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Comparative Analysis of Major Stock Markets in Relation to Global Financial Crises: Evidence from 2000 to the COVID-19 Pandemic

Doobae Jun[1], Jinsu Kim[2], Gwangil Kim[3*]

[1,2,3]Department of Mathematics, Gyeongsang National University
Jinju, 52828, Republic of Korea
E-mail: [1]dbjun@gnu.ac.kr; [2]urgoodjob@naver.com; [3]gikim@gnu.ac.kr [*]Corresponding author*

[1,3]Research Institute of Natural Science, Gyeongsang National University
Jinju, 52828, Republic of Korea

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We analyze the impact of financial crises on major stock markets from 2000 to the COVID-19 pandemic using Fourier series. Analyzing the behaviors of the spectra obtained from monthly returns of their indices, we identify three global financial crises from 2000 to 2015, with different characteristics. In addition, applying Z-test and the color-contour plotting method to monthly propagations of the spectra of major frequencies from the monthly returns of each index, we analyze the developments in each market around the crises by comparing patterns in the color-contour plots. Using recent status analysis, we identify an instability around 2016 close to a real crisis; starting in 2020, the markets, which had already recovered from this instability have generated abnormal signals of an approaching crisis. Applying Z-test and color-contour plotting to monthly propagations from the recent status, we show that recent developments in major markets might be more serious than those occurring around previous financial crises.

Keywords: *Stock Index Return; Financial Crisis; Fourier Series; Spectral Analysis; COVID-19.*

Introduction

In financial phenomena, various signals are generated by participants and systems engaged in economic activity. When the normal rhythms of financial systems and processes are disrupted or abnormal behaviors of participants are reinforced and become frequent, strong signals emerge in many markets, indicating the occurrence of unusual events. By accurately capturing these signals and carefully analyzing them, we can receive warnings about crises that are likely to hit the market; furthermore, if these crises actually occur, countermeasures and strategies for the “rainy season” can be effectively planned and implemented in advance. This perspective has been widely generalized and used in various fields that study financial phenomena (Feng & Palomar, 2015; Akansu *et al.*, 2016; Edison, 2003; Chaudhuri, 2018). However, it has primarily focused on financial time series analysis or research on models that accurately reproduce financial signals and portfolio optimization or financial index behavior tracking as applications (Feng & Palomar, 2015; Akansu et al., 2016), including the development of an early warning system (Edison, 2003). Unlike previous studies that only focused on financial signals or their performance, this study focuses on the internal components of financial signals that are believed to contain information about the participants and systems that produce them. We attempt to explain what happened in the markets in greater depth by precisely analyzing their structural and dynamic characteristics during abnormal conditions such as financial crises and by performing comprehensive cross-crisis and cross-country comparisons among the major stock markets. We confirm whether they have the potential to be used as warning indicators for an impending financial crisis.

Concretely, in this study, we analyze the impact of financial crises on major stock markets since 2000 as well as of the recent COVID-19 pandemic. For the 2000–2020 period, several significant global financial crises have occurred: the 2000–2002 dot com bubble burst (Goodnight *et al.*, 2010), the 2008–2009 global financial crisis (Stiglitz, 2010; Helleiner, 2011), the 2010 European sovereign crisis (Lane, 2012), and the ongoing crisis due to COVID-19. We find that several unusual signals occurred in several markets around these crises, which was deeply related to the structural imbalance in the system and the subsequent behavior of market participants, represented by the abnormal behavior of key financial indicators (Lane, 2012; Wheale & Amin, 2003; Lane & Milesi-Ferretti, 2011). However, as observed in previous studies, such key financial indicators are strongly related to macroeconomics in the event of a financial crisis (Lane, 2012; Lane & Milesi-Ferretti, 2011) or are directly related to financial signals in stock markets (Wheale & Amin, 2003). Therefore, it appears to be the most effective to intensively analyze the signals hidden in the time series of the returns of each stock index, which is one of the most important indicators of stock markets, to extract useful and key information related to financial crises from the signals occurring in the stock market. To achieve such goals, we use the Fourier series approximation method, which is one of the most popular and powerful tools in extracting the internal components of a target signal and analyzing their structural and behavioral characteristics by observing the behavior of their spectra.

Through detailed analysis of Fourier series approximations and their spectra to all the signals within a period of one year from the monthly returns of each stock index, we examine the developments in major stock markets, the differences between them, and the characteristics of each financial crisis around the global financial crises. In addition, to assess the behavioral characteristics of the spectra obtained by the Fourier series approximations, we perform complete cross-mode and cross-country comparisons, identifying three global financial crises, all with different behavioral characteristics, in the spectra propagations from 2000 to 2015. Applying color-contour plotting (Jun *et al.*, 2019) and visualizing the monthly propagations of major spectra for all major markets, we confirm that the behavioral patterns in their propagations can help identify the crises. In our previous work (Jun *et al.*, 2019), we examined the characteristics of the behavioral patterns of spectra in the US market in the early 2000s, including the year 2008. However, in this study, by improving the visualization precision of the previous method, we simultaneously perform the cross-crisis and cross-country comparison of the structural and behavioral characteristics of the spectral patterns for the major stock markets; from the results, we identify in depth the differences and commonalities in the responses of the major markets to financial crises. Additionally, from these analyses, we show that the spectra in lower-frequency modes can serve as sensitive indicators of impending crises, including how the abnormal instability of markets has transitioned along the modes.

Furthermore, through recent status analysis (since 2015), we show that there was significant instability around 2016, which could have amplified to become a real global crisis. While major markets had almost recovered from this instability by the second half of 2019, since the start of 2020, all major spectra of these markets started behaving violently and abnormally, as if suddenly facing an unexpected immense shock. We show that the scale of the abnormality in 2020 is much larger than that in 2008 by using the monthly propagations of the major spectra of each stock index and color-contour plotting, which might be a strong warning sign of an approaching crisis. This strongly suggests that our behavioral pattern analysis method for the spectra in major modes could also be useful as a tool for diagnosing and providing an early warning of the crisis.

The results of this research have two major contributions. First, we contribute to the literature by establishing a new way for researchers and market participants to use the Fourier series in order to detect the early warning signs of an impending economic crisis. Second, we contribute to market information research by analyzing the internal components of the financial signals deriving information from market participants or economic systems.

To the best of our knowledge, until date, no research has used spectra in the lower-frequency modes of Fourier series as sensitive indicators of an approaching crisis.

The rest of this paper is organized as follows: In Section 2, we present a review of the related literature. In Section 3, we introduce our research methodologies. In Section 4, we discuss our first result: cross-mode and cross-country comparisons for major mode spectra obtained from the Fourier series approximation and analyze abnormal behaviors around global financial crises. In Section 5, we present our second result: comparative analysis of the patterns of crisis propagations observed in major stock markets and discuss the characteristics of the crises. In Section 6, we elaborate on our third result: analysis of the current states of major markets, including the current unstable situation caused by the COVID-19 pandemic, and discuss their implications. In Section 7, we conclude the paper and propose directions for further research.

Literature Review

Global financial crises, from the tulip mania in 1636 to the recent European sovereign debt crisis, have been studied from various perspectives and have provided meaningful lessons (Kindleberger & Aliber, 2005). For example, several studies have examined the 2008 financial crisis, widely considered to be the worst since the 1930s (Eigner & Umlauft, 2015). While Blanchard et al. (2010) and Berkmen et al. (2012) attribute the crisis to levels of trade and financial exposure, Claessens et al. (2010) find that asset price bubbles and current account deficits explain the intensity of this crisis, and that increased financial integration explains its global propagation. Frankel and Saravelos (2012) argue that international reserves and overvaluations of the real exchange rate were relevant indicators for predicting the crisis. Lane and Milesi-Ferretti (2011) suggest that the ratio of private credit to GDP, current account deficits, and openness to trade were also key determinants. Rose and Spiegel (2010, 2012) explain differences in the intensity of the crisis' effects between countries. Trancoso (2014) find that high levels of real and financial interdependence between economies were determinants of the global recession after the crisis.

Since the global financial crises of the 2000s, studies analyzing financial crises from traditional social-scientific perspectives (see Edison, 2003; Goodnight *et al.*, 2010; Helleiner, 2011, and references therein), as well as more structural and dynamic natural scientific perspectives (Scalas, 2005), have increased. This is probably because the world economy is deeply related to the overall situation of the modern world and is organized, globalized, and linked through diverse networks of relationships. In addition, recently, based on various financial engineering and network models, many studies have attempted to explain global financial crises from the perspective of the dynamics of structures/networks and their evolution (Fang & Oosterlee, 2011; Angabini & Wasuzzaman, 2011; Lee & Nobi, 2018; Coquide *et al.*, 2020).

From a dynamic point of view, which is strongly related to our work and to artificial economics (Palmer *et al.*, 1994; Mathieu *et al.*, 2005, p. 255), a financial crisis can be attributed to severe imbalances and instability in financial systems and processes (Landau, 2009; Choi & Douady, 2012). This suggests that we need to understand the dynamic states of financial systems, including market structures and agent behaviors (Mathieu *et al.*, 2005, p. 255), to analyze changes in various financial process measures. Therefore, it is worth paying attention to studies on financial signals such as economic time series. For instance, Baxter and King (1999) isolate cyclic fluctuations in economic time series using band-pass filters. Malliavin and Mancino (2002) compute the time series volatility of semi-martingale market data using Fourier series analysis. Addo et. al. (2013) analyze the dynamics of financial time series using delay vector variance. Manchado et al. (2011) analyze the dynamic properties of financial data series, using Fourier transforms. The authors of this paper (Jun et al., 2019) perform signal analysis of DJI behaviors around financial crises using Fourier series. From the viewpoint of artificial economics, LeBaron et al. (1999) show that the artificial stock market can replicate certain time series features from real markets, and Aydin and Cavdar (2015) develop an early warning system to predict financial crises with artificial neural networks empirically analyzing the Turkish economy during 1990 and 2014. Finally, Caccioli et al. (2018) summarize recent developments in the network modeling of financial risk. Our research follows these dynamic perspectives and focuses on analyzing signals generated in stock markets.

Methodology

To analyze the developments in major stock markets between 2000 and 2015, we observe how the indices of these markets behaved during that period. For the stock indices, we select the Dow Jones index (DJI; US), FTSE (UK/GB), CAC (France/FR), CDAX (Germany/DE), and TOPIX (Japan/JP) from five major countries in the top six of the financial asset ranking list. The raw data for each index are taken from the IMF's International Financial Statistics (IMF, 2015).

Next, we introduce our four main methodologies: First, we compute the monthly returns of each index from the raw data as follows: if $I_n(k)$ is the stock index for the kth month of year n, the return of the stock index, $R_n(k)$, is expressed as:

$$R_n(k) = \frac{I_n(k+1) - I_n(k)}{I_n(k)} \ (1 \leq k \leq 11), \tag{1}$$

$$R_n(12) = \frac{I_n(1) - I_n(12)}{I_n(12)}. \tag{2}$$

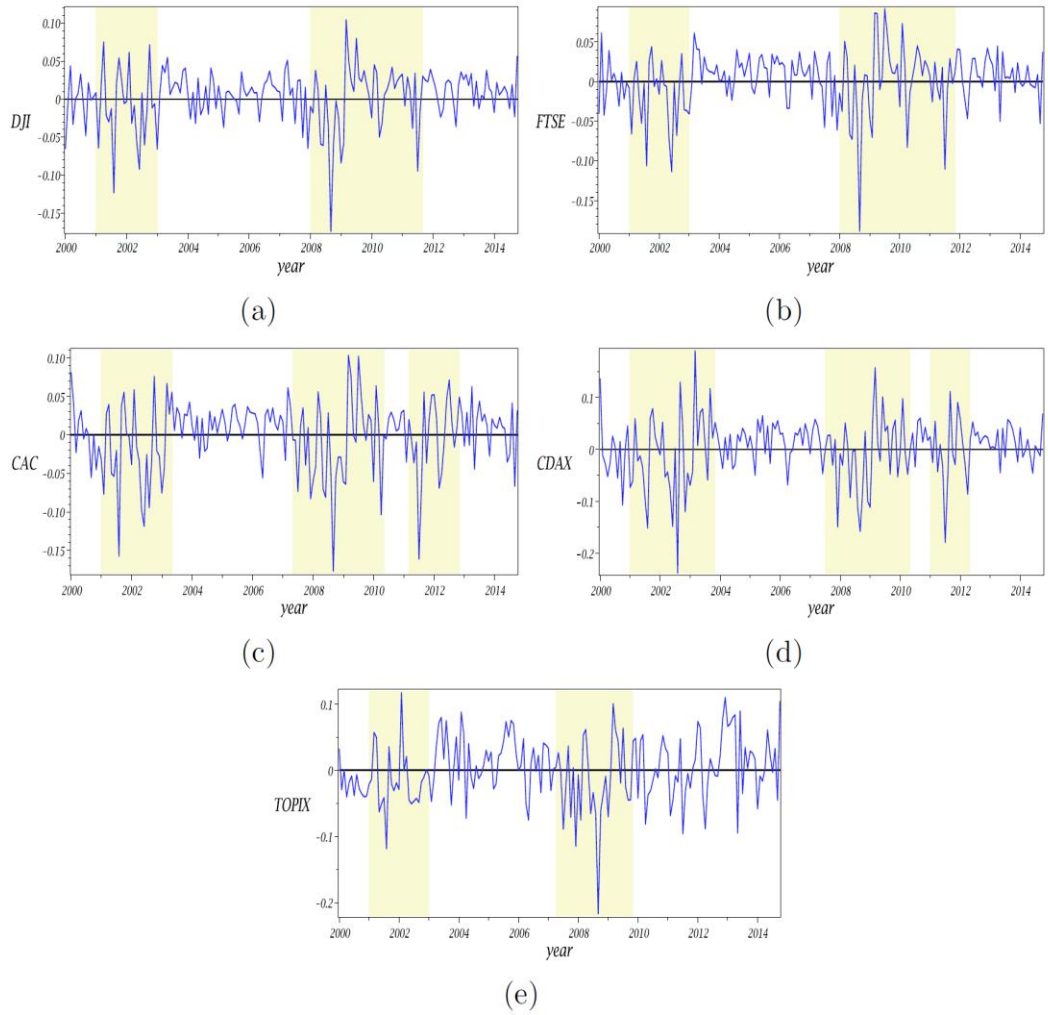

Figure 1. Comparison of the time series of monthly returns for the stock indices of major countries: (a) USA (DJI); (b) UK (FTSE); (c) France (CAC); (d) Germany (CDAX); (e) Japan (TOPIX)

Figure 1 shows the time series of the monthly returns of the indices, from which we can observe frequent large fluctuations, with different patterns and scales around specific periods. Comparing the colored regions, we find some interesting facts. The DJI and FTSE show similar behaviors during the entire period, while fluctuations of the CAC and CDAX differ in terms of scale despite the similarity in their patterns. Further, TOPIX behaves differently from the other indices, except for the period around 2008. These findings imply that important information might be hidden in the time series.

Next, to analyze these facts more closely, we introduce Fourier series approximation to the time series of monthly returns as follows: Let $f(t)$ be a real valued integrable function defined on $[-L, L]$. Then, the Fourier series of $f(t)$ is given by:

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\left(\frac{n\pi}{L}t\right) + b_n \sin\left(\frac{n\pi}{L}t\right)\right), \quad (3)$$

where

$$a_0 = \frac{1}{L}\int_{-L}^{L} f(t)dt,$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(t)\cos\left(\frac{n\pi}{L}t\right)dt,$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(t)\sin\left(\frac{n\pi}{L}t\right)dt.$$

We can transform Eq. (3) to the following cosine series as:

$$f(t) = C_0 + \sum_{n=1}^{\infty} C_n \cos\left(\frac{n\pi}{L}t - p_n\right), \quad (4)$$

where $C_0 = \frac{a_0}{2}, C_n = \sqrt{a_n^2 + b_n^2}$ $(n \geq 1)$, and $p_n = tan^{-1}(b_n/a_n)$. $C_0 = \frac{a_0}{2}$ shows the average behavior of $f(t)$ on the given interval, and $C_n = \sqrt{a_n^2 + b_n^2}$ means the weight of the wave component with frequency $\frac{n\pi}{L}$ in $f(t)$, called the spectrum of Fourier series for frequency $\frac{n\pi}{L}$ (or period $T = \frac{2\pi}{\frac{n\pi}{L}} = \frac{2L}{n}$).

Then, we apply the Fourier series approximation to the time series of monthly returns as follows: First, we select 12 consecutive monthly returns for a target index. Second, we create a signal consisting of the 12 returns, a piecewise linear function on an interval of length 12, represented by a broken line (see Figure 2 (a)). Finally, we complete the Fourier series approximation for the signal within the expected accuracy given by a suitable n.

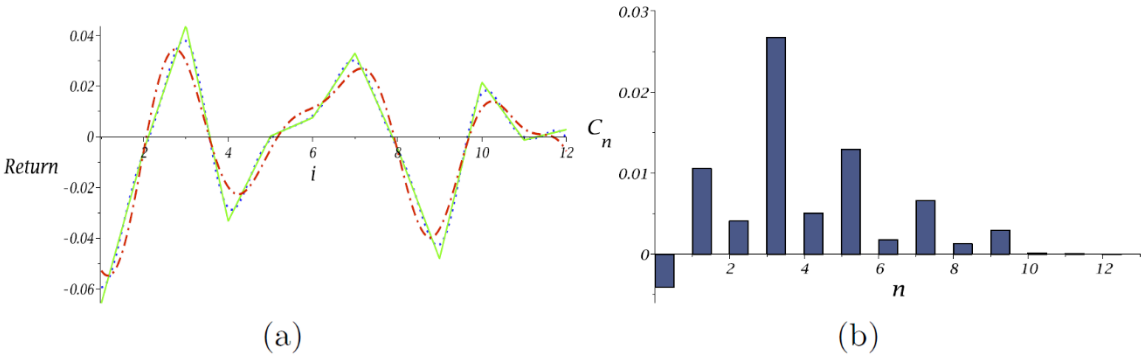

Figure 2. Fourier Series Approximation of a Signal Consisting of 12 Monthly Returns. (a) the Red Dash-Dot (Blue-dot) Curve Denotes the Fourier Series Approximation to the Original Signal Represented by the Green Broken Line When $0 \leq n \leq 6$ ($0 \leq n \leq 12$). (b) the Spectra in the Approximation When $0 \leq n \leq 12$

Figure 2 shows the Fourier series approximation to the signal consisting of 12 monthly returns of the DJI in 2000 and the 12 dominant spectra from the approximation. From Figure 2 (a), the approximation is good enough if $n \geq 6$ and almost perfect when $n = 12$. Additionally, from Figure 2 (b), the spectra of odd n are relatively larger with large fluctuations, while those with even n are generally small. For our analysis, it is enough when $n \leq 7$. In the next sections, by analyzing the behaviors of the spectra C_n($n \leq 7$ and sometimes $n \leq 4$), we determine the regional and behavioral characteristics of the major markets around global financial crises.

Additionally, we introduce the monthly propagation of signals with period 12 to analyze the time series of the spectra from the Fourier series approximation to them, as follows. We choose a signal consisting of 12 consecutive data points from a given time series. Next, after deleting the first observation from the original signal and adding a 13th observation at the end of the raw time series, we obtain the new signal that replaces the old one. We repeat this process for each month. Applying the Fourier series approximation to this monthly propagation, we can obtain the monthly propagations of the spectra of the major modes for all markets and then perform cross-mode and cross-country comparisons (see Figures 3 and 4).

Furthermore, to properly analyze the monthly propagation of the spectra of major modes, it is necessary to specify their observation times, since each original signal consists of 12 consecutive monthly returns. Basically, the spectra show information about the states of a market during the period of each signal. Therefore, to detect changes in the state of a market, we analyze their monthly propagations. To determine when abnormal signals start, we measure them at the end of that period and assign the end-time of each signal to the observation time of the spectra obtained from the signal; this criterion for the observation time point is equally applied to all subsequent measurements in this analysis.

Finally, to more precisely analyze the behavior of C_n around global crises, we first perform a Z-test for the time series of C_n $(0 \leq n \leq 7)$ for each market: a two-sided Z-test for the time series of C_0 and a one-sided Z-test for the others. In performing these Z-tests, we use the mean and standard deviation obtained from the 60 consecutive monthly data observations of each C_n between January 2000 and December 2004, and the confidence levels for determining the abnormal states (large and exceptionally large values) of C_n are specified by the statistics from the distribution of the reference dataset. (The practical details of the statistics for each market are given in Table 1.) An exceptionally large value in each time series of C_n is chosen as follows: First, we select the large values of C_n over the 95% confidence level computed with the given statistics. Second, from the selected large values, we again choose the values over the 99% confidence level, called exceptionally large values. Third, for the large and exceptionally large values of C_n, we assign the levels of their abnormality proportional to the reference value given by $(C_n - \mathrm{L})/\mathrm{SD} + 1$, where L denotes the value of a 95% confidence level and SD is the standard deviation of the reference dataset. Note that in performing the two-sided Z-test, especially for the time series of C_0, large and exceptionally large values of C_0 are also selected on both sides over the two-sided confidence levels.

In addition, for large and exceptionally large values of C_n, we apply color-contour plotting (Jun et al., 2019) as follows: First, we select a market and chronologically arrange all C_n values of its index. Second, to all these large and exceptionally large values, we assign a specific color. Next, we adjust the assigned color so that it gradually becomes darker in proportion to the level determined by the reference value for the original. Third, applying these procedures to all the markets, we obtain the results in Figure 5. Finally, we perform the cross-market and cross-mode comparison simultaneously on how crises started differently and spread differently.

Main Result 1: Cross-Mode and Cross-Country Comparisons for Major Spectra Propagations

First, for the monthly propagation of signals with period 12 of each stock market, we construct their time series of the spectra from the Fourier series approximation to them. We then obtain the monthly propagations of the spectra of the major modes for all markets as shown in Figures 3 and 4. Here, we perform cross-mode and cross-country comparisons.

Figure 3 shows the time series of the spectra of major modes obtained by the Fourier series approximation to the monthly propagations of the signals for the major markets. First, we can easily observe at first glance that during some periods (2001–2003, 2008–2010, and partially 2011–2013), $C_n(n \geq 1)$ fluctuates sharply while C_0 crashes, which is a sign of crises. Additionally, the TOPIX shows different behaviors in the monthly propagations of the spectra, except for 2008–2010, which is a sign of crises. Furthermore, the TOPIX shows different behaviors in the monthly propagations of the spectra, except for 2008–2010, which implies that the Japanese stock market worked independently of the other markets except for that period.

Analyzing in more detail, from Figure 3 (a)–(d), we can observe that the characteristics of the three crises— the dot com bubble, the 2008–2009 global financial crisis, and European sovereign debt crisis—clearly differ according to the behaviors of C_1, C_2, and C_3. During the first crisis, in the order of $C_2 \rightarrow C_1 \rightarrow C_2$, spectra C_n showed sharp peaks. During the second one, such peaks were observed in the order of $C_2 \rightarrow C_3$ and C_1, while during the third one, they were observed in the order of $C_1 \rightarrow C_2$ or C_3. Additionally, while C_1 showed commonly sharp peaks in all three crises, C_2 and C_3 behaved remarkably similarly, generating particular patterns during the first and second crises. This implies the C_2 and C_3 with lower frequencies than C_1 can be used as identifiers of the corresponding crisis and can function as crisis indicators. We also find that, unusually, in the case of European markets, C_2 reacted sensitively to the crises, which suggests that the impacts of the crises were concentrated in actual stocks with large weights of mode 2 in their return behaviors.

Next, we analyze how the markets responded to the crises across major modes (see Figure 4). From Figure 4 (a), we find that around 2014, except for TOPIX (JP), the propagations of C_0 in all countries were similar and showed three synchronized crashes, while in the other modes, despite the similarity in propagations, the scales and patterns of the observed fluctuations varied. Moreover, comparing Figure 4 (b)-(e), the peaks in the propagations were much larger for CDAX (DE) and CAC (FR) than for DJI (US) and FTSE (GB), which means that CDAX and CAC were more sensitive to the crises and responded more strongly to them. Furthermore, around the first crash of C_0, all indices showed sharp peaks in the other modes in the order of the spectrum with the highest frequency to that with the lowest frequency: $C_4 \rightarrow C_3$ or $C_2 \rightarrow C_1$. CDAX (DE) responded most strongly, with intense fluctuations in all modes, which implies that the crisis might have partially originated in European markets. Around the second crash, while the DJI (US) and FTSE (GB) showed small peaks, CDAX (DE) and CAC (FR) exhibited very large peaks. All markets simultaneously showed large peaks for C_1 and C_3. Additionally, large peaks for C_1 and C_3 are evident prior to the very large peaks in the subsequent period, which might have been a warning sign of the crisis. Finally, around the third small crash, based on the behaviors of C_n $(n \geq 1)$, the crisis/instability seems to have been largely limited to European markets. All observations reveal the characteristics of the three crises. Finally, unlike the other indices, TOPIX (JP) often exhibited a number of fairly large peaks even during non-crisis periods, which seems to be induced by the unique internal energy of the market.

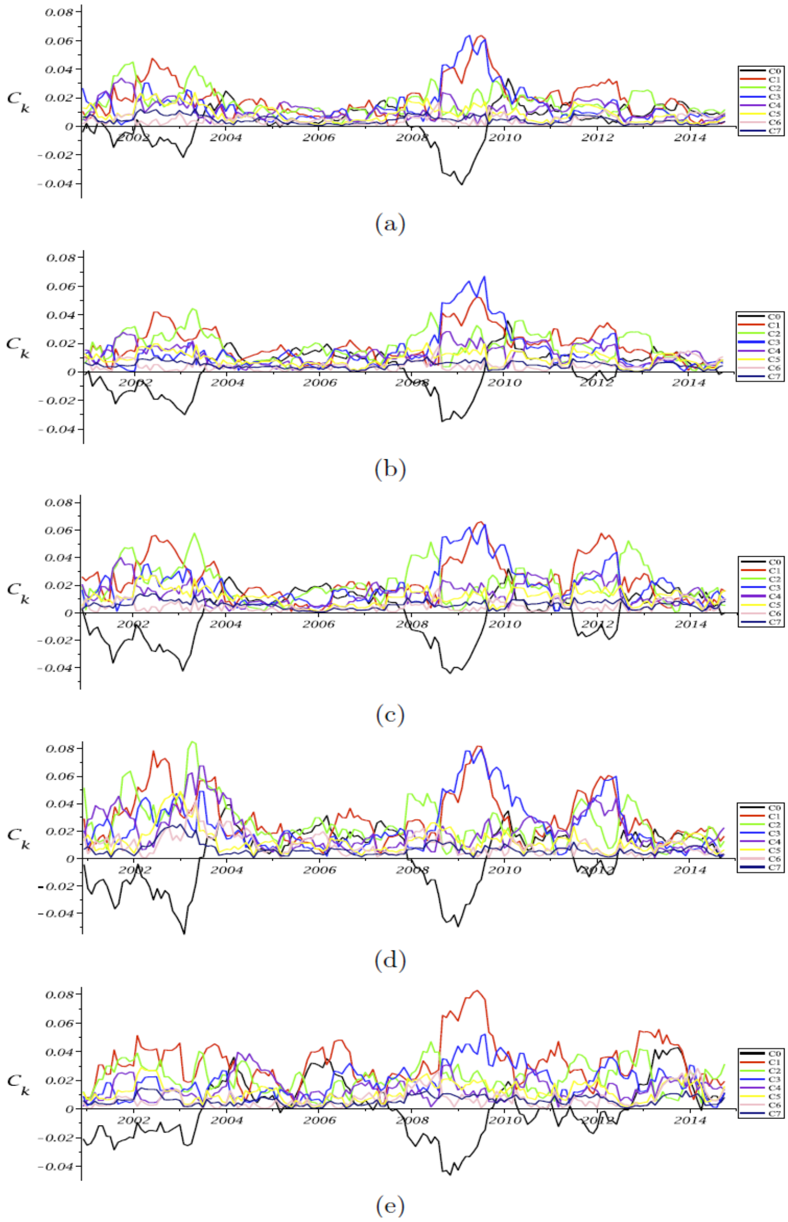

Figure 3. Comparisons of the Monthly Propagations of the Spectra for the Major Indices
(a) DJI; (b) FTSE; (c) CAC; (d) CDAX; (e) TOPIX

Main Result 2: Comparative Analysis of the Patterns of Crisis Propagation

To analyze the characteristics and patterns of crisis propagation that occurred in the major stock markets, we perform Z-tests for all the time series C_n ($1 \leq n \leq 7$) for each market. We use 60 consecutive monthly data observations of each C_n between January 2000 and December 2004 as the reference dataset. The statistics of each market for this test are shown in Table 1. Next, we select exceptionally large values in each time series of C_n and apply color-contour plotting visualization to them, obtaining Figure 5.

From this figure, we can observe the developments during the crises with contour lines on the topographic map of time as the fingerprints of the crises: First, we clearly observe that, in the beginning of crises, exceptionally large values of C_n ($n \geq 1$) appear in even modes such as C_2, C_4 and C_6, as found in previous analyses, which rapidly switch to odd modes like C_1, C_3, C_5, and C_7. Next, we re-confirm that around financial crises, the distributed patterns of exceptionally large values of C_n are similar to each other except that of TOPIX.

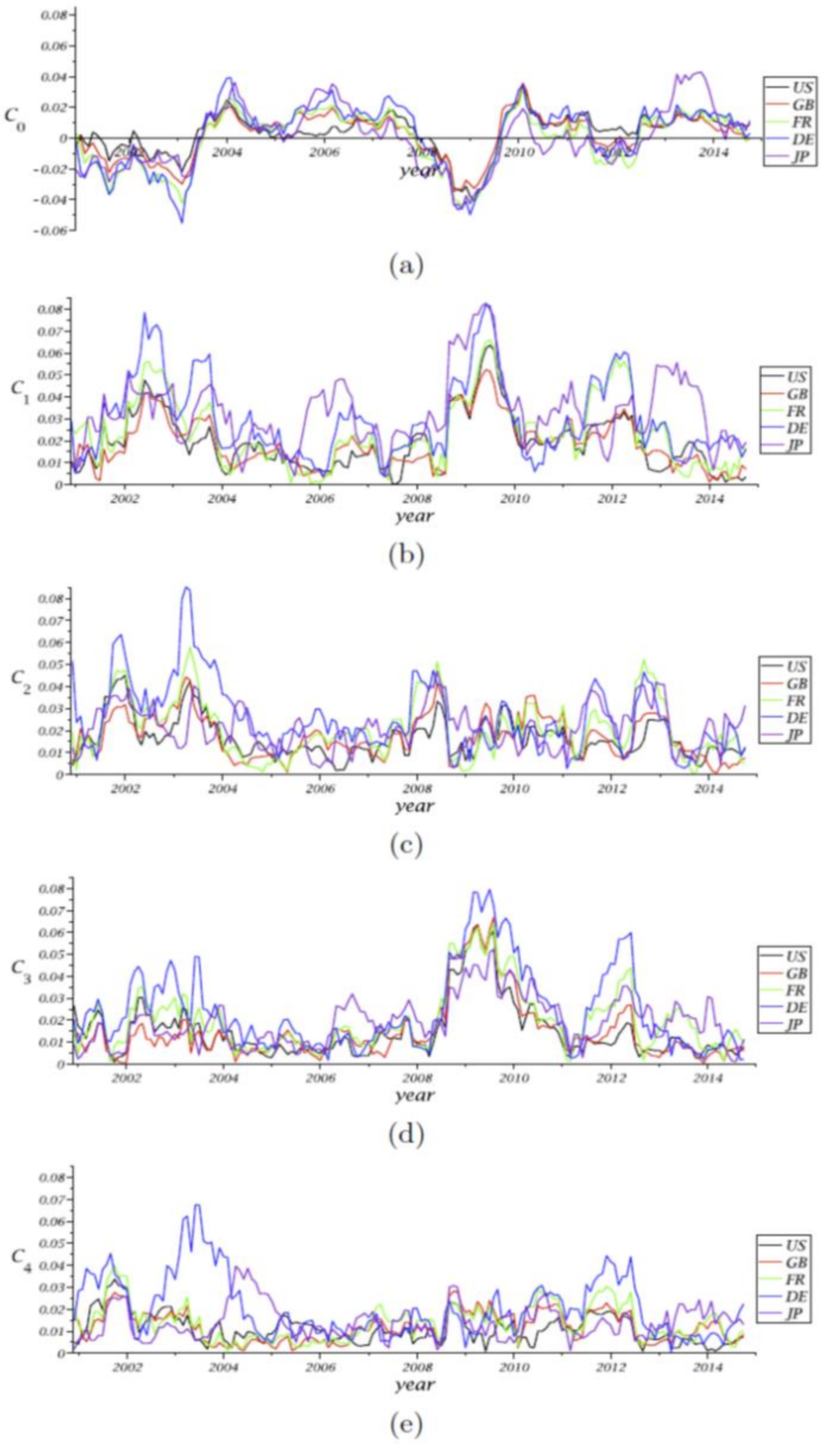

Figure 4. Cross-Country Comparisons of the Monthly Propagations of the Spectra

Additionally, we can see that the Japanese stock market reacts to financial crises independently of the other stock markets, and that it has been exposed more frequently to unhealthy instability, which seems to represent internal vulnerability. Next, around the 2008 crisis, unlike the previous crisis, we find that the distribution of exceptionally large values from CDAX shows the most simplified pattern focused on the C_0, C_1, and C_3 modes, which implies that the German stock market was more capable of responding to crises than the others. Additionally, there exist certain repetitions in the darkness of the density of the exceptionally large values. This interesting phenomenon seems to be related to the financial cycle of each stock market.

Table 1

Statistics (Colors) for C_i ($0 \leq i \leq 7$) of Each Index

Country	Statistics	C_0	C_1	C_2	C_3	C_4	C_5	C_6	C_7
DJI(US)	Mean	0.00042	0.01906	0.01965	0.01275	0.01423	0.01100	0.00409	0.00561
	SD	0.01006	0.01101	0.01095	0.00758	0.00780	0.00600	0.00242	0.00285
FTSE(GB)	Mean	-0.00212	0.01834	0.01814	0.00970	0.01105	0.00963	0.00343	0.00491
	SD	0.01416	0.01018	0.01051	0.00493	0.00757	0.00434	0.00312	0.00222
CAC(FR)	Mean	-0.00532	0.02346	0.02172	0.01586	0.01285	0.01285	0.00397	0.00656
	SD	0.01906	0.01491	0.01498	0.00892	0.00967	0.00633	0.00319	0.00323
CDAX(DE)	Mean	-0.00464	0.03210	0.03451	0.02217	0.02824	0.01614	0.01408	0.00823
	SD	0.02289	0.01897	0.01850	0.01170	0.01681	0.01261	0.00885	0.00644
TOPIX(JP)	Mean	-0.00232	0.02820	0.02181	0.01207	0.01524	0.01072	0.00455	0.00547
	SD	0.01781	0.01229	0.00935	0.00623	0.00999	0.00676	0.00382	0.00345

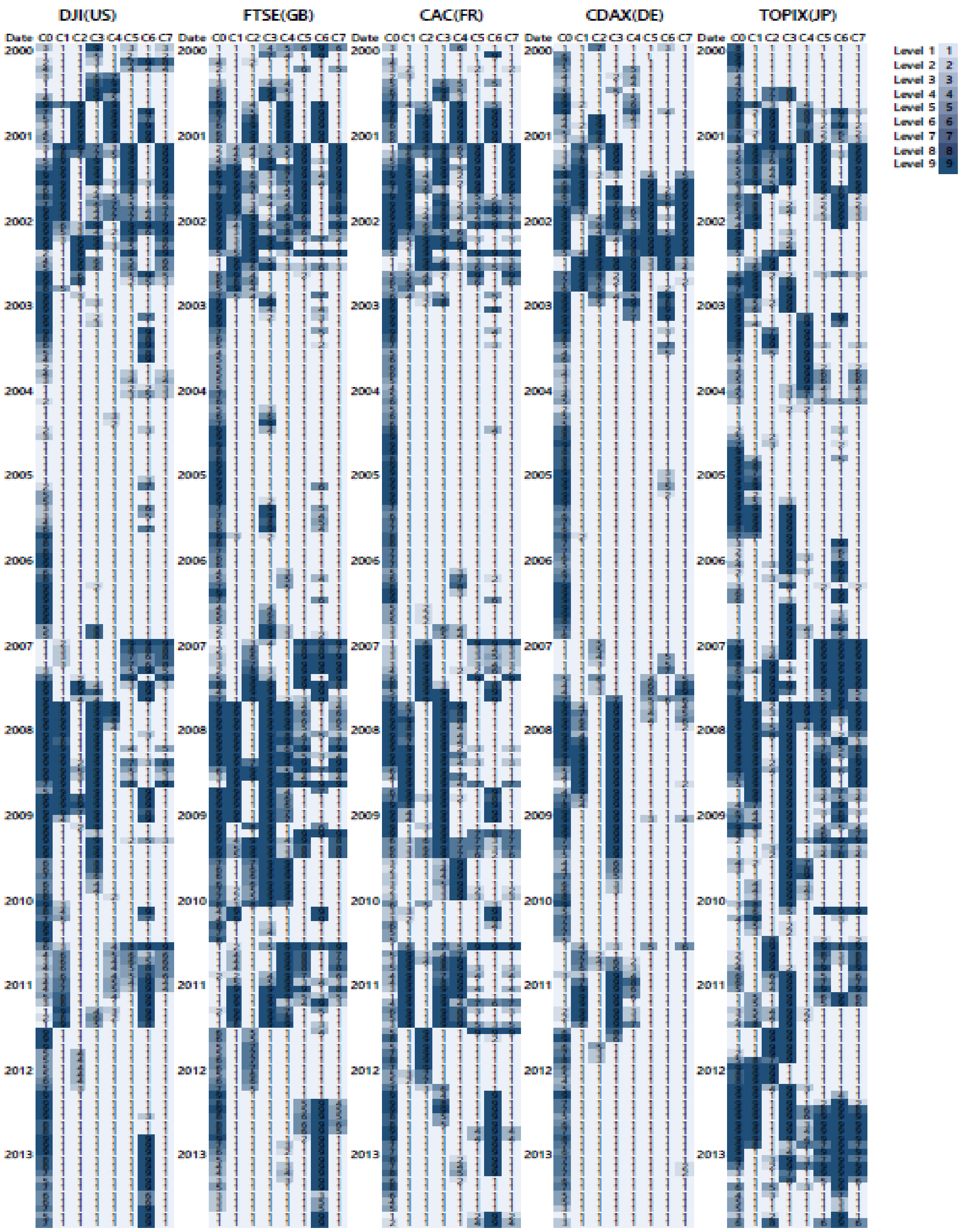

Figure 5. Color-Contour Plots for the Monthly Propagations of the Spectra

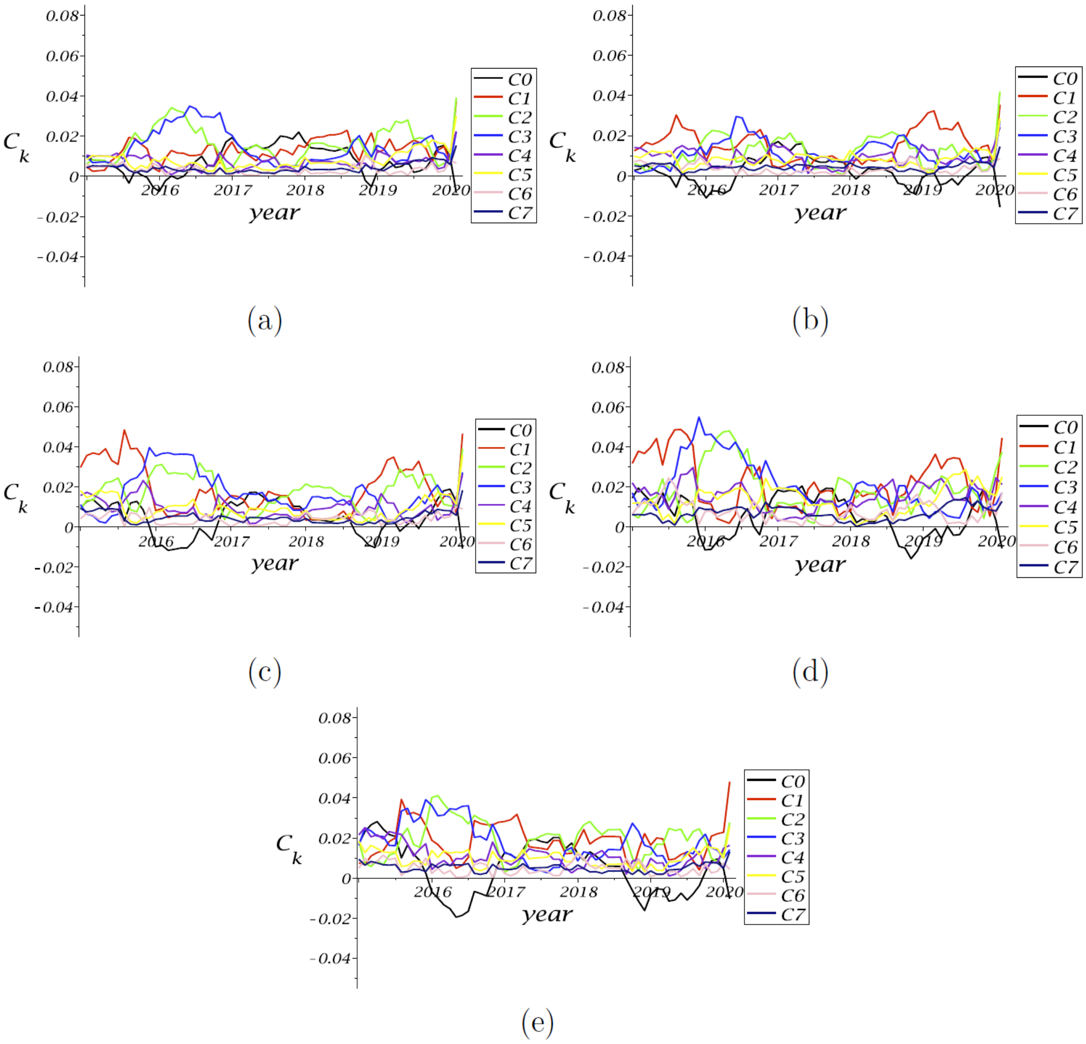

Figure 6. Cross-Mode Comparisons of the Current Monthly Propagations of the Spectra
(a) DJI; (b) FTSE; (c) CAC; (d) CDAX; (e) TOPIX

Main Result 3: Recent Status Analysis

Here, we analyze the current state of the major markets by investigating the monthly propagations of the spectra for their indices from January 2015 to March 2020 (Figure 6).

Figure 6 shows that around 2016, all markets simultaneously created large peaks for C_1, C_2, and C_3 in propagations with sufficiently large crashes of C_0 of the indices except for DJI, which implies a serious instability that could have become a real global crisis. Around 2019, we find small-scale phenomena similar to those observed around 2016: large crashes of C_0 for all indices except DJI and large peaks of C_1, C_2, and C_3 for most indices. Around the end of 2019, the spectra of all indices were stabilized to the level of 2017 and 2018, which means that the global financial state was recovering from the instability. However, from the start of 2020, we witness a startling phenomenon caused by the COVID-19 pandemic: sharp crashes of C_0 and explosive increases of the other C_n for all indices, which seems to be a warning sign for a subsequent global crisis. To confirm this possibility, we perform cross-country comparisons for the monthly propagations of the spectra (see Figure 7).

In Figure 7, around 2020, we observe explosive increases for all the other C_n ($1 \leq n \leq 4$) with accompanying sharp crashes of C_0, which strongly suggest the possibility of a global financial crisis. Comparing the degrees of the fluctuations of all C_n in this period with those around 2008 (Figure 4), we infer that the current state of major stock markets could be worse than the 2008 crisis. In other words, warning signs seem to encompass all markets.

Finally, to more precisely analyze the behaviors of recent C_n values, we again perform a Z-test for their recent time series of C_n and apply color-contour plotting visualization to their exceptionally large values. Accordingly, we obtain Figure 8, which shows two prominently interesting features in the distributed patterns of exceptionally large values: One is that the emergence of exceptionally large values of C_n around the 2016 semi-crisis is concentrated only in the C_3 mode, unlike the previous crises. Then, as seen in Figure 5, in the 2008 global financial crisis, exceptionally large values of C_3 also appeared in the outstanding patterns, in darker colors. This implies that the behavior of the exceptionally large values of C_3 can play a role in warning of an approaching crisis.

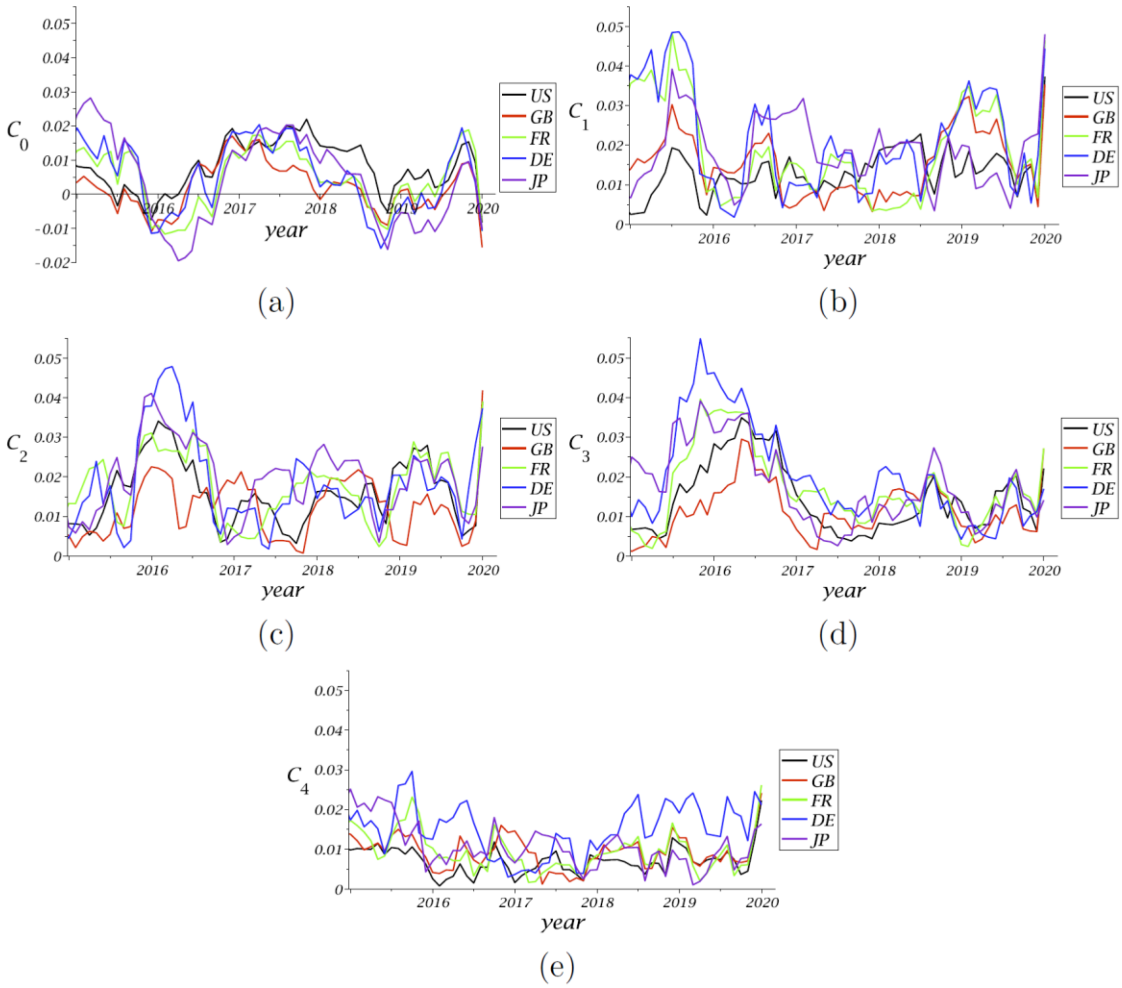

Figure 7. Cross-Country Comparisons of the Current Monthly Propagations of the Current State Spectra

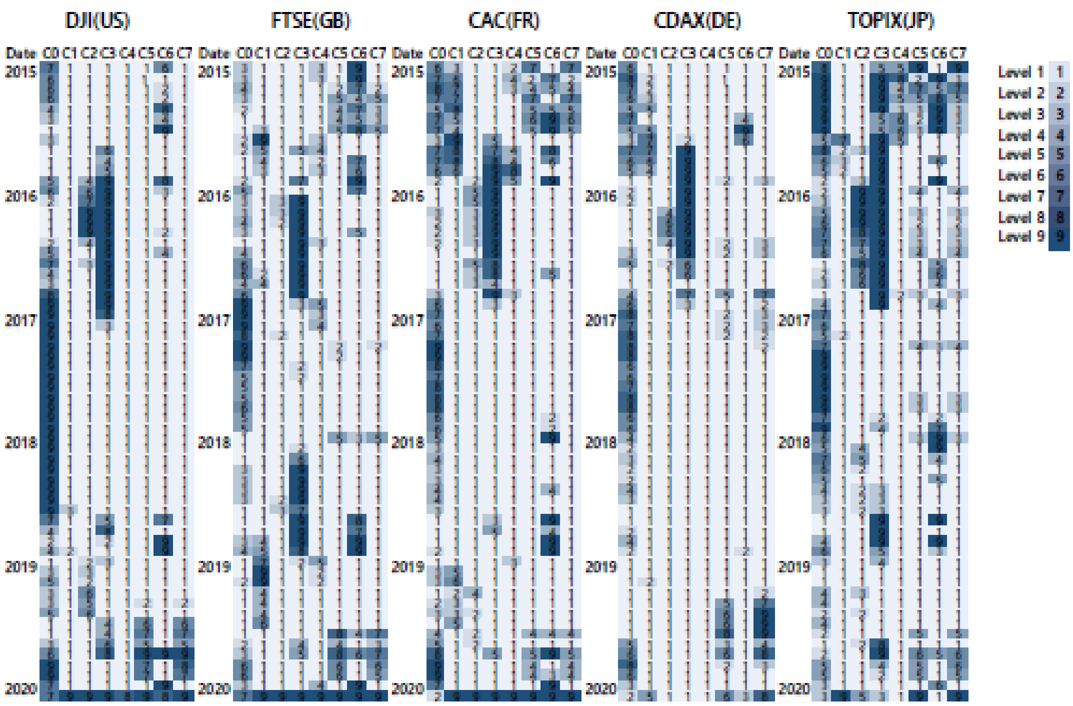

Figure 8. Color-Contour Plots for Monthly Propagations of the Spectra from Recent Datasets

The other interesting feature is the strange behavior of all the exceptionally large values of C_n observed in all markets at the beginning of 2020. In Figures 6 and 7, we observe that all the values of C_n behave seriously abnormally. Then, in Figure 8, while all the exceptionally large values of all C_n suddenly appear with fairly dark colors in the case of the DJI, FTSE, and CAC, in the case of the German stock market, the emergence of the exceptionally large values of C_n seems to be well controlled in that period, estimated by the confidence level based on data from 2000 to 2004. However, comparing the distributed patterns of the exceptionally large values of C_n with those in the previous crises, the pattern observed in recent data is unusually different from those in previous ones, which could be a warning sign of a unexpected big crisis. This could be an important topic for future research.

Conclusions

We analyze the impact of financial crises on major stock markets from 2000 till the COVID-19 pandemic, identifying three global financial crises during 2000–2015, all with different characteristics. By analyzing the monthly propagations of the major spectra C_n of each market's annual signals, we show that during the three crises, each market reacted interestingly depending on continent, country, and crisis. Regarding the scale of the crises' impact, it was generally larger in the European markets than in the US market. Furthermore, unusually, in the case of the European markets, C_2 reacted sensitively to the crises, which suggests that their impacts were concentrated in actual stocks with large weights of mode 2 in their return behaviors. Regarding the behavior patterns of C_n, the US and UK markets were very similar to each other during the first two crises, but during the European sovereign crisis after 2010, the impact was observed mainly in the French and German markets. Through comparative analysis of the patterns of crisis propagation, Z-test, and color-contour plotting, we confirm that the major markets responded differently to each financial crisis, showing different patterns in the propagations of extraordinary values of C_n. In the case of the 2008–2009 global financial crisis, the propagations in the US and German markets were centered on modes 1 and 3, whereas those in the UK and French markets were observed in almost all modes of low frequencies (the French market reacted particularly strongly); however, in the case of the 2000–2002 financial crisis, all the markets behaved very similarly. Interestingly, we find that the Japanese market was exposed to and reacted to crises almost independently. We show that the spectra of lower-frequency modes can be used as sensitive indicators of an approaching crisis. Moreover, analyzing the behaviors of the spectra of odd modes with lower frequencies, which responded more sensitively to unusual situations in markets, we show that C_1 (or C_3) can function as a useful, common early-warning indicator in major markets.

We also observe a significant instability around 2016, which could have become a real global crisis. Furthermore, since the start of 2020, all major recovered markets started to generate abnormal signals, as if suddenly facing an unexpected immense shock, which might be warning signs of an approaching crisis. Comparing the patterns of crisis propagation in this case with those of previous ones by market, we infer that the kind of instability passing through the market right now could be exacerbated by an unprecedented crisis.

Finally, we suggest some avenues for further research. In preparing for an approaching crisis and establishing response strategies and countermeasures in each market, it is important to understand why the spectra of particular modes in each stock market respond to a particular crisis, as well as the nature of and solution for the recent instability, by analyzing the behaviors of the spectra. Moreover, it is important to analyze these all in conjunction with the behaviors of the stock prices of all component companies in the stock index corresponding to each market. It will also be interesting to analyze the developments over the entire period of the financial crisis triggered by the COVID-19 pandemic and how the crisis has propagated, using the methods adopted in this study.

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Authors' Biographies

Doobae Jun, PhD, is an Associate Professor at Department of Mathematics, Gyeongsang National University, Republic of Korea. He achieved his PhD in Applied Mathematics from Sungkyunkwan University, Republic of Korea. His research interest covers financial data and pricing financial derivatives.

Jinsu Kim is a Graduate student at Department of Mathematics, Gyeongsang National University, Republic of Korea.

Gwangil Kim, PhD, is a Professor at Department of Mathematics, Gyeongsang National University, Republic of Korea. He achieved his PhD in Applied Mathematics from Pohang University of Science and Technology(POSTECH), Republic of Korea.

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