaz, *et al.*, 2013; Shane, *et al.*, 2013). The types of PDSs in construction industry practice can be selected, including design-bid-build (DBB), design-build (DB), construction management at risk (CM-at risk), engineering-procurement-construction (EPC), and integrated project delivery (IPD) (Chen, *et al.*, 2010; Shi, *et al.*, 2014; Qiang *et al.*, 2015; Li *et al.*, 2015) Selecting a suitable PDS for a construction project is a key task for an owner in the planning stage.

Quite a lot of research works focus on PDS selection decision making (Liu *et al.*, 2015; Li *et al.*, 2015). Analytical hierarchical process (AHP) was widely applied in the topic of PDS selection (Alhazmi & Mccaffer, 2000; Khalil, 2002; Mafakheri *et al.*, 2007; Mahdi & Alreshaid, 2005). But Belton and Stewart (2002) argued that AHP lacks sound statistical theory and is incapable to adequately handle uncertainty information. Some researchers applied multi-attribute utility to tackle the PDSs decision problem (Love *et al.*, 1998; Chan *et al.*, 2001; Oyetunji & Anderson, 2006; Chen *et al.*, 2011). However, the utility values of the indicators cannot totally reflect the project characteristics. Fuzzy set is employed to deal with PDSs selection problem (Ng *et al.*, 2005; Hong *et al.*, 2008; Chen *et al.*, 2011; Wang *et al.*, 2014). Actually, the PDSs selection problem can be considered as a multi-criteria decision making (MCDM) problems. Li *et al.* (2015) developed a MCDM model integrating information entropy and unascertained set to select PDS for a construction project. A group decision making model using Interval-valued intuitionistic fuzzy set (IVIFS) theory for PDS selection was proposed by An *et al.* (2018).

Recently, MCDM problems have been applied in a wide research field (Podviezko & Podvezko, 2014; Zhou *et al.*, 2015; Chen 2015; Hanine *et al.*, 2016; Pourahmadi *et al.*, 2017; Fahmi *et al.*, 2017; Zhang *et al.*, 2017; Gitinavard *et al.*, 2017; Potharaju & Sreedevi, 2018; Ren *et al.*, 2018).

From the existing research, decision making methods and preference information are important factors in decision making process, in which desirable alternative solution can be chosen by selecting a decision making method and providing preference information from a decision maker.

As one of the popular MCDM methods, Technique for Order Preference by Similarity to an Ideal Solution (TOPSIS) method, firstly introduced by Hwang and Yoon (Hwang & Yoon, 1981), is introduced to many decision making topics. The essential goal of the TOPSIS technique is that the most desired alternative should have not only the farthest distance from the negative ideal solution, but also the shortest distance from the positive ideal solution (Onat *et al.*, 2016). Over the past decades, a lot of researchers have applied TOPSIS, modified or extended TOPSIS method to solve decision making problems in a different field (Boran *et al.*, 2009; Chen *et al.*, 2016; Joshi & Kumar, 2016; Li & Wu, 2016; Liang *et al.*, 2018; Liu, 2014; Nehi & Keikha, 2016; Tian *et al.*, 2015). Zadeh (1965) introduced Fuzzy set theory. It has been widely employed to model uncertainty in real-world. Fuzzy sets were extended to Atanassov's intuitionistic fuzzy sets (IFSs) (Atanassov, 1986; Krassimir & Atanassov, 1989). The Pythagorean fuzzy set (PFS) (Yager, 2013, 2014) has become an effective tool for solving the multiple attribute decision making (MADM) problems under uncertainty. Many different cases under uncertainty information have been investigated in the literature.

From the existing research results, we can easily find that distance and similarity measures are vital tools for calculating the relative closeness coefficient. Liang and Xu (2017) established a new improved TOPSIS decision making method under hesitant Pythagorean fuzzy sets, in which the geometric distance was used to measure the closeness degree between ideal solution and each alternative. Aikhuele and Turan (2017) developed an intuitionistic fuzzy TOPSIS model based on exponential-related function. In their study, the exponential-related function served as a mean to compute relative closeness coefficients by distance from each alternative to the intuitionistic fuzzy negative and positive ideal solutions. Zhang and Xu (2015) gave an extension of TOPSIS with PFS, and defined a distance measure from each alternative to the Pythagorean fuzzy negative ideal solution and the Pythagorean fuzzy positive ideal solution, respectively.

An abundant theoretical support is provided through reviewing the existing research, while further research should be developed on two main aspects. (1) The existing similarity measure ignored the confidence degrees of experts when they give evaluation values in the evaluation process. Actually, the confidence degrees from decision experts play an important role to ensure the effective evaluation information for all types of PDSs in the selection process of PDSs. Generally, it needs to find a simple and intuitive way to make a reliable decision making. (2) The existing similarity measure, which is used to depict the "closeness" degree between each alternative and ideal alternative, is generally considered from the problem rather than considering the psychological behavior of experts or decision maker. To bridge this gap, this paper aims to develop a Pythagorean fuzzy TOPSIS approach applied to PDSs selection. For the purpose, this paper firstly presents three new similarity measures under PFSs based on minimum and maximum operators. Secondly, an improved TOPSIS method using the proposed similarity measures is established. Thirdly, improved TOPSIS approach is employed to PDS selection under Pythagorean fuzzy environment.

The remainder of the paper is organized as follows. Decision making framework for selection PDSs is given in Section 2. In Section 3, methodology for PDSs selection is introduced, including preliminaries about Pythagorean fuzzy sets and three new similarity measures for PFSs are presented in this paper. In Section 4, a Pythagorean fuzzy TOPSIS method based on similarity measure for PDSs selection is developed. A case study about selection of PDS is presented to illustrate the effectiveness of the proposed method in section 5. The conclusions are drawn in Section 6.

Decision Making Framework for Selection PDSs

For a proposed construction project, the alternative PDS can be selected including DBB, DB, CM-at risk, and EPC. The study of Mafakheri *et al.* (2007) had shown that there were many factors that affecting the selection of an appropriate PDS. The selection of PDSs is a typical decision making problem, the decision making indicators for selection PDS can be illustrated as Figure 1. Based on them, the appropriate PDS can be obtained through choosing a matching approach, and the selection decision making process is shown as in Figure 2.

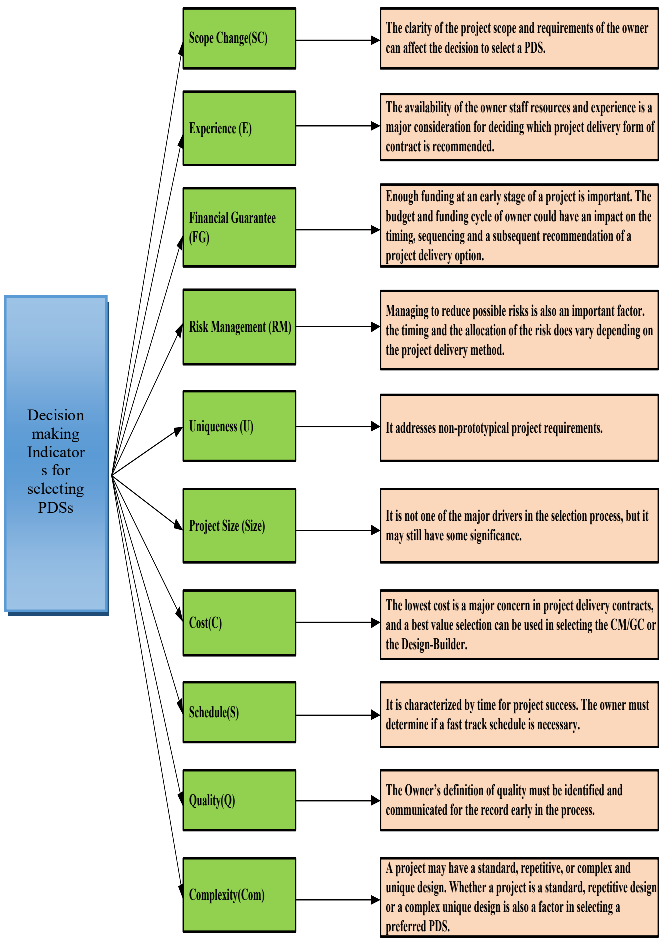

Figure 1. Decision Making Indicators for Selection PDSs

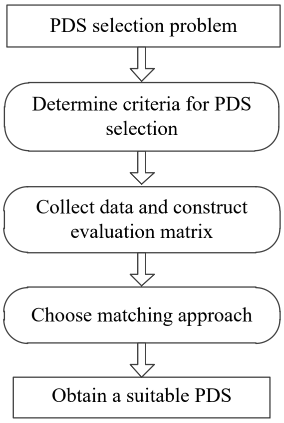

Figure 2. The Flow Chart for PDSs Selection

Methodology for PDSs Selection

This section presents the methodology for selecting PDSs. Following the preliminaries about PFSs and new similarity measures are introduced.

Preliminaries about PFSs

This subsection will give the concepts of PFSs and their relevant operations.

Definition 1 (Yager, 2013, 2014) Assume that X is a universe of discourse. The PFS P in X denoted as

$$P=\left\{\left\langle x,u_P(x),v_P(x)\right\rangle \middle| x\in X\right\} \tag{1}$$

in which $u_P(x): X\to[0,1]$ is membership degree and $v_P(x): X\to[0,1]$ denotes the degree of non-membership of the element $x\in X$ to P respectively, the degree of indeterminacy is $\pi_P(x)=\sqrt{1-u_P^2(x)-v_P^2(x)}$. And the PFS P satisfies the condition $0\le u_P^2(x)+v_P^2(x)\le 1$.

For convenience, $(u_P(x),v_P(x))$ is called a PFN and denoted as $\bar{P}=(u_{\bar{P}},v_{\bar{P}})$.

Definition 2 A Pythagorean fuzzy set P_1 contains the other Pythagorean fuzzy set P_2, i.e., $P_2\subseteq P_1$, if and only if $u_{P_1}(x)\ge u_{P_2}(x)$, $v_{P_1}(x)\le v_{P_2}(x)$ for all $x\in X$.

Definition 3 (Zhang & Xu, 2015) Assume that $p=(u_p,v_p)$ is a PFN, then we define the score function of p can be defined as follows:

$$\mathrm{S}(p)=(u_p)^2-(v_p)^2$$

where $\mathrm{S}(p)\in[-1,1]$.

Definition 4 (Peng & Yang, 2015) Let $p=(u_p,v_p)$ be a PFN, then the accuracy function of p can be defined as follows:

$$\mathrm{K}(p)=(u_p)^2+(v_p)^2 \tag{2}$$

where $\mathrm{K}(p)\in[0,1]$.

For any two PFNs p_1 and p_2, the comparison rules are shown as follows (Peng and Yang, 2015):

(I) if $\mathrm{S}(p_1)<\mathrm{S}(p_2)$, then $p_1\prec p_2$;

(II) if $\mathrm{S}(p_1)=\mathrm{S}(p_2)$, then,

(a) if $\mathrm{K}(p_1)<\mathrm{K}(p_2)$, then $p_1\prec p_2$;

(b) if $\mathrm{K}(p_1)=\mathrm{K}(p_2)$, then $p_1\sim p_2$.

Definition 5 (Yager, 2014) Assume that $p_i=(u_i,v_i)$ $(i=1,2,\ldots,n)$ is a PFN set and $w=(w_1,w_2,\ldots,w_n)^T$ be the weight vector from p_i $(i=1,2,\ldots,n)$, where $\sum_{i=1}^{n}w_i=1$, then a PFWA operator is expressed as:

$$\mathrm{PFWA}(p_1,p_2,\ldots,p_n)=\left(\sum_{i=1}^{n}w_iu_i,\sum_{i=1}^{n}w_iv_i\right).$$

New Similarity Measures Between PFSs

In this subsection, three new similarity measures between PFSs based on maximum and minimum operators, and their properties are presented.

Generally, a similarity measure between two sets X_1 and X_2 is a function defined as $\mathrm{r}: F(X)\times F(X)\to[0,1]$, which satisfies the following properties:

(P1) $\mathrm{r}(X_1,X_2)=\mathrm{r}(X_2,X_1)$;

(P2) $\mathrm{r}(X_1,X_2)=1$ if $X_1=X_2$;

(P3) $\mathrm{r}(X_1,X_2)=\mathrm{r}(X_2,X_1)$;

(P4) $\mathrm{r}(X_1,X_3)\le\mathrm{r}(X_1,X_2)$ and $\mathrm{r}(X_1,X_3)\le\mathrm{r}(X_2,X_3)$ if $X_1\subseteq X_2\subseteq X_3$ for a set X_3.

Proposition 1 Assume that $X=\{x_1,x_2,\ldots,x_n\}$ is a given set, P_1 and P_2 are two PFSs, then 1-type PFSs similarity measure

$$\mathrm{r}_1(P_1,P_2)=\frac{1}{2n}\sum_{i=1}^{n}\left(\frac{\min\left(u_{P_1}(x_i),u_{P_2}(x_i)\right)}{\max\left(u_{P_1}(x_i),u_{P_2}(x_i)\right)}+\frac{\min\left(v_{P_1}(x_i),v_{P_2}(x_i)\right)}{\max\left(v_{P_1}(x_i),v_{P_2}(x_i)\right)}\right) \tag{3}$$

should satisfy the following properties:

(P1) $0\le\mathrm{r}_1(P_1,P_2)\le 1$;

(P2) $\mathrm{r}_1(P_1,P_2)=1$ if $P_1=P_2$;

(P3) $\mathrm{r}_1(P_1,P_2)=\mathrm{r}_1(P_2,P_1)$;

(P4) $\mathrm{r}_1(P_1,P_3)\le\mathrm{r}_1(P_1,P_2)$ and $\mathrm{r}_1(P_1,P_3)\le\mathrm{r}_1(P_2,P_3)$ if $P_1\subseteq P_2\subseteq P_3$ for a PFS P_3.

Proof. Properties (P1)-(P3) can be easily verified. Therefore, we only prove the property (P4). Let $P_1 \subseteq P_2 \subseteq P_3$, then

$$u_{P_1}(x_i) \le u_{P_2}(x_i) \le u_{P_3}(x_i), \tag{4}$$

$$v_{P_1}(x_i) \ge v_{P_2}(x_i) \ge v_{P_3}(x_i) \tag{5}$$

for every $x_i \in X$. So, we can obtain that

$$\min\{u_{P_1}(x_i), u_{P_2}(x_i)\} = u_{P_1}(x_i);$$

$$\max\{u_{P_1}(x_i), u_{P_2}(x_i)\} = u_{P_2}(x_i);$$

$$\min\{v_{P_1}(x_i), v_{P_2}(x_i)\} = v_{P_2}(x_i);$$

$$\max\{v_{P_1}(x_i), v_{P_2}(x_i)\} = v_{P_1}(x_i),$$

thus, the 1-type PFSs similarity measure between two PFSs P_1 and P_2 can be obtained as follows:

$$\begin{aligned} r_1(P_1,P_2) &= \frac{1}{2n}\sum_{i=1}^{n}\left(\frac{\min\{u_{P_1}(x_i),u_{P_2}(x_i)\}}{\max\{u_{P_1}(x_i),u_{P_2}(x_i)\}} + \frac{\min\{v_{P_1}(x_i),v_{P_2}(x_i)\}}{\max\{v_{P_1}(x_i),v_{P_2}(x_i)\}}\right) \\ &= \frac{1}{2n}\sum_{i=1}^{n}\left\{\frac{u_{P_1}(x_i)}{u_{P_2}(x_i)} + \frac{v_{P_2}(x_i)}{v_{P_1}(x_i)}\right\}. \end{aligned} \tag{6}$$

Similarly, the 1-type PFSs similarity measure between two PFSs P_1 and P_3 is:

$$\begin{aligned} r_1(P_1,P_3) &= \frac{1}{2n}\sum_{i=1}^{n}\left(\frac{\min\{u_{P_1}(x_i),u_{P_3}(x_i)\}}{\max\{u_{P_1}(x_i),u_{P_3}(x_i)\}} + \frac{\min\{v_{P_1}(x_i),v_{P_3}(x_i)\}}{\max\{v_{P_1}(x_i),v_{P_3}(x_i)\}}\right) \\ &= \frac{1}{2n}\sum_{i=1}^{n}\left\{\frac{u_{P_1}(x_i)}{u_{P_3}(x_i)} + \frac{v_{P_3}(x_i)}{v_{P_1}(x_i)}\right\} \end{aligned} \tag{7}$$

and the 1-type PFSs similarity measure between two PFSs P_2 and P_3 is:

$$\begin{aligned} r_1(P_2,P_3) &= \frac{1}{2n}\sum_{i=1}^{n}\left(\frac{\min\{u_{P_2}(x_i),u_{P_3}(x_i)\}}{\max\{u_{P_2}(x_i),u_{P_3}(x_i)\}} + \frac{\min\{v_{P_2}(x_i),v_{P_3}(x_i)\}}{\max\{v_{P_2}(x_i),v_{P_3}(x_i)\}}\right) \\ &= \frac{1}{2n}\sum_{i=1}^{n}\left\{\frac{u_{P_2}(x_i)}{u_{P_3}(x_i)} + \frac{v_{P_3}(x_i)}{v_{P_2}(x_i)}\right\}. \end{aligned} \tag{8}$$

For the proof of (P4), we only comparison the right terms of formulas (6) and (7) in curly braces. From formulas (4) and (5), we can easily obtain the result through comparing numerator or denominator in the corresponding terms. Therefore, $r_1(P_1,P_3) \le r_1(P_1,P_2)$.

Similarly, from formulas (7) and (8), we get $r_1(P_1,P_3) \le r_1(P_2,P_3)$.

So the 1-type PFSs similarity measure $r_1(P_1,P_2)$ satisfies the property (P4).

When we consider the importance in the two terms, i.e., membership, non-membership, in a PFN, we should take the weights of the two terms in formula (3) into account. Therefore, we develop another similarity measure between PFSs.

Proposition 2 Assume that $X = \{x_1, x_2, \ldots$ is a given set, P_1 and P_2 be two PFSs, then 2-type PFSs weighted similarity measure

$$r_2(P_1,P_2) = \frac{1}{n}\sum_{i=1}^{n}\left(\alpha\frac{\min(u_{P_1}(x_i),u_{P_2}(x_i))}{\max(u_{P_1}(x_i),u_{P_2}(x_i))} + \beta\frac{\min(v_{P_1}(x_i),v_{P_2}(x_i))}{\max(v_{P_1}(x_i),v_{P_2}(x_i))}\right) \tag{9}$$

should satisfy the following properties:

(P1) $0 \le r_2(P_1,P_2) \le 1$;

(P2) $r_2(P_1,P_2) = 1$ if $P_1 = P_2$;

(P3) $r_2(P_1,P_2) = r_2(P_2,P_1)$;

(P4) $r_2(P_1,P_3) \le r_2(P_1,P_2)$ and $r_2(P_1,P_3) \le r_2(P_2,P_3)$ if $P_1 \subseteq P_2 \subseteq P_3$ for a PFS P_3, where α, β are the weights of the two elements (i.e., membership degree, non-membership degree) in a PFS and $\alpha + \beta = 1$. Especially, when $\alpha = \beta = 1/2$, formula (9) reduces to formula (3).

With the help of the proof of Proposition 1, Proposition 2 can be proved.

Furthermore, if the important differences are considered in the elements in a universe of discourse $X = \{x_1, x_2, \ldots$, the weight w_i from every element x_i $(i = 1, 2, \cdots$ is needed to be taken into account. If w_i is the weight from element x_i $(i = 1, 2, \cdots$, $w_i \in [0,1]$, and $\sum_{i=1}^{n} w_i = 1$, and then the weighted similarity measure is given in next proposition.

Proposition 3 Assume that $X = \{x_1, x_2, \ldots$ is a given set, P_1 and P_2 are two PFSs, the 3-type PFSs weighted similarity measure

$$r_3(P_1,P_2) = \sum_{i=1}^{n} w_i\left(\alpha\frac{\min(u_{P_1}(x_i),u_{P_2}(x_i))}{\max(u_{P_1}(x_i),u_{P_2}(x_i))} + \beta\frac{\min(v_{P_1}(x_i),v_{P_2}(x_i))}{\max(v_{P_1}(x_i),v_{P_2}(x_i))}\right) \tag{10}$$

should satisfy the following properties:

(P1) $0 \le r_3(P_1,P_2) \le 1$;

(P2) $r_3(P_1,P_2)=1$ if $P_1 = P_2$;

(P3) $r_3(P_1,P_2)=r_3(P_2,P_1)$;

(P4) $r_3(P_1,P_3) \le r_3(P_1,P_2)$ and $r_3(P_1,P_3) \le r_3(P_2,P_3)$ if $P_1 \subseteq P_2 \subseteq P_3$ for a PFS P_3, where α, β are the weights of the two elements (i.e., membership, non-membership) in a PFS and $\alpha+\beta=1$. Especially, when $w_1 = w_2 = \cdots /n$, formula (10) reduces to formula (9).

Proof of Proposition 3 can be obtained from the proof of Proposition 1.

Example 1. Assume that there are three PFSs in a universe of discourse $X=\{x_1,x_2,x_3\}$:

$$P_1=\{\langle x_1,0.3,0.8\rangle,\langle x_2,0.4,0.7\rangle,\langle x_3,0.5,0.6\rangle\};$$

$$P_2=\{\langle x_1,0.5,0.7\rangle,\langle x_2,0.5,0.4\rangle,\langle x_3,0.7,0.5\rangle\};$$

$$P_3=\{\langle x_1,0.9,0.3\rangle,\langle x_2,0.8,0.3\rangle,\langle x_3,0.8,0.4\rangle\},$$

and $P_1 \subseteq P_2 \subseteq P_3$. By using formula (2), the 1-type similarity measures are as follows:

$$r_1(P_1,P_2)=\frac{1}{6}\left(\frac{0.3}{0.5}+\frac{0.7}{0.8}+\frac{0.4}{0.5}+\frac{0.4}{0.7}+\frac{0.5}{0.7}+\frac{0.5}{0.6}\right)\approx 0.7323;$$

$$r_1(P_1,P_3)=\frac{1}{6}\left(\frac{0.3}{0.9}+\frac{0.3}{0.8}+\frac{0.4}{0.8}+\frac{0.3}{0.7}+\frac{0.5}{0.8}+\frac{0.4}{0.6}\right)\approx 0.4881;$$

$$r_1(P_2,P_3)=\frac{1}{6}\left(\frac{0.5}{0.9}+\frac{0.3}{0.7}+\frac{0.5}{0.8}+\frac{0.3}{0.4}+\frac{0.7}{0.8}+\frac{0.4}{0.5}\right)\approx 0.6724,$$

thus, $r_1(P_1,P_3) \le r_1(P_1,P_2)$ and $r_1(P_1,P_3) \le r_1(P_2,P_3)$ are obtained.

Let the weight values of the two terms in a PFS are $\alpha=0.55$ and $\beta=0.45$, by applying formula (9), then 2-type PFSs weighted similarity measures are as follows:

$$r_2(P_1,P_2)=\frac{1}{3}\left(0.55\times\frac{0.3}{0.5}+0.45\times\frac{0.7}{0.8}+0.55\times\frac{0.4}{0.5}+0.45\times\frac{0.4}{0.7}+0.55\times\frac{0.5}{0.7}+0.45\times\frac{0.5}{0.6}\right)\approx 0.7296;$$

$$r_2(P_1,P_3)=\frac{1}{3}\left(0.55\times\frac{0.3}{0.9}+0.45\times\frac{0.3}{0.8}+0.55\times\frac{0.4}{0.8}+0.45\times\frac{0.3}{0.7}+0.55\times\frac{0.5}{0.8}+0.45\times\frac{0.4}{0.6}\right)\approx 0.4879;$$

$$r_2(P_2,P_3)=\frac{1}{3}\left(0.55\times\frac{0.5}{0.9}+0.45\times\frac{0.3}{0.7}+0.55\times\frac{0.5}{0.8}+0.45\times\frac{0.3}{0.4}+0.55\times\frac{0.7}{0.8}+0.45\times\frac{0.4}{0.5}\right)\approx 0.6736.$$

Therefore, we have $r_2(P_1,P_3) \le r_2(P_1,P_2)$ and $r_2(P_1,P_3) \le r_2(P_2,P_3)$.

Assume that three criteria's weight vector is $w=(0.4,0.3,0.3)$, and weight values of two terms (i.e., membership and non-membership degrees) in a PFS, are $\alpha=0.55$ and $\beta=0.45$. By applying formula (10), the 3-type PFSs weighted similarity measures are as follows:

$$r_3(P_1,P_2)=\left(0.55\times0.4\times\frac{0.3}{0.5}+0.45\times0.3\times\frac{0.7}{0.8}+0.55\times0.3\times\frac{0.4}{0.5}+0.45\times0.4\times\frac{0.4}{0.7}+0.55\times0.3\times\frac{0.5}{0.7}+0.45\times0.3\times\frac{0.5}{0.6}\right)\approx 0.7153;$$

$$r_3(P_1,P_3)=\left(0.55\times0.4\times\frac{0.3}{0.9}+0.45\times0.3\times\frac{0.3}{0.8}+0.55\times0.3\times\frac{0.4}{0.8}+0.45\times0.4\times\frac{0.3}{0.7}+0.55\times0.3\times\frac{0.5}{0.8}+0.45\times0.3\times\frac{0.4}{0.6}\right)\approx 0.4767;$$

$$r_3(P_2,P_3)=\left(0.55\times0.4\times\frac{0.5}{0.9}+0.45\times0.3\times\frac{0.3}{0.7}+0.55\times0.3\times\frac{0.5}{0.8}+0.45\times0.4\times\frac{0.3}{0.4}+0.55\times0.3\times\frac{0.7}{0.8}+0.45\times0.3\times\frac{0.4}{0.5}\right)\approx 0.6706.$$

Thus, there are $r_3(P_1,P_3) \le r_3(P_1,P_2)$ and $r_3(P_1,P_3) \le r_3(P_2,P_3)$.

A Pythagorean Fuzzy TOPSIS Method Based on Similarity Measure for PDSs Selection

In Pythagorean fuzzy environment, a given PDSs selection problem can be described as below.

We assume that $O=\{o_1,o_2,\ldots$ is a given PDSs set, $C=\{c_1,c_2,\ldots$ is a criteria set affecting PDSs selection, and $W=\{w_1,w_2,\ldots$ is weight vector of criteria with $0 \le w_j \le 1$ and $\sum_{j=1}^{m} w_j = 1$. Let $c_j(o_i)=p(u_{ij},v_{ij})$ denotes the evaluation values of the ith PDS o_i $(i=1,2,\ldots$ with regard to the criteria c_j $(j=1,2,\ldots$, then $U=(c_j(o_i))_{n\times m}$ is a Pythagorean fuzzy matrix. Therefore, the PDSs selection problem with Pythagorean fuzzy setting is represented as:

$$U=(c_j(o_i))_{n\times m}=\begin{pmatrix} p(u_{11},v_{11}) & p(u_{12},v_{12}) & \cdots & \ldots,v_{1m}) \\ p(u_{21},v_{21}) & p(u_{22},v_{22}) & \cdots & \ldots,v_{2m}) \\ \vdots & \vdots & \vdots & \vdots \\ p(u_{n1},v_{n1}) & p(u_{n1},v_{n1}) & \cdots & \ldots,v_{nm}) \end{pmatrix}$$

According to the principle of TOPSIS, the distance between the optimal alternative and the positive ideal solution should be as close as possible, and the distance between the optimal alternative and the negative ideal solution should be as far as possible.

Thus, we firstly identify the best suitable PDS and the worst suitable PDS based on the following formulas:

Best suitable PDS:

$$O^{+} = \{o_1^{+}, \dots \quad (11)$$

where $o_j^{+} = \max_i \left(\mathrm{p}(u_{ij}, v_{ij}) \right)$ for benefit criteria, while $o_j^{+} = \min_i \left(\mathrm{p}(u_{ij}, v_{ij}) \right)$ for cost criteria:

Worst suitable PDS:

$$O^{-} = \{o_1^{-}, \dots \quad (12)$$

where $o_j^{-} = \min_i \left(\mathrm{p}(u_{ij}, v_{ij}) \right)$ for benefit criteria, while $o_j^{-} = \max_i \left(\mathrm{p}(u_{ij}, v_{ij}) \right)$ for cost criteria. For brevity, the best and worst suitable PDSs are rewritten as $o_j^{+} = \{c_j, (u_j^{+}, v_j^{+}) | j = 1, 2, \dots$ and $o_j^{-} = \{c_j, (u_j^{-}, v_j^{-}) | j = 1, 2, \dots$, respectively. And the PFN $\mathrm{p}(u_{ij}, v_{ij})$ is represented as (u_{ij}, v_{ij}) $(i = 1, 2, \dots \quad j = 1, 2, \dots$.

Applying the similarity measure in Proposition 3, the 3-type PFSs weighted similarity measures between the PDS o_i and the best suitable PDS O^{+} can be calculated as follows:

$$\mathrm{r}_4(o_i, O^{+}) = \sum_{j=1}^{m} w_j \left(\alpha \frac{\min(u_{ij}, u_j^{+})}{\max(u_{ij}, u_j^{+})} + \beta \frac{\min(v_{ij}, v_j^{+})}{\max(v_{ij}, v_j^{+})} \right) \quad (13)$$

Analogously, the 3-type PFSs weighted similarity measures between the PDS o_i and the worst suitable PDS O^{-} can be calculated as:

$$\mathrm{r}_4(o_i, O^{-}) = \sum_{j=1}^{m} w_j \left(\alpha \frac{\min(u_{ij}, u_j^{-})}{\max(u_{ij}, u_j^{-})} + \beta \frac{\min(v_{ij}, v_j^{-})}{\max(v_{ij}, v_j^{-})} \right) \quad (14)$$

So, the relative closeness value of the alternative o_i with regard to the best suitable PDS O^{-} as follows:

$$r_i = \frac{\mathrm{r}_4(o_i, O^{-})}{\mathrm{r}_4(o_i, O^{-}) + \mathrm{r}_4(o_i, O^{+})}. \quad (15)$$

Obviously, $0 < r_i < 1$. Therefore, the ranking of all PDSs can be obtained by the relative closeness coefficient r_i. The greater the value r_i, the better suitable the corresponding PDS.

Based on the above statements, we further give a selection process using the proposed decision making method for PDS selection, as shown in Figure 3.

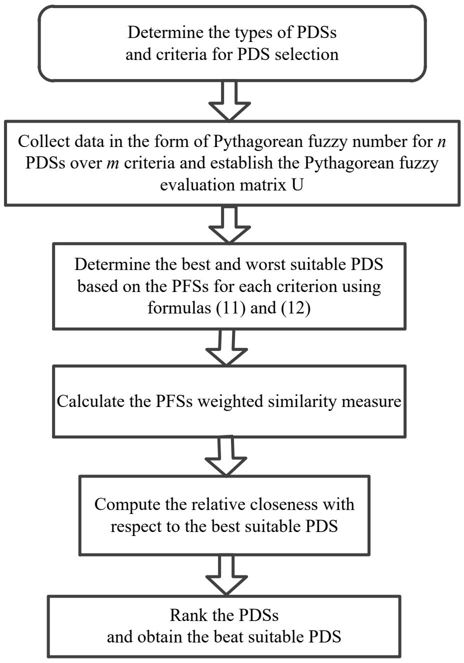

Figure 3. The Selection Process for PDS Selection Using the Proposed Method

Step 1: For a selection problem for PDS, the set of PDSs $O = \{o_1, o_2, \dots$ and the set of criteria $C = \{c_1, c_2, \dots$ are firstly determined. Furthermore, let the weight vector of criteria be $W = \{w_1, w_2, \dots$. Then, the Pythagorean fuzzy matrix $U = \left(c_j(o_i)\right)_{n \times m}$ is constructed, where $c_j(o_i) = \mathrm{p}(u_{ij}, v_{ij})$ is the evaluation value of PDS under the *j*th criteria c_j $(i = 1, 2, \dots \quad \dots$.

Step 2: Using formulas (11) and (12), we identify the best suitable PDS $O^{+} = \{o_1^{+}, o_2^{+}, \dots$ and the worst suitable PDS $O^{-} = \{o_1^{-}, o_2^{-}, \dots$.

Step 3: By formulas (13) and (14), the 3-type PFSs weighted similarity measures between each PDS o_i and the best suitable PDS O^{+} and the worst suitable PDS O^{-} are calculated, $i = 1, 2, \dots$.

Step 4: From formula (15), we compute the relative closeness value r_i of PDS o_i about the best suitable PDS O^{+}, $i = 1, 2, \dots$.

Step 5: Ranking the *m* PDSs from the results obtained in Step 4.

Case Study

There is a real-world infrastructure project, the owner intends to select the most applicable delivery system from four delivery systems including design-build (DB), engineering-procurement-construction (EPC), construction management at risk method (CM at-Risk), and design-bid-build (DBB), and their criteria and decision making framework are discussed in Section 2. To ensure the reliability and availability of data, the experienced experts from different fields should be invited to evaluate the project before carrying out the work.

Generally speaking, five or seven experts are invited in traditional engineering project to select PDSs. Firstly, the owners introduced their capacity and the goal of project. Secondly, further investigation to the construction site was conducted, and the related principals described the whole project in detail. Finally, according to the score chart and score criterion in advance, the evaluation results of the project from experts were obtained. Considering the practical situation of this project, five experts including engineers, academics, contractors and owners with rich experiences in this filed were invited, and aggregated all the evaluation information. The final result will be the evaluation matrix.

In this selection process, the four project delivery systems (DB, DBB, EPC, CM) form the set of delivery options, which is written as $O=\{o_1,o_2,o_3,o_4\}$. Similarly, the ten criteria (i.e., C, S, Q, Com, SC, E, FG, RM, U, Size) make up the criteria set $C=\{C_1,C_2,C_3,C_4,C_5,C_6,$ $C_7,C_8,C_9,C_{10}\}$ .For convenience, weight vector of criteria will be followed by equal weight method, that is, $w_1 = w_2 = \cdots$ 0.1, though there are many approaches for calculating weight. We assume that $\left(u_{ij}^{(l)},v_{ij}^{(l)}\right)$ $(i=1,2,3,4$, $j=1,2,\ldots$ $,l=1,\ldots$ is the evaluation value from the lth expert to delivery option o_i with respect to criteria c_j , and $U_{4\times10}^{(l)}=\left(U_{ij}^{(l)}\right)_{4\times10}=\left(u_{ij}^{(l)},v_{ij}^{(l)}\right)_{4\times10}$ denotes a PFN evaluation matrix from the lth expert. Every expert should give the evaluation value $\left(u_{ij}^{(l)},v_{ij}^{(l)}\right)$ $(i=1,2,3,4, j=1,\ldots$, the weight of each expert is a convenient way to set $W_1=\cdots$ 0.2 . To utilize the proposed approach, we also assume that weights of membership and non-membership degrees are α=0.55 and β=0.45, respectively.

We assume that the Pythagorean fuzzy evaluation values matrixes are $U_{4\times10}^{(l)}=\left(U_{ij}^{(l)}\right)_{4\times10}=\left(u_{ij}^{(l)},v_{ij}^{(l)}\right)_{4\times10}$ $(l=1,\ldots$, where

$$U_{4\times10}^{(1)}=\begin{pmatrix} p(0.7,0.3) & p(0.7,0.5) & p(0.6,0.5) & p(0.6,0.2) & p(0.6,0.2) \\ p(0.8,0.4) & p(0.8,0.4) & p(0.7,0.4) & p(0.7,0.4) & p(0.8,0.4) \\ p(0.6,0.4) & p(0.5,0.3) & p(0.6,0.3) & p(0.8,0.3) & p(0.5,0.3) \\ p(0.5,0.2) & p(0.7,0.5) & p(0.7,0.3) & p(0.6,0.3) & p(0.7,0.3) \end{pmatrix};$$

$$\begin{pmatrix} p(0.6,0.3) & p(0.7,0.4) & p(0.6,0.4) & p(0.4,0.4) & p(0.6,0.3) \\ p(0.6,0.2) & p(0.8,0.5) & p(0.8,0.5) & p(0.6,0.3) & p(0.7,0.2) \\ p(0.8,0.4) & p(0.7,0.3) & p(0.8,0.2) & p(0.7,0.2) & p(0.4,0.5) \\ p(0.7,0.5) & p(0.8,0.4) & p(0.7,0.3) & p(0.4,0.2) & p(0.6,0.4) \end{pmatrix}$$

$$U_{4\times10}^{(2)}=\begin{pmatrix} p(0.5,0.7) & p(0.6,0.7) & p(0.8,0.4) & p(0.8,0.4) & p(0.6,0.7) \\ p(0.8,0.5) & p(0.9,0.2) & p(0.7,0.4) & p(0.7,0.6) & p(0.7,0.5) \\ p(0.4,0.8) & p(0.3,0.6) & p(0.7,0.5) & p(0.8,0.3) & p(0.5,0.7) \\ p(0.4,0.2) & p(0.8,0.3) & p(0.8,0.5) & p(0.6,0.3) & p(0.4,0.8) \end{pmatrix};$$

$$\begin{pmatrix} p(0.7,0.3) & p(0.6,0.4) & p(0.8,0.5) & p(0.8,0.3) & p(0.6,0.7) \\ p(0.8,0.3) & p(0.8,0.5) & p(0.8,0.3) & p(0.6,0.7) & p(0.8,0.5) \\ p(0.6,0.7) & p(0.6,0.5) & p(0.9,0.3) & p(0.6,0.4) & p(0.4,0.6) \\ p(0.8,0.4) & p(0.6,0.7) & p(0.7,0.7) & p(0.8,0.5) & p(0.9,0.2) \end{pmatrix}$$

$$U_{4\times10}^{(3)}=\begin{pmatrix} p(0.7,0.7) & p(0.7,0.6) & p(0.7,0.6) & p(0.6,0.7) & p(0.6,0.6) \\ p(0.7,0.5) & p(0.7,0.6) & p(0.6,0.6) & p(0.5,0.7) & p(0.5,0.6) \\ p(0.6,0.7) & p(0.4,0.8) & p(0.8,0.5) & p(0.3,0.8) & p(0.3,0.8) \\ p(0.6,0.7) & p(0.6,0.7) & p(0.7,0.5) & p(0.6,0.7) & p(0.5,0.6) \end{pmatrix};$$

$$\begin{pmatrix} p(0.8,0.5) & p(0.5,0.8) & p(0.7,0.7) & p(0.7,0.7) & p(0.7,0.6) \\ p(0.5,0.5) & p(0.5,0.5) & p(0.7,0.6) & p(0.6,0.7) & p(0.6,0.6) \\ p(0.6,0.7) & p(0.5,0.7) & p(0.5,0.7) & p(0.4,0.8) & p(0.3,0.8) \\ p(0.6,0.7) & p(0.6,0.8) & p(0.7,0.7) & p(0.4,0.8) & p(0.6,0.6) \end{pmatrix}$$

$$U_{4\times10}^{(4)}=\begin{pmatrix} p(0.4,0.5) & p(0.4,0.5) & p(0.4,0.4) & p(0.6,0.5) & p(0.6,0.5) \\ p(0.9,0.4) & p(0.8,0.3) & p(0.8,0.7) & p(0.6,0.4) & p(0.4,0.4) \\ p(0.6,0.7) & p(0.6,0.6) & p(0.6,0.6) & p(0.4,0.3) & p(0.2,0.3) \\ p(0.1,0.3) & p(0.2,0.7) & p(0.2,0.3) & p(0.8,0.4) & p(0.8,0.3) \end{pmatrix};$$

$$\begin{pmatrix} p(0.4,0.5) & p(0.6,0.3) & p(0.6,0.3) & p(0.6,0.3) & p(0.4,0.4) \\ p(0.8,0.5) & p(0.2,0.3) & p(0.4,0.4) & p(0.5,0.4) & p(0.8,0.3) \\ p(0.6,0.5) & p(0.4,0.3) & p(0.8,0.5) & p(0.8,0.3) & p(0.6,0.5) \\ p(0.2,0.3) & p(0.8,0.2) & p(0.2,0.3) & p(0.7,0.5) & p(0.2,0.5) \end{pmatrix}$$

$$U_{4\times10}^{(5)}=\begin{pmatrix} p(0.7,0.5) & p(0.4,0.6) & p(0.5,0.3) & p(0.3,0.5) & p(0.8,0.3) \\ p(0.5,0.2) & p(0.6,0.4) & p(0.6,0.4) & p(0.6,0.3) & p(0.5,0.4) \\ p(0.6,0.3) & p(0.5,0.2) & p(0.6,0.3) & p(0.5,0.4) & p(0.6,0.3) \\ p(0.6,0.4) & p(0.7,0.2) & p(0.7,0.3) & p(0.7,0.4) & p(0.4,0.2) \end{pmatrix}.$$

$$\begin{pmatrix} p(0.4,0.3) & p(0.7,0.6) & p(0.3,0.8) & p(0.4,0.5) & p(0.3,0.6) \\ p(0.7,0.4) & p(0.5,0.3) & p(0.5,0.4) & p(0.7,0.3) & p(0.6,0.4) \\ p(0.5,0.3) & p(0.6,0.5) & p(0.3,0.7) & p(0.4,0.3) & p(0.4,0.5) \\ p(0.7,0.2) & p(0.5,0.2) & p(0.7,0.3) & p(0.8,0.2) & p(0.7,0.2) \end{pmatrix}$$

Using the proposed method, the steps of the PDS selection are as follows:

Step 1: Construction of Pythagorean fuzzy evaluation matrix through aggregating the evaluation information of five experts. By Definition 5, the Pythagorean fuzzy evaluation matrix is determined

$$U_{4\times10}=\left(p_{ij}\left(\sum_{l=1}^{5}W_l u_{ij}^{(l)},\sum_{l=1}^{5}W_l v_{ij}^{(l)}\right)\right)_{4\times10}$$

$$=\begin{pmatrix} p(0.60,0.54) & p(0.56,0.58) & p(0.60,0.44) & p(0.58,0.46) & p(0.64,0.46) \\ p(0.74,0.40) & p(0.76,0.38) & p(0.68,0.50) & p(0.62,0.48) & p(0.58,0.46) \\ p(0.56,0.58) & p(0.46,0.50) & p(0.66,0.44) & p(0.56,0.42) & p(0.42,0.48) \\ p(0.44,0.36) & p(0.60,0.48) & p(0.62,0.38) & p(0.66,0.42) & p(0.56,0.44) \end{pmatrix}$$

$$\begin{pmatrix} p(0.58,0.38) & p(0.62,0.50) & p(0.60,0.54) & p(0.58,0.44) & p(0.52,0.52) \\ p(0.68,0.38) & p(0.56,0.42) & p(0.64,0.44) & p(0.60,0.48) & p(0.70,0.40) \\ p(0.62,0.52) & p(0.56,0.46) & p(0.66,0.48) & p(0.58,0.40) & p(0.42,0.58) \\ p(0.60,0.42) & p(0.66,0.46) & p(0.60,0.46) & p(0.62,0.44) & p(0.60,0.38) \end{pmatrix}$$

Step 2: Based on formulas (11) and (12), the best suitable PDS and the worst suitable PDS can be calculated as follow:

$$O^{+}=\{(0.56,0.58),(0.76,0.38),(0.66,0.44),$$
$$(0.56,0.42),(0.42,0.48),(0.68,0.38),$$
$$(0.66,0.46),(0.64,0.44),(0.60,0.48),(0.70,0.40)\};$$

$$O^{-}=\{(0.74,0.40),(0.46,0.50),(0.60,0.44),(0.66,0.42),(0.64,0.46),(0.62,0.52),(0.56,0.46),(0.60,0.54),(0.62,0.44),(0.42,0.58)\}.$$

Step 3: By formula (13), the 3-type PFSs weighted similarity measures between each PDS o_i and the best suitable PDS O^+ are calculated below:

$$r_5(o_1,O^+)=0.8806;\ r_5(o_2,O^+)=0.9309;$$

$$r_5(o_3,O^+)=0.8857;\ r_5(o_4,O^+)=0.8989.$$

Analogously, we obtain that the similarity measures between the PDS o_i and the worst suitable O^- are as follows:

$$r_5(o_1,O^-)=0.9080;\ r_5(o_2,O^-)=0.8674;$$

$$r_5(o_3,O^-)=0.9209;\ r_5(o_4,O^-)=0.8844.$$

Step 4: From formula (15), the relative closeness value r_i from the PDS o_i with regard to the best suitable PDS O^+ is calculated:

$$r_1=0.5077;\ r_2=0.4823;\ r_3=0.5097;\ r_4=0.4959.$$

Step 5: According to the results in Step 4, we obtain $r_3 > r_1 > r_4 > r_2$, that is, EPC > DB > DBB > CM. Therefore, EPC is the best choice among the four options. According to the sorted results, it is suitable to accept for practical application.

The research plays an important role to give a reasonable reference for owners using the proposed ways in selection of PDSs. The existing literature about PDS selection methods, such as Alhazmi and McCaffer (2000), Chen *et al*. (2011) and Li *et al*. (2015), always require the experts to provide the fuzzy characteristic of criteria with the sum of non-membership and membership degrees smaller than one. It actually constraints the judgement of experts in practical. However, it is an inevitable reality when experts give their evaluation preferences with a sum from membership and non-membership degrees is larger than one, unlike discussed in intuitionistic fuzzy set. PFS, which extends IFS, gives a wide thinking space for experts with a more general condition, that is, $u_A^2+v_A^2\le 1$. As shown in Figure 4, the filed I is the thinking space using IFS theory, and the filed I+II when using PFS theory.

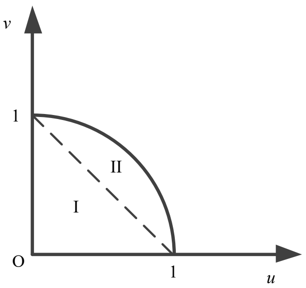

Figure 4. The Fields of IFS and PFS

Conclusions

In construction projects, PDS defined the relationship between these stakeholders. It includes contractual and arrangements that allow the owner to gain a complete facility that meet their needs. The success or failure of a construction project is the selection approach of a PDS. Multi-criteria decision making method was an efficient tool for PDS selection problem.

This research reviewed the literature on PDS selection decision making methods and found a research gap in knowledge related to PDS selection through a Pythagorean fuzzy TOPSIS approach using the proposed similarity measure. Based on it, the main contribution of this paper are as follows. (1) Three new similarity measures under PFSs based on minimum and maximum operators are presented, and their properties are investigated. (2) An improved TOPSIS method using the proposed similarity measures was established. (3) To obtain suitable PDS considering completely the "true psychological" behavior of decision experts, the selection model of PDS under Pythagorean fuzzy setting is given based on improved TOPSIS approach.

In general, the PDS selection problem is one key task for owners, and the MCDM problem is a hotspot in academic filed because of uncertainty in practical. The Pythagorean fuzzy set has aroused more and more attention. Meanwhile, similarity measure, which describes the relatedness between PFSs, is an important tool to measure similarity degree between two objects. Integrated their advantage, this paper established the selection model of PDS and applied it to a case study about selection of project delivery system. It has great theoretical and realistic significance for owners to select appropriate PDS. In the later study research, interval Pythagorean fuzzy environment and others application areas could be considered for the proposed method.

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Conflicts of Interest

The authors declare that they have no conflicts of interest.

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