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Modelling Risk under Volatile Conditions: Tail Index Estimation and Validation

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The subject of the research is to analyse and evaluate methods of investment risk modelling in dynamic, changing market circumstances, with a special focus on the assessment success of the expected effects of investment activities in 'extreme' return points. In that sense, different Value at Risk models were used: the Historical Simulation (HS VaR), the Delta Normal VaR (D VaR) and the Extreme Value Theory model (EVT). The research objective is to test the performance of these models in specific, volatile, market circumstances, in terms of estimating the maximum possible losses from these activities. The basic hypothesis of the research is that it is possible to successfully anticipate the maximum possible losses from the investment activities in the extreme points of the return function by applying different methods of investment risk modelling in volatile market circumstances. The analysed financial data comprise daily stock returns of the BELEX15 (Serbia), BUX (Hungary), CROBEX (Croatia) and SBITOP (Slovenia) stock exchange indices in the period 2012–2019, which is relatively long time period suitable for the sound analyses. The main findings of the research point to the superior application adequacy of the Extreme Value Theory model (EVT) for successful risk modelling, i.e. for making optimal investment decisions. The research results represent innovated, concrete knowledge in the field of understanding the behaviour of the return function in its extremes, and consequently are of great importance to both the academic and professional public in the process of generating decisions on investment activities in volatile market conditions.

Keywords: *Modelling Risk; Value at Risk; Volatile Conditions; Tail Index Estimation; Investment Validation.*

Introduction

Observing investment as a process presently, in actual market circumstances, and doing it ten years ago, might be identified as an attempt to analyse identical phenomena and processes; in fact, the differences between investment processes today and a decade ago are so significant that these two processes can be and should be observed completely isolated from one another. After the outbreak of the financial crisis in 2008, it took some time to realise this fact and reach a consensus of both the academic and professional public on it. It is questionable whether the consensus still exists today, but the facts speak for themselves and irrefutably confirm the market reality.

In their 2019 paper, the Authors confirmed the thesis that portfolio analysts can no longer rely on one of the key assumptions of fundamental analysis that history is repetitive. Following the same concept line, in this paper, the authors want to analyse and evaluate the possibilities of applying different methods of investment risk modelling in dynamic, changing, market circumstances, with the special focus on the successful assessment of the expected effects in "extreme" points of investment returns. Namely, in addition to numerous other altered circumstances related to investment activities, the 2008 crisis also affected the change in the movement of returns from these activities. Thus, it was found, tested, and confirmed in practice, that the return function no longer behaved like a sinusoid, but took the form of Poisson distribution, i.e. the majority of the investment effects took place in the return extremes, the so-called tails of distribution.

Dealing with the return extremes presupposes research which is evidently directed to the distribution tails, with an attempt to set the confidence interval as realistically as possible, i.e. to include the widest range of potential investment returns as possible. It is not easy to achieve such a task in practice, and the researchers are required to observe and analyse carefully the return movements in different market circumstances. In this study, the authors deliberately focus on volatile market conditions because there is relatively little research in the area with such a focal point. Thus, the basic research objective is to test the performance of different risk assessment models in specific, volatile, market circumstances, in terms of estimating the maximum possible investment losses. Such an objective will also gain practice-proven knowledge about the behaviour of the investment return in its extremes. Only in this way, it is possible to test the performance of the risk assessment model in modern, volatile market circumstances.

Such an approach is original in the field because the authors have not heretofore encountered other research that was set up and realised likewise. It is very important to point out the fact that presently, in real-time, one can primarily talk about extreme return movements so the attempt to anticipate the performance of the investment effects must take it into account.

This is also the reason why the basic hypothesis is focused on the assessment and evaluation of the investment effects in volatile market circumstances, observing the movement of the return function in its extremes. Accordingly, the basic hypothesis of the research is that it is possible to successfully anticipate the maximum possible losses from these activities in the extreme points of the return function by applying different methods of investment risk modelling in volatile market circumstances.

The need for research conducted in this paper is significant because any expansion of knowledge about the behaviour of the return function of investment activities in its extremes is a big step forward towards successful anticipation of the effects of these activities. It is especially important to analyse the behaviour of the return function in its extremes in the case of volatile market circumstances because in such markets there are numerous variables related to the return behaviour. Undoubtedly, it cannot be expected that different models of risk assessment and evaluation will magically solve all the unknowns, but it can be expected that the research realised in this paper will significantly contribute to understanding the possibilities of effective application of different assessment and evaluation models, focusing in particular on the extreme points of the return function. That is the reason why the research realised in this paper is set to observe primarily the extremes of the return function, the so-called 'tails', and test different methods for assessing and evaluating the effects of investment activities, i.e. anticipating the maximum possible losses.

This paper is a continuation of the authors' research in the subject area, intending to expand the knowledge base (primarily practical, but also theoretical) on the possibilities of applying different methods of investment risk modelling in dynamic, changing, market circumstances, focused on the successful assessment of the expected investment effects in the "extreme" return points. Extending the research time horizon, the authors want to provide a scientific basis for a quality assessment of the performance of the above models in specific, volatile, market circumstances. A relatively small number of studies have been realised so far with this approach, i.e. focusing on the widest possible period of market observation in order to achieve the most significant validation of the results obtained by the research.

The specific research realised in the paper covers the period from 2012 to 2019 for four financial markets of developing countries, thus providing an optimal database for its successful implementation.

The paper is structured as follows. The introductory part gives an overview of the subject and objectives of the research, while the next part includes the relevant research in the field. This is followed by the used methodology with explanations and the presentation of the research results with a discussion, respectively. The final part is the conclusion and the review of the references.

Theoretical Background

Analysing the investment processes in the modern market environment, the consequences of the global economic crisis, and the characteristics i.e. the nature of the tested data, both from the theoretical and practical aspect, it is necessary to study a wide range of recent literature in the field. Consequently, it enables optimal investment decision-making and adequate assessment of the investment effects, which are of great importance for both the academic and professional public.

Investment risk modelling issues, with special emphasis on the application of parametric and non-parametric Value-at-Risk (VaR) models in volatile business conditions, have been addressed by many authors, e.g. Begenau, J. (2020); Embrechts et al. (2018); Kim et al. (2018); Lin, Z. (2018); Patton et al. (2019); Tiwari et al. (2018); Yu et al. (2018). Additionally, the authors' focus is the analysis of the nature of the tested data, i.e. tail index estimation, as well as backtesting validation of the VaR model application in investment processes, e.g. Ahmad et al. (2019); Bekiros et al. (2019); De Luca et al. (2020); Jia et al. (2018); Kratz et al. (2018). This topic is even more crucial in emerging markets with their inherent abrupt changes in volatility regimes, especially characterized by lower liquidity, frequent internal and external shocks (Zikovic & Aktan, 2009).

Danielsson et al. (2016) explore the possibility of risk modelling with special reference to the application of different risk management models. Risk prediction is especially analysed in business conditions determined by market risk and extreme volatility. The environmental conditions induce successful application of the observed risk management model, i.e. the biggest challenge of the performance adequacy of the tested models is increasing uncertainty, which represents the threshold of risk management capabilities in general. The authors' concluding remarks indicate the necessity of risk prediction in investment decision-making and the necessity of applying a wide range of tested VaR risk management models.

Boucher et al. (2014) analyse the effects of investment activities in the context of the global economic crisis, which negatively affect the success of standard risk management models. The results of the application of different VaR models (CAViAR, GARCH VaR, etc.) are influenced by extreme events that affect the distribution tails of the research sample. The desired degree of success in predicting the maximum possible loss is especially influenced by the frequency, independence, and magnitude of exceeding the established threshold. The authors conclude that it is necessary to continuously incorporate corrections when adjusting VaR models and using standard backtesting methodologies. This research is important because it indicates the justification of using the simplest VaR models, such as historical simulation (HS VaR).

Embrechts et al. (2013) focus on uncertainty as a key challenge in determining the application adequacy of a particular VaR model. Consequently, the numerical estimation of the VaR affects the risk capital calculation, since VaR is known and recognised as an industry and regulatory standard. The authors especially emphasise the necessity of different dependence scenarios on the observed risk factors, as well as the chosen level of confidence. The research is significant because it presents a numerical algorithm that allows the calculation of reliable bounds of the VaR calculations and concludes that the incorporation of additional information does not greatly contribute to the success of the tested model.

Fontana et al. (2021) indicate the importance of the choice and calibration of the VaR model in the credit risk analysis and the consistency related to its application. Accordingly, the authors test the application performance of VaR and Expected Shortfall (ES) methodology as a basis for determining risk prediction errors. The study results of multivariate distributions and the VaR threshold imply the necessary size of the tested risk management model, especially if the marginal default probability is high.

Bernard et al. (2017) estimate the magnitude of the uncertainty modelling while determining the maximum and minimum VaR values accordant with the available information. The authors identify the gap between worst-case and best-case VaR performances in the context of identifying cause-and-effect information when defining the VaR threshold. This is evident at high levels of confidence of VaR calculation. This research is important because the initial assumptions affect the success of risk modelling, so it is necessary to pay attention to possible extreme events.

Lwin et al. (2017) use nonparametric models of VaR calculation and optimal level of capital. The authors focus on risk/return characteristics in investment processes. Since VaR is non-linear, non-convex and non-differentiable, it is necessary to consider the risk exposure, as well as the fluctuations of the investment assets. The authors' concluding remarks point to the need to establish a realistic, practically tested and validated, risk management framework, which will not require assumptions about the research sample distribution.

Echaust et al. (2020) emphasise the need to select the optimal distribution tail of the research sample to calculate VaR and apply it successfully. The authors apply a two-stage hybrid risk management model that involves the pre-specification of the VaR calculation threshold using various advanced algorithms. The comparative analysis in the paper indicates the advantages and disadvantages of forecast concerning the traditional approach for determining the threshold. Based on the results of the research, the authors conclude that the optimisation of the sample distribution tail does not improve the accuracy of VaR estimates compared to standard risk management models.

Chen, J. M. (2018) analyses existing financial regulations with special reference to Basel standards and adequate risk management. The author explores the regulatory implementation and practical challenges of backtesting. Thereby, the paper provides a unique approach to the requirements of balancing risk management measures and changing regulations. The significance of the research is reflected in the fact that no risk management model can achieve adequacy, i.e. accuracy of regulations and standards used.

BenSaïda et al. (2018) determine how volatility affects investment performance in a globalised business environment. The authors identify factors and dynamics of volatility in periods of crisis and periods of tranquillity. They analyse a large number of variables and the impact they have on existing risk management procedures. Their research indicates that in more stable business conditions volatility is moderately transmitted, globally.

Barrieu et al. (2015) deal with risk assessment and modelling, in particular, absolute, relative and local risk measure. The authors focus on quantitative risk modelling measures while providing flexible application. They additionally point out that it is necessary to primarily specify and select an individual risk management model according to the existing risk factors in order to adequately manage risk. The authors emphasise that it is important to pay attention to the empirical distribution of the risk management reference model, especially in light of the global economic crisis.

The topicality of the research is evident, especially having in mind the existing disturbances and rising volatility due to the frequency of extreme events, which have a significant impact on adequate risk management and selection of the appropriate VaR model. Backtesting results show that VaR models commonly used in developed stock markets are not well suited for measuring market risk in these markets (Zikovic, 2007).

Research Methodology

The research realized within this paper included the analysis of the following stock exchange indices: BELEX15 (Serbia), BUX (Hungary), CROBEX (Croatia) and SBITOP (Slovenia). Dynamics of emerging financial markets show substantial differences as compared to developed countries. These markets experience larger "financial earthquakes" than developed economies and can be labeled as "markets with many fault lines" (Gencay and Selcuk, 2004). Thus, specific, volatile market circumstances are included in the research sample, by default. For each of the four observed indices, daily stock returns were calculated during the eight-year period, i.e. from January 3, 2012, to December 30, 2019, with the data from the previous years, (2011 and 2010) being used to assess the necessary parameters for the beginning of the examined period. On an annual level, the data volume ranged from n=244 to n=253 days, depending on the year and the stock exchange index. Accordingly, the choice of analysed data from four stock exchange indices in relatively long time period is induced by its specificities and the novelty of the research. In order to enable insight of dynamics of the tested markets, the research was conducted and its results were standardly presented in separate years, by the application of wide array of rolling windows, in an appropriate and comprehensive manner. In order to standardise and compare the results, the VaR results were scaled by the linear transformation to a standard value of n=252 working days per year. It should be noted that this alteration did not, in any case, cause a change in integer value of the number of days per year in which the VaR break was registered with a given confidence level of 97.5 %. The empirical analysis of the data included (i) descriptive analysis of the sample; (ii) normality tests; (iii) VaR analysis; and (iv) comparative analysis of the results obtained by the VaR methodology. Below is the summary of the methodology and theoretical foundations related to each of these research phases.

The descriptive analysis of the sample, both stock indices closing values and relative daily returns, included standard measures of central tendency (the mean and 95 % confidence interval for the mean), measures of dispersion (minimum, maximum, standard deviation, and coefficient of variation), and measures of shape (skewness and kurtosis) for the observed stock exchange indices in the whole period 2012–2019. In addition to the initial description of the realised sample, this analysis aimed to identify leptokurtic i.e. fat-tailed variables, since they were appropriate for the application of the Extreme Value Theory (EVT) model for the risk analysis.

For the purpose of quantitative risk modelling, three classical Value at Risk methods were applied in parallel: historical simulation (HS VaR), delta normal VaR (D VaR) and the Extreme Value Theory (EVT). The objective of each of these methods was to model a value of risk based on the realised values of daily returns for the previous k days, i.e. a threshold value that would not be exceeded by the relative loss on a given day with a probability expressed by a predefined level of confidence. The research was done with the confidence level of 97.5 %, and the VaR calculation was performed for 100, 200, and 300 prior days. Using the so-called rolling window, i.e. by shifting the observed interval of k days by one day, the calculation was repeated for each day during the eight-year interval covered by the study. The realised and modelled daily return values were subsequently compared for each day. If the realised value was higher (i.e. relative loss was less) than estimated, it was considered a successful day, while otherwise, if a loss was greater than anticipated, it was considered as a VaR break. The total number of VaR breaks per year, for each method, each stock exchange index, and each of the three observed rolling window volumes was summarised and tabulated.

Due to its simple application and universality, the historical simulation VaR model is a well-known non-parametric model for risk calculation, which is used frequently although it is neither sophisticated nor very precise. Based on the sample from the previous k days, the calculation of the VaR value with confidence level c is performed by determining (1- c)·100 % quantile for the sorted non-decreasing sample and it represents the risk limit value for the day k+1. Specifically, in this case, the HS VaR value was calculated as a Q2.5 % quantile.

Delta normal VaR belongs to the class of parametric models for risk calculation and assumes the agreement of the empirical distribution of daily returns with the normal distribution N(m, σ). Point estimates of the parameters of the normal distribution are obtained by the method of moments or by the method of maximum reliability from the realised sample from the previous k days: m is estimated by the mean, and σ by the standard deviation of the sample. Then D VaR for the day k + 1 and for a given confidence level c is computed as the inverse value of the distribution function for argument 1-c: F-1(1-c), where F(x) is the distribution function for normal distribution with estimated values of the parameters m and σ. Despite its simplicity, D VaR is considered very effective if the assumption of the normally distributed sample is satisfied, but its reliability decreases if the data significantly deviate from the normal distribution.

Contrary to the D VaR model, which assumes fitting of the entire daily returns' distribution for a given time window, Extreme Value Theory focuses on fitting only the tail of the distribution. The major advantage of EVT is that it enables extrapolation, that is the estimation of the probability of events that are more extreme than the ones contained in the data set. Also, it is more suitable when fitting leptokurtic or platykurtic data, which are frequently found in financial market risk research. There are numerous empirical pieces of evidence of the superiority of EVT in comparison with other classical VaR models, see for example Fernandez, 2003. By convention, negative daily returns i.e. extreme losses are presented as positive values, thus the right tail of the distribution is the subject of modelling. There are two classic approaches to EVT: the Block Maxima approach, which is usually modelled by General Extreme Value (GEV) distribution, and Peaks Over a Threshold approach (POT), which uses Generalized Pareto (GP) distribution, introduced by Picklands, 1975 and defined as follows:

$$G(x) = \begin{cases} 1 - \left(1 + \frac{x-\mu}{\sigma}\right)^{-\frac{1}{\xi}}, & \xi \neq 0 \\ 1 - \exp\left(-\frac{x-\mu}{\sigma}\right), & \xi = 0 \end{cases} \tag{1}$$

G(x) is the cumulative distribution function of the random variable X. It depends on three parameters, namely shape parameter $\xi, \xi \in R$, location parameter $\mu, \mu \in R$, and scale parameter $\sigma, \sigma > 0$. Location parameter μ also represents the threshold i.e. the starting point of the right tail. Since in this case the values exceeding the threshold are of main interest, the excess distribution function, depending on the threshold μ, is defined in terms of the conditional probability:

$$F(\mu, y) = P(X - \mu < y \mid X > \mu)\,. \tag{2}$$

In order to effectively calculate VaR as the quantile of GP distribution, two nontrivial questions should be answered: (i) the threshold detection and (ii) shape and scale parameter estimation.

The selection of the threshold is of crucial importance because it requires finding a balance between two opposed requests. If the threshold is too high, there will not be enough data for precise parameter estimation in the remaining (threshold exceeding) subsample. On the other hand, too-low threshold yields to the inclusion of data points that do not belong to the right tail of the parent distribution, which may lead to the violation of the GPD assumption might. There are a plethora of methods in the literature for the threshold selection, from graphical methods such as Mean Residual Life (MRL) plot and Threshold Choice (TC) plot to the Goodness-of-Fit based methods using Cramer-von Mises and Anderson-Darling statistics (see Choulakian & Stephens, 2001). As commented by Langousis et al, 2016, there is no universally superior method, each of them showing strengths or weaknesses under certain circumstances, but graphical methods tend to be the most frequent choice. In this research, due to the very large number of calculations, the visual inspection of plots was an inappropriate choice, so the more robust approach is used based on quantiles of the empirical distribution, which is in lines with the results of McNeil and Frey, 2000, who used the 90th quantile of the innovation distribution obtained by the historical simulation to set the number of exceedances over the threshold.

After the threshold (location parameter μ) has been selected, the estimation of the remaining two parameters (shape ξ and scale σ) takes place. This is a less challenging part because numerous estimation methods have been developed, from traditional moments, maximum likelihood, probability-weighted moments (PWM), over the family of Bayesian-based methods and specially tailored generalizations of PWM, to the very recently published amalgamated improvements of Zhao et alt, 2019. The extensive review of estimation procedures can be found in De Zea Bermudez and Kotz, 2010. For example, the R-package POT (Ribatet & Dutang, 2019) itself offers seventeen estimators for the univariate case. Our method of choice was the Maximum Likelihood Estimator (MLE) because its implementation in the POT package allows for varying threshold argument.

In order to validate the obtained risk modelling results, the backtesting is performed by applying the ”proportion-of-failure” (POF) test, proposed by Kupiec, 1995. This test is the most well-known representative of the family of the coverage backtest methodologies, aiming to check whether the frequency of VaR breaks corresponds to the desired quantile of loss of a Value-at-Risk measure (Holton, 2014). According to Holton, the null-hypothesis of coverage tests is formulated as H0(q=q*), where q represents the projected quantile of loss in the VaR analyses, whereas the so-called coverage q* corresponds to the actual, empirical failure rate i.e. the observed frequency of VaR breaks. For a given significance level α, 100(1-α)% confidence interval is constructed for the appropriate number of VaR breaks in the observed period of n+1 returns. This confidence interval serves as a non-rejection region of the test, therefore null-hypothesis is rejected if the actual number of VaR breaks falls out of it. In other words, the risk modelling is considered successful if the number of exceedances is neither too small nor too big. There are several ways to computationally execute this idea. A direct way to compute the confidence interval limits is to utilize the fact that a random variable X representing a number of VaR breaks is binomial, with n trials and 1- q probability of larger loss than projected:

$$X \sim B(n, 1-q) \tag{3}$$

This approach is used in a standard coverage test, recommended by Holton. The other way, used by Kupiec, is to formulate a likelihood ratio Λ:

$$\Lambda = \frac{q^{n-X}(1-q)^X}{(\frac{n-X}{n})^{n-X}(\frac{X}{n})^X} \tag{4}$$

and then apply the finding of Lehmann and Romano, 2005, that a logarithmic transformation of Λ is approximately chi-square distributed with one degree of freedom:

$$-2\log\Lambda \sim \chi^2(1,1-\alpha). \tag{5}$$

Now, for a given significance level α, using the critical value from Pearson's chi-square distribution, it is possible to solve the following equation for X:

$$-2\log\Lambda = 2\log((\frac{n-X}{qn})^{n-X}(\frac{X}{(1-q)n})^X). \tag{6}$$

The obtained two solutions are x_1 and x_2 such that

$$P(X < x_1) = P(x_2 < X) = \frac{\alpha}{2}. \tag{7}$$

By rounding x_1 to lower integer value, and x_2 to higher integer value, the limits of the confidence interval which represents the non-rejection region for the Kupiec POF test are derived with the significance level α. Particularly, for n = 252 annual working days, q equals the desired confidence level of 97.5 % and α=0.025, 97.5 % confidence interval for POF test yields [2,12] VaR break days per year. To conclude, in this framework, a Value-at-Risk model is considered successful, if the number of losses larger than predicted is between 2 and 12 per year, limits included.

Finally, in order to analyse the impact of particular factors such as the VaR model, length of the rolling window, or kind of stock index on the calculated number of VaR breaks per year, the univariate comparative analysis is performed. The applied tests are Kruskal-Wallis ANOVA by ranks for independent groups and Friedman ANOVA, which is a nonparametric counterpart of one-way repeated measures ANOVA, for dependent (paired) samples. In the cases where some statistically significant differences are determined by ANOVA, post-hoc testing is performed by using multiple comparisons of mean ranks. The between-group comparisons are illustrated by the box-and-whiskers plot, representing median\quartile\range of the selected subsamples.

The calculations are performed using software Statistica 13.0 and R (R Core Team, 2020), particularly packages POT (Ribatet & Dutang, 2019) and Dowd (Acharya, 2016).

Results and Discussion

The preliminary analysis, containing common descriptive measures of the sample of both stock indices (value) and daily returns (%) for the entire period 2012–2019, is presented in Table 1.

Table 1

Descriptive Statistics of Stock Indices and Daily Returns for 2012–2019

	Valid N	Mean	SD	Min	Max	CV (%)	95% CI for mean		Skewness	Kurtosis
SBITOP value	1982	739.36	102.268	501.27	926.30	13.832	734.86	743.87	-0.387	-0.837
SBITOP %	1982	0.02	0.811	-5.46	3.41	4159.0	-0.02	0.06	-0.356	3.709
CROBEX value	1990	1814.46	108.147	1576.47	2246.34	5.960	1809.70	1819.21	0.906	1.669
CROBEX %	1990	0.01	0.557	-3.16	3.33	9473.41	-0.02	0.03	-0.287	3.834
BELEX15 value	2015	639.35	97.216	426.80	801.69	15.206	635.10	643.59	-0.512	-0.873
BELEX15 %	2015	0.02	0.665	-4.09	3.67	3116.27	-0.01	0.05	0.024	3.512
BUX value	1981	27177.72	9222.132	15686.69	46082.82	33.933	26771.37	27584.08	0.382	-1.487
BUX %	1981	0.04	1.044	-6.46	4.85	2322.72	0.00	0.09	-0.154	1.813

Source: the authors

All analysed indices have balanced daily returns in the considered time interval, i.e. means are approximately close to 0 and 95 % confidence intervals for mean contain 0, with BUX being slightly leaned towards the positive side (mean =

0.04, mean CI95 % 0-0.09). Standard deviations range from 0.55, in the case of CROBEX, to 1.04, in the case of BUX. apart from the largest SD, the BUX index also contains the largest extreme values: its maximal observed daily loss equals -6.46 % whereas its maximal daily gain equals 4.85 %. Coefficients of variation are extremely large (measured in thousands of per cent) for all daily returns, which is expected due to the fact that mean values are close to zero. On the other hand, CVs for stock values are small to moderate, ranging from 5.96 % (CROBEX) to 33.93 % (BUX). Finally, from the analysis of shape descriptors, i.e. skewness and kurtosis, the following is observed: three out of four index daily returns are negatively skewed, only the BELEX15 index is characterized by positively skewed returns. Kurtosis is larger than 3 for all returns except for the BUX, indicating that the observed variables are leptokurtic (i.e. fat-tailed), thus appropriate for risk analysis by means of Extreme Value Theory.

The results of Kolmogorov-Smirnov normality tests for daily returns, both for annual and cumulative data, are presented in Table 2. The results show that the returns for the entire observed period for all four indices significantly deviate from the normal distribution ($p<0.01$). On the other hand, when analysed on the annual level, most of the returns data fit well to normal distribution. Only 3 out of 32 samples (8 years x 4 indices) deviate at the significance level of 0.01. The obtained results confirm that in this case, the usage of Delta Normal Value-at-Risk (D VaR) is the appropriate model for the risk analysis.

Table 2

Kolmogorov-Smirnov Normality Test for Stock Daily Returns

Year	CROBEX	BUX	SBITOP	BELEX15
2012	n.s.	n.s.	p<0.01	p<0.10
2013	n.s.	n.s.	n.s.	n.s.
2014	n.s.	n.s.	n.s.	n.s.
2015	n.s.	p<0.15	p<0.10	n.s.
2016	n.s.	n.s.	n.s.	n.s.
2017	p<0.01	n.s.	n.s.	p<0.15
2018	n.s.	n.s.	n.s.	n.s.
2019	p<0.10	n.s.	n.s.	p<0.01
2012-2019	p<0.01	p<0.01	p<0.01	p<0.01

n.s. stands for "non-significant"

Source: the authors

The results of the tail index estimation and validation are presented in Tables 3 – 10. Each table corresponds to one year, i.e. to 252 working days period, and contains a number of estimated VaR breaks for four stock index daily returns: CROBEX, BUX, SBITOP and BELEX15, three applied VaR models: Historical Simulation (HS VaR), Delta Normal VaR (D VaR) and Extreme Value Theory (EVT), and three applied rolling windows: 100, 200 and 300 days. All calculations are based on the 97.5 % confidence level. The results of the validation, performed by the Kupiec POF test, are also included in the following way: numbers of VaR breaks that are rejected by the POF test at the significance level 0,025 are marked by (*). Therefore, unmarked numbers correspond to successful estimations for the given confidence level.

Table 3

Number of VaR Breaks in the Year 2012 with the Results of the POF Kupiec Backtest

2012	rolling window 100			rolling window 200			rolling window 300		
	HS VaR	D VaR	EVT	HS VaR	D VaR	EVT	HS VaR	D VaR	EVT
CROBEX	9	9	3	4	4	2	3	3	0*
BUX	5	4	0*	3	1*	0*	3	0*	0*
SBITOP	9	9	3	5	8	1*	5	6	1*
BELEX15	9	4	5	2	1*	1*	1*	0*	0*

** denotes that the result is rejected by the POF test (at the significance level p=0,025)*

Source: the authors

The results for the year 2012 show a significant number of unsuccessful estimations – 13 out of 36. It can be observed that all model failures are due to the fact that the number of VaR breaks is smaller than the lower limit of the Kupiec POF confidence interval, i.e. too low. The number of model failures is directly proportional to the size of rolling windows: the rolling window of 100 days provided only 1/12 bad estimations, the proportion of bad estimations for the rolling window 200 is 5/12, and for the rolling window 300 there were 7/12 model failures. This yields to the conclusion that the prior two years (2011 and 2010) were significantly different and that influenced unbiased risk analyses for the year 2012. In terms of VaR models, HS VaR provided the most adequate estimations out of three applied models, whereas the EVT was overly harsh and thus the least successful one.

Table 4

Number of VaR Breaks in the Year 2013 with the Results of the POF Kupiec Backtest

2013	rolling window 100			rolling window 200			rolling window 300		
	HS VaR	D VaR	EVT	HS VaR	D VaR	EVT	HS VaR	D VaR	EVT
CROBEX	5	2	0*	3	1*	1*	1*	1*	0*
BUX	8	7	3	5	6	2	5	3	1*
SBITOP	8	4	3	6	6	3	8	7	3
BELEX15	10	8	4	12	10	6	8	7	6

** denotes that the result is rejected by the POF test (at the significance level p=0,025)*

Source: the authors

In the year 2013, all three applied VaR models were fairly successful, except for the case of CROBEX stock index daily returns, where 6 out of 9 estimations were overly low. For this index, EVT provided bad estimations for all three rolling windows, D VaR for rolling windows 200 and 300, and HS VaR only for the largest rolling window. Apart from that, there was only one model failure: in the case of BUX, modelled by EVT with the rolling window of 300 days. In total, 80.55 % (29/36) of VaR models were successful, which is a significantly better rate of success in comparison with the previous year.

Table 5

Number of VaR Breaks in the Year 2014 with the Results of the POF Kupiec Backtest

2014	rolling window 100			rolling window 200			rolling window 300		
	HS VaR	D VaR	EVT	HS VaR	D VaR	EVT	HS VaR	D VaR	EVT
CROBEX	9	9	8	8	9	6	8	9	2
BUX	10	11	8	7	9	4	7	7	3
SBITOP	8	10	4	9	7	3	6	6	2
BELEX15	14*	6	5	5	6	4	6	5	2

** denotes that the result is rejected by the POF test (at the significance level p=0,025)*

Source: the authors

The results presented in Table 5, referring to the year 2014, show that risk modelling was very successful for that period. The rate of success was 97,22 % (35/36) since there was only one failure determined by the backtesting. There were 14 exceedances in the case of BELEX15 when risk was estimated by HS VaR with rolling window 100, which is above the upper limit of the Kupiec confidence interval.

Table 6

Number of VaR Breaks in the Year 2015 with the Results of the POF Kupiec Backtest

2015	rolling window 100			rolling window 200			rolling window 300		
	HS VaR	D VaR	EVT	HS VaR	D VaR	EVT	HS VaR	D VaR	EVT
CROBEX	9	4	2	5	4	2	3	2	2
BUX	7	5	4	6	5	4	5	5	4
SBITOP	7	7	4	6	5	3	5	5	3
BELEX15	13*	11	7	10	12	8	14*	13*	10

** denotes that the result is rejected by the POF test (at the significance level p=0,025)*

Source: the authors

In the year 2015, risk modelling by applied VaR models was completely successful for CROBEX, BUX and SBITOP daily returns. It was less successful for the BELEX15 index, where the number of days with losses higher than expected for the 97.5 % confidence level exceeded the Kupiec limits in three occurrences. HS VaR was unsuccessful two times (rolling windows 100 and 300), while D VaR failed to provide an adequate model for the rolling window 300. It yields that BELEX15 showed the highest volatility, making it less suitable for predictions. It is noteworthy to observe that EVT turned out to be the most precise risk modelling technique for this and the previous year, contrary to the years 2012 and 2013 where the other two models gave better estimations.

Table 7

Number of VaR Breaks in the Year 2016 with the Results of the POF Kupiec Backtest

2016	rolling window 100			rolling window 200			rolling window 300		
	HS VaR	D VaR	EVT	HS VaR	D VaR	EVT	HS VaR	D VaR	EVT
CROBEX	9	8	7	8	8	6	7	8	5
BUX	6	7	5	6	5	6	6	6	7
SBITOP	6	6	5	7	8	6	5	6	5
BELEX15	7	8	4	5	7	4	5	7	5

** denotes that the result is rejected by the POF test (at the significance level p=0,025)*

Source: the authors

VaR analyses for the year 2016, presented in Table 7, are characterized by the absolute 100 % rate of successful estimations. This is the only year in the period covered by the research that all models were validated by the Kupiec POF backtest without exceptions.

Table 8

Number of VaR Breaks in the Year 2017 with the Results of the POF Kupiec Backtest

2017	rolling window 100			rolling window 200			rolling window 300		
	HS VaR	D VaR	EVT	HS VaR	D VaR	EVT	HS VaR	D VaR	EVT
CROBEX	11	14*	12	12	11	8	12	15*	12
BUX	10	10	9	9	8	7	7	7	7
SBITOP	7	6	4	5	4	3	4	5	3
BELEX15	6	7	4	5	5	2	2	3	2

** denotes that the result is rejected by the POF test (at the significance level p=0,025)*

Source: the authors

The results of the VaR analyses for the year 2017 are presented in Table 8. The success rate of the applied models was very high: 94.44 % (34/36). The only two determined estimation failures happened in the case of CROBEX stock index returns, and the D VaR tail estimation method, for rolling windows 100 and 300. The possible reason for these bad estimations might be found in the results of normality tests (Table 2). There, it can be seen that the distribution of CROBEX significantly deviates from the normal distribution ($p<0.01$). Thus, it is not surprising that D VaR, which is a parametric model that assumes a fairly normal distribution of the returns, failed to provide a good estimation in that case.

Table 9

Number of VaR Breaks in the Year 2018 with the Results of the POF Kupiec Backtest

2018	rolling window 100			rolling window 200			rolling window 300		
	HS VaR	D VaR	EVT	HS VaR	D VaR	EVT	HS VaR	D VaR	EVT
CROBEX	9	7	5	6	5	5	4	5	3
BUX	11	9	6	11	12	8	10	11	10
SBITOP	10	9	9	8	9	9	9	9	8
BELEX15	13*	10	6	10	10	5	9	9	5

** denotes that the result is rejected by the POF test (at the significance level p=0,025)*

Source: the authors

The results presented in Table 9, referring to the year 2018, show that risk modelling was very successful for that year. The rate of success was 97,22 % (35/36) since there was only one failure determined by the backtesting. There were 13 exceedances in the case of BELEX15 when risk was estimated by HS VaR with rolling window 100, which is above the upper limit of the Kupiec confidence interval.

Table 10

Number of VaR Breaks in the Year 2019 with the Results of the POF Kupiec Backtest

2019	rolling window 100			rolling window 200			rolling window 300		
	HS VaR	D VaR	EVT	HS VaR	D VaR	EVT	HS VaR	D VaR	EVT
CROBEX	7	6	5	5	6	5	5	6	4
BUX	6	5	2	2	2	2	1*	1*	1*
SBITOP	6	4	2	4	3	2	3	3	2
BELEX15	10	8	6	6	7	5	8	8	5

** denotes that the result is rejected by the POF test (at the significance level p=0,025)*

Source: the authors

Finally, the results of the stock returns in the last of the analysed years are presented in Table 10. In the year 2019, there is one group of results that differs from others: it is the BUX index for rolling window 300. All three VaR models consistently estimate only 1 VaR break, which is too low for the given 97.5 % confidence interval level and thus rejected by the Kupiec backtest. In this case, the explanation of all three model failures can be found in the behaviour of BUX returns in the previous year (Table 9), because these returns are input for parameter estimation. It can be seen that numbers of VaR breaks were high for BUX in 2018, and those low negative returns made the impact to overly harsh estimations for 2019 for this index. Apart from this, models provided good estimations, resulting in a high total rate of 33 out of 36 successes.

After the annually presented and discussed results, the results of a brief univariate comparative analysis of the obtained sample of 288 numbers of VaR breaks per year are presented. This sample contains accumulated results of the VaR modelling (Tables 3-10): 4 indices x 3 Var models x 3 rolling windows x 8 years = 288 VaR breaks. The objective is to investigate whether there is a significant impact of the following factors: (i) applied VaR model, (ii) applied length of the rolling window and (iii) observed stock index. Subsamples generated by grouping according to the first two factors are considered as dependent, whereas the third factor generates four independent groups.

Differences between three applied VaR models on the same stock returns data (N=96) are presented in Figure 1 and analysed in Table 11.

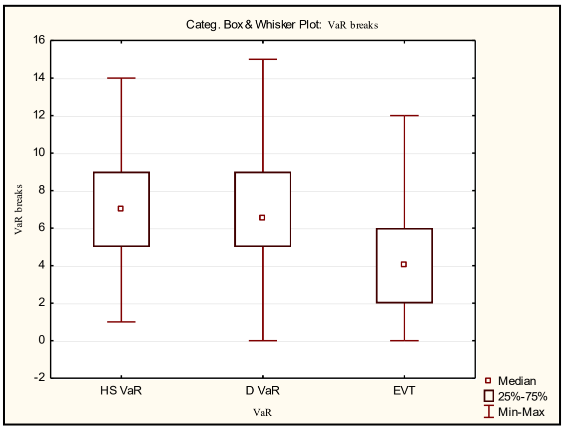

Figure 1. Box-and-Whiskers plot for the Comparison of three VaR Models
Source: the authors

Table 11

Friedman ANOVA for the Comparison of three VaR Models

VaR model	Average rank	Sum of ranks	Mean	Std. Dev.	Chi-square (N=96, df=2)	p-value
HS VaR	2.4844	238.5000	6.9167	2.9296	114.2866	< 0.0001
D VaR	2.3438	225.0000	6.5208	3.0881		
EVT	1.1718*	112.5000	4.2708	2.7125		

** significantly different by post-hoc test at the level 0.01*
Source: the authors

From Figure 1 it can be seen that the median number of VaR breaks for HS VaR was 7, for D VaR 6.5 and for EVT 4. Inter-quartile range for HS VaR was 5–9, for D VaR 5–9 and for EVT 2–6. Results of the Friedman ANOVA test, reported in Table 11, suggest that there exists a statistically significant difference in the number of VaR breaks depending on the applied VaR model. More precisely, results of the mean comparison by ranks show that the application of EVT leads to the significantly smaller number of VaR breaks than the other two models: for HS VaR vs. EVT z-statistics = 5.98 ($p < 0.0000$); for D VaR vs. EVT z-statistics = 5.17 ($p < 0.0000$). The difference between HS VaR and D VaR is not significant: z-statistics = 0.80 ($p > 0.1$). This can be in general interpreted as a positive side of the EVT model, however, it can sometimes lead to overly harsh and thus wrong estimations, as observed in the case of years 2012 and 2013.

Differences between three lengths of rolling windows applied on the same stock returns data (N=96) are presented in Figure 2 and analysed in Table 12.

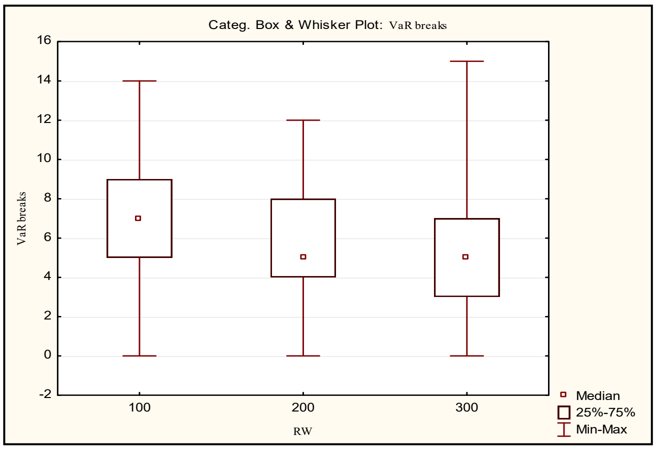

Figure 2. Box-and-Whiskers Plot for the Comparison of three Rolling Window Lengths
Source: the authors

Table 12

Friedman ANOVA for the Comparison of three Rolling Window Lengths

rolling window	Average rank	Sum of ranks	Mean	Std. Dev.	Chi-square (N=96, df=2)	p-value
100	2.5364*	243.5000	6.8958	2.9325	61.5987	< 0.0001
200	1.9271	185.0000	5.6458	2.8873		
300	1.5365	147.5000	5.1667	3.3235		

** significantly different by post-hoc test at the level 0.05*
Source: the authors

From Figure 2 it can be seen that the median number of VaR breaks for the rolling window of 100 days was 7, and for both 200 and 300 days, the median was 5. The inter-quartile range for 100 days long rolling window was 5-9, for 200 days 4-8 and for 300 days 3-7. Results of the Friedman ANOVA test, reported in Table 12, suggest that there exists a statistically significant difference in the number of VaR breaks depending on the applied length of the rolling window. More precisely, results of the mean comparison by ranks show that the usage of 100 days long rolling window leads to a significantly larger number of VaR breaks than the other two lengths: 100 vs. 200 days z-statistics = 2.82 ($p < 0.05$); for 100 vs. 300 days z-statistics = 3.98 ($p < 0.001$). The difference between 200 and 300 days long rolling window is not significant: z-statistics = 1.15 ($p > 0.1$). The general conclusion is that it is more preferable to use longer rolling windows, although the presented research also offers evidence against the generalized application of this rule (Tables 3 and 4).

Finally, differences between four stock indices (CROBEX, BUX, SBITOP, BELEX15) are analysed in Table 13 and illustrated by Figure 3.

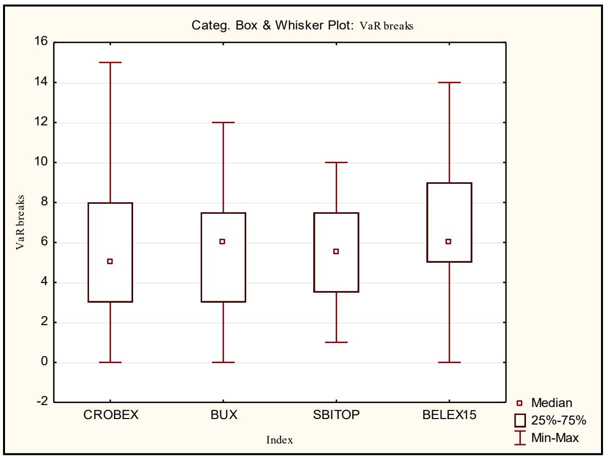

Figure 3. Box-and-Whiskers Plot for the Comparison of four Stock Indices

Source: the authors

Table 13

Kruskal-Wallis ANOVA for the Comparison of Four Stock Indices

stock index	Valid N	Sum of ranks	Mean rank	H-statistics (N=288, df=3)	p-value
CROBEX	72	10201.50	141.6875	3.5968	0.3084
BUX	72	10057.00	139.6806		
SBITOP	72	9822.50	136.4236		
BELEX15	72	11535.00	160.2083		

Source: the authors

The result of Kruskal-Wallis ANOVA by ranks confirms that there is no statistically significant difference between the four indices in terms of the calculated number of VaR breaks per year ($p>0.3$).

Conclusions

The results obtained by the research have both academic and practical importance. Academic, because they provide specific, empirical knowledge about the specifics of the application of different models of investment risk modelling in the observed markets of developing countries, thus expanding the cognitive base in the field. Practical, because the knowledge gained can be used by investment policymakers to optimise and maximise the effects of investment activities.

The basic research hypothesis was confirmed, i.e. it is possible to successfully anticipate the maximum possible investment losses in the extreme return points by applying different methods of risk modelling in volatile market circumstances.

Based on the research results, it can be concluded that there is a statistically significant difference in the performance of the tested VaR models and that EVT is to some extent dominant in relation to HS VaR and D VaR. However, it is necessary to continuously monitor the performance of their implementation with special emphasis on the possible risk overestimation or underestimation.

The research results also indicate the need to optimise the number of days (rolling windows) used in the calculation of the tested VaR models. Namely, although the general conclusion is that it is desirable to use wider rolling windows, it is necessary to pay attention to the frequency of extreme events and crises when determining the maximum possible loss from investment activities. Bearing in mind that the research did not establish a statistically significant difference between the analysed CROBEX, BUX, SBITOP and BELEX15 stock exchange indices, the necessity of further analytical study of extreme return function in the markets of developing countries, especially in volatile market conditions, was confirmed.

The analysed performance of risk modelling under volatile conditions emphasises the necessary and justified application of the Extreme Value Theory (EVT) models as an adequate basis for estimating the return tails, i.e. the extremes, which determine volatile market circumstances.

Specifically, the focus of EVT is on modelling the tails of the returns, and it is necessary to continuously monitor the situation and opportunities in the observed markets, in order to promptly identify extreme events and timely fit the tested model. Having in mind the specifics of the markets of developing countries and their sensitivity to extreme events, the application of the Extreme Value Theory (EVT) model is the basis for successful risk modelling, i.e. making optimal investment decisions. In practice, however, it is suggested to apply the risk modelling methods using the entire sample simultaneously with those that model only tails. For example, EVT could be combined with computationally simpler VaR estimation methods such as HS VaR and D VaR, in order to obtain more reliable results.

The challenges that the authors encountered during the research primarily stem from the specificities of the analysed markets since these are emerging markets that can be considered as low-efficient, highly volatile, and insufficiently liquid. The data collection needed for statistical processing of these markets was additionally challenging.

Suggestions for further research are primarily directed to additional analytical studies of the return function in its extremes, because only in this way it is possible to maximise the effects of investment activities in volatile circumstances and consequently anticipate the maximum possible losses from these activities.

All the above-mentioned data conclude the risk modelling calculation, validation, and analysis in this research. The next logical step, i.e a more advanced multivariate analysis of the obtained risk modelling results, is envisaged for the continuation of the research.

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