

## The Comparative Analysis of MCDA Methods *SAW* and *COPRAS*

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**crossref** <http://dx.doi.org/10.5755/j01.ee.22.2.310>

In recent years, quantitative multicriteria methods have been widely used for comparative evaluation of complicated technological and social-economic processes, as well as for determining the best alternative among the available options and ranking the alternatives based on their significance for a particular purpose. Professor of Vilnius Gediminas Technical University E.K. Zavadskas was the first to use these methods in Lithuania in the mid-eighties of the last century for evaluation, substantiation and choosing of optimal technological solutions at various stages of construction (Zavadskas 1987). In this period, new multicriteria evaluation methods were being developed and widely used in the world in various scientific and practical areas. Later, numerous disciples and colleagues of prof. Zavadskas as well representatives of various scientific schools extensively used the considered methods in Lithuania.

The main concept behind the quantitative evaluation methods is integration of the values of the criteria describing a particular process and their weights (significances) into a single magnitude, i.e. the criterion of the method. For some particular (maximizing) criteria the largest value is the best, while for others (minimizing criteria) the smallest value is the best. The units of criteria measurement are also different. The alternatives compared are ranked according to the calculated values of the criterion of the method. Great numbers of multicriteria evaluation methods, based on different logical principles and having different complexity levels and the inherent features, have been created in the world. There is hardly any 'best' multicriteria evaluation method. Therefore, a parallel use of several multicriteria evaluation methods as well as the analysis of the spread of estimates and averaging of the values obtained may be recommended for evaluating complicated multifaceted objects and processes.

The method *SAW* (Simple Additive Weighting) is one of the simplest, natural and most widely used multicriteria evaluation methods. It clearly demonstrates the idea of integrating the values and weights of criteria into a single estimating value – the criterion of the method. However, *SAW* uses only maximizing evaluation criteria, while minimizing evaluation criteria should be converted into the maximizing ones by the respective formulas prior to their application. This limitation is eliminated in the method *COPRAS* (Complex Proportional Assessment). The authors of the method, E. K. Zavadskas and his disciple A.Kaklauskas suggested that the influence of maximizing and minimizing evaluation criteria should be assessed separately. In this case, the component, taking into account

the effect of maximizing criteria, matches the estimate yielded by the method *SAW*.

Despite the fact that the method *COPRAS* is most commonly used in Lithuania, its main characteristics and properties have not been clearly defined and demonstrated. However, the awareness of these properties allows us to show the benefits of the method's application, to predict the influence of minimizing criteria values on the final result (estimate), to check the calculations and to take into account possible instability of estimates yielded by the method due to the specific character of the actual data.

The paper describes the main features of multicriteria evaluation methods *SAW* and *COPRAS* and their common and diverse characteristics, as well as defining and demonstrating the properties of the method *COPRAS*, which are of great theoretical and practical value.

All theoretical statements are illustrated by numerous examples and calculations.

Keywords: decision making, MCDA, *SAW* method, *COPRAS* method, comparative analysis

### Introduction

In practice, a decision-making person (DM) is often faced with the problem of choosing the best alternative from the available options. This may be the choice of the best technological or investment variants. In particular, the choice of the best technological or investment project or determination of an enterprise which is the best according to its financial and commercial activities or strategic potential, etc. should be made. Besides, the above problems may embrace the evaluation of the development of the state regions or various states, etc. None of these processes or phenomena can be evaluated by a single magnitude because it is hardly possible to find a characteristic which could integrate all relevant aspects of the considered issue.

In recent years, multicriteria methods have been increasingly used for quantitative evaluation of complicated economic or social processes (Figueira *et al.* 2005; Ginevicius 2008; Ginevicius, Podvezko 2004; Ulubeyli, Kazaz 2009; Kaklauskas *et al.* 2007; Kracka *et al.* 2010; Liaudanskiene *et al.* 2009; Plebankiewicz 2009; Podvezko 2007, 2009; Podvezko, Podviezko 2010; Selih *et al.* 2008; Turskis *et al.* 2009; Ustinovichius *et al.* 2007; Urbanavicienė *et al.* 2009a,b; Zavadskas, Vaidogas 2008, Zavadskas *et al.* 2007a,b, 2010; Zavrli *et al.* 2009).

The considered methods are based on the matrix  $R = \left\| r_{ij} \right\|$  of the criteria, describing the alternatives (objects)

$A_j$  ( $j=1,2,\dots,n$ ) compared, statistical data or expert estimates and the criteria weights (significances)  $\omega_i$  ( $i=1,2,\dots,m$ ), where  $m$  is the number of criteria and  $n$  is the number of the alternatives (objects) compared ( $\sum_{i=1}^m \omega_i = 1$ ).

The aim of the evaluation is to choose the best alternatives, ranking the alternatives  $A_j$ , i.e. arranging them in the order of their significance to the research object by using quantitative multicriteria evaluation methods. None of these methods can be used formally without a preliminary analysis. Each method is characterized by specific features and has some advantages. To apply quantitative multicriteria evaluation methods, the type of criteria (minimizing or maximizing) should be determined. The best values of maximizing criteria are the largest values, while the smallest values are the best for minimizing criteria. The criteria of quantitative evaluation methods usually integrate normalized (dimensionless) criteria values  $\tilde{r}_{ij}$  and weights  $\omega_i$ .

In Lithuania, such multicriteria evaluation methods as SR (Sum of Ranks), GM (Geometric Mean), SAW (Simple Additive Weighting), TOPSIS (Technique for Order Preference by Similarity to an Ideal Solution), a compromise classification approach VIKOR (Visekriterijumsko Kompromisino Rangiranje (in Serbian)), COPRAS (Complex Proportional Assessment) and PROMETHEE (Preference Ranking Organisation Method for Enrichment Evaluation) are used, but the most commonly used methods are SAW, COPRAS and TOPSIS. The first two have much in common possessing, however, quite a few different properties. Their advantages also differ. Though these methods are widely used, many of their features have not been analysed yet. In the present work, the methods SAW and COPRAS are thoroughly investigated and compared. The main features of COPRAS are defined and demonstrated, and their stability with respect to data variation is investigated. The possibilities of using SAW and COPRAS for evaluating the criteria of hierarchically structured composite numbers of the same level are defined. All theoretical statements are illustrated by examples and calculations.

### The method SAW

SAW (Simple Additive Weighting) is the oldest, most widely known and practically used method (Hwang, Yoon 1981; Chu *et al.* 2007; Ginevicius, Podvezko 2008 a,b,c; Ginevicius *et al.* 2008c; Podvezko 2008; Ginevicius, Gineviciene 2009; Zavadskas *et al.* 2007c; Chu *et al.* 2007; Jakimavicius, Burinskiene 2009; Podvezko *et al.* 2010; Sivilevicius *et al.* 2008). The criterion of the method  $S_j$  clearly demonstrates the main concept of multicriteria evaluation methods – the integration of the criteria values and weights into a single magnitude. This is also reflected in its name.

The sum  $S_j$  of the weighted normalized values of all the criteria is calculated for the  $j$ -th object:

$$S_j = \sum_{i=1}^m \omega_i \rho_{ij}^0, \quad (1)$$

where  $\omega_i$  is weight of the  $i$ -th criterion ( $\sum_{i=1}^m \omega_i = 1$ );  $\tilde{r}_{ij}$  is normalized  $i$ -th criterion's value for  $j$ -th object;  $i=1,\dots,m$ ;  $j=1,\dots,n$ ;  $m$  is the number of the criteria used,  $n$  is the number of the objects (alternatives) compared.

The largest value of the criterion  $S_j$  corresponds to the best alternative. The alternatives compared should be ranked in the decreasing order of the calculated values of the criterion  $S_j$ .

SAW may be used if all the criteria are maximizing. This is a drawback of this method, though minimizing criteria can be easily converted to the maximizing ones by the formula:

$$\bar{r}_{ij} = \frac{\min_j r_{ij}}{r_{ij}}, \quad (2)$$

where  $r_{ij}$  is  $i$ -th criterion's value for  $j$ -th alternative,  $\min_j r_{ij}$  is the smallest  $i$ -th criterion's value for all the alternatives compared,  $\bar{r}_{ij}$  denotes the converted values.

Thus, the smallest criterion value  $r_{ij} = \min_j r_{ij}$  acquires the largest value equal to unity.

In many papers (Hwang, Yoon 1981; Zavadskas, Kaklauskas 1996, etc.), normalization  $z$  (or transformation) of the initial data is used, so that the best criterion value (the largest one for a maximizing criterion and the smallest one for a minimizing criterion) would get the largest value equal to unity. As mentioned above, it is recommended to use formula (2) for transforming minimizing criteria. The transformation formula used for maximizing criteria is as follows:

$$\bar{r}_{ij} = \frac{r_{ij}}{\max_j r_{ij}}, \quad (3)$$

where  $\max_j r_{ij}$  is the largest  $i$ -th criterion's value of all alternatives.

Another SAW limitation is the requirement that all criteria values  $r_{ij}$  should be positive. In the opposite cases (migration balance and similar cases), negative values are transformed to positive values, using, for example, the formula (Ginevicius, Podvezko 2007a):

$$\bar{r}_{ij} = r_{ij} + \left| \min_j r_{ij} \right| + 1. \quad (4)$$

Due to this transformation, the smallest negative value is turned to unity.

To illustrate, compare and analyse the methods used in the present paper, a case study based on the statistical data of economic development in 2003 of four countries – Estonia, Latvia, Lithuania and Poland is provided (Ginevicius *et al.* 2006). The data obtained are presented in Table 1.

Table 1

The criteria of economic growth of different countries (2003)

Criteria	Country	Types of criteria	Estonia		Latvia		Lithuania		Poland	
			value	rank	value	rank	value	rank	value	rank
1	Annual growth of the GDP, %	max	5.1	3	7.5	2	9.7	1	3.8	4
2	Annual growth of production, %	max	9.8	2	6.5	4	16.1	1	8.4	3
3	Average annual salary, euro, %	max	430	2	298	4	306	3	501	1
4	Unemployment rate, %	min	9.3	1	10.3	2	11.6	3	19.3	4
5	Export/import ratio, %	max	0.70	3	0.55	4	0.73	2	0.79	1
	Sum of ranks		-	11	-	16	-	10	-	13
	Rank of the country		-	2	-	4	-	1	-	3

We can see that four criteria are maximizing, while one (unemployment rate) is minimizing. A typical case of the use of multicriteria evaluation methods is considered, when none of the countries seems to be the best, and the ranks of particular criteria range to a great extent.

Let us transform the data provided in Table 1 by using

formulas (2)-(3).

The transformed values of five criteria describing the economic development of four countries are given in Table 2. The criteria weights (column 3 in Table 2) are determined by the experts of the Finance Ministry of Lithuanian Republic (Ginevicius *et al.* 2006).

Table 2

The data of Table 1 transformed by using formulas (2)-(3)

Criteria	Country	Weights	Estonia	Latvia	Lithuania	Poland
1	Annual growth of the GDP	0.35	0.526	0.773	1.0	0.392
2	Annual growth of production	0.10	0.609	0.404	1.0	0.522
3	Average annual salary, euro	0.17	0.858	0.595	0.599	1.0
4	Unemployment rate	0.25	1.0	0.903	0.802	0.482
5	Export/import ratio	0.13	0.886	0.696	0.924	1.0

The values of SAW criterion  $S_j$ , calculated based on the data taken from Table 2 by formula (1), are given in Table 3.

Table 3

SAW criterion  $S_j$  values obtained by transforming the data by formulas (2)-(3)

Method	Estonia	Latvia	Lithuania	Poland
$SAW(S_j)$	0.756	0.728	0.872	0.610
Rank	2	3	1	4

This is a common application of multicriteria evaluation methods: two out of five results (3rd and 5th) obtained for Poland are the best, while two other values (1st and 4th) are the worst. Their weights are the largest, therefore, the country is ranked the last according to the evaluation by the method SAW.

The values of the criterion  $S_j$  range from zero to unity when the transformation of the data by formulas (2)-(3) is used:  $0 < S_j \leq 1$

The method SAW, like some other multicriteria methods, can yield the distorted evaluation data, e.g. the value of an alternative of one of the criteria greatly exceeds the values of other alternatives, while the weight (significance) of this criterion is the largest. In this case, the alternative may be assessed as the best, though the values of its other criteria are relatively small.

SAW with its data transformation by formulas (2)-(3) has some disadvantages: the largest value of the criterion

of the method  $S_j$  may be about the unity, while the smallest value may approach zero. However, the difference in estimates of the compared alternatives can hardly be determined from the first sight. The relative  $S_j$  values can be determined by normalizing them by the formula:

$$\mathcal{S}_j^0 = \frac{S_j}{\sum_{j=1}^n S_j}, \tag{5}$$

where  $\mathcal{S}_j^0$  is the normalized value of the criterion  $S_j$ ,  $n$  is the number of alternatives ( $\sum_{j=1}^n \mathcal{S}_j^0 = 1$ ). Thus, the value  $\mathcal{S}_j^0$

of the criterion  $A_j$  of the j-th alternative may be easier compared to its average value equal to  $1/n$  (in the considered case,  $1/4 = 0.25$ ).

Normalized values of the criterion  $S_j$  taken from Table 3 are given in Table 4 ( $\sum_{j=1}^4 S_j = 2.966$ ).

Table 4

Normalized values  $S_j^{\%}$  of the criterion  $S_j$  from Table 3

$S_j$	0.756	0.728	0.872	0.610
$S_j^{\%}$	0.255	0.245	0.295	0.206

In practice, it is more convenient to use *SAW* with ‘classical’ normalization ( $\sum_{j=1}^n \rho_{ij}^{\%} = 1$ ) (Ginevicius, Podvezko 2007a,b):

$$\rho_{ij}^{\%} = \frac{r_{ij}}{\sum_{j=1}^n r_{ij}}, \quad (6)$$

where  $r_{ij}$  is  $i$ -th criterion’s value for  $j$ -th alternative.

In using this type of normalization, the *SAW* criterion’s  $S_j$  sum of values of all  $n$  alternatives is equal to unity:

$$\sum_{j=1}^n S_j = \sum_{j=1}^n \sum_{i=1}^m \omega_i \rho_{ij}^{\%} = \sum_{i=1}^m \omega_i \sum_{j=1}^n \rho_{ij}^{\%} = 1, \quad (7)$$

i.e. normalized values of the criterion  $S_j$  of *SAW*, which may be used for evaluating hierarchically structured alternatives of the same level are calculated (Ginevicius, Podvezko 2003, 2007b). In this case, it is also easier to compare different estimates of the alternatives.

The values of Table 1, normalized by using formula (6) are given in Table 5. The values of the minimizing 4th criterion were preliminarily transformed by formula (2) (row 4, Table 2). Normalized by using formula (6) values of the maximizing criteria, do not depend on the fact whether they were preliminarily transformed by formula (3) or not.

Table 5

The data of Table 1 normalized by formulas (6) and (2)

		Country			
Criterion		Estonia	Latvia	Lithuania	Poland
1	Growth of the gross national product	0.195	0.287	0.372	0.146
2	Annual growth of industrial production	0.240	0.159	0.395	0.206
3	Average annual work payment	0.280	0.194	0.199	0.326
4	Unemployment rate	0.314	0.283	0.252	0.151
5	Export/import ratio	0.253	0.199	0.264	0.285

The values of *SAW* criterion  $S_j$ , based on the normalized values taken from Table 5, are given in Table 6.

Table 6

The values of *SAW* criterion  $S_j$  based on the normalization performed by using formulas (3) and (5)

Method	Estonia	Latvia	Lithuania	Poland
SAW( $S_j$ )	0.251	0.246	0.301	0.202
Rank	2	3	1	4

The ranks of the countries investigated have not changed as before (Table 3), and the values of the criteria  $S_j^{\%}$  (Table 4) and  $S_j$  (Table 6) are practically the same.

Let us consider the main features of the method *SAW*.

It has the following positive characteristics and features:

1) The criterion  $S_j$  of the method *SAW* reflects the main concept underlying quantitative multicriteria evaluation methods, consisting in integrating the criteria values and weights into a single magnitude – the criterion of the method.

2) The calculation algorithm of the method is not complicated, being implemented either without the help of a computer or by applying very simple computer programs.

3) Normalized values of the evaluation *SAW* criterion  $S_j$  (or  $S_j^{\%}$ ) help visually determine the differences between the alternatives compared.

However, *SAW* also has some disadvantages:

1) All the values of the criteria  $R_i$  ( $i=1, \dots, m$ ) should be maximizing. Minimizing criteria should be transformed to maximizing ones, for example, by formula (2) before being used in the analysis.

2) All the values of the criteria  $R_i$  ( $i=1, \dots, m$ ) should be positive. The evaluation results, i.e. the values of the criterion  $S_j$ , depend on the type of their transformation to positive values.

3) The estimates yielded by *SAW* do not always reflect the real situation. The result obtained may not be logical, with the values of one particular criterion largely differing from those of other criteria.

### The method *COPRAS*

In 1996, the researchers of Vilnius Gediminas Technical University (Zavadskas, Kaklauskas 1996) created a method of complex proportional evaluation *COPRAS* (Complex Proportional Assessment). It is used for multicriteria evaluation of both maximizing and minimizing criteria values. This is the advantage of the method *COPRAS* over the *SAW* method. The method

*COPRAS* is widely used by its authors, their disciples and specialists evaluating complex processes by quantitative multicriteria methods (Kaklauskas *et al.* 2005, 2006, 2008, 2010; Zavadskas *et al.* 2008a,b,c, 2009a,b; Zavadskas, Antucheviciene 2007; Banaitiene *et al.* 2008; Lepkova *et al.* 2008; Ginevicius, Podvezko 2007d; Ginevicius *et al.* 2008a,b; Podvezko 2008; Sarka *et al.* 2008; Sliogeriene *et al.* 2009; Datta *et al.* 2009; Hofer 2009; Karbassi *et al.* 2008; Mickaityte *et al.* 2008; Mazumbar 2009; Mazumbar *et al.* 2010; Bindu Madhuri *et al.* 2010a,b; Schieg 2009; Tupenaite *et al.* 2010). In this method, the influence of maximizing and minimizing criteria on the evaluation result is considered separately. The evaluation component  $S_{+j}$  of the  $j$ -th alternative of maximizing criteria matches the sum  $S_j$  (1) of normalized weighted values in the method *SAW*. This implies that if only maximizing criteria and classical normalization (6) of criteria values are used, the calculation results obtained by the method *COPRAS* match the data yielded by the method *SAW* (Ginevicius, Podvezko 2007a).

The values of the criterion  $Z_j$  in *COPRAS* are obtained by the formula:

$$Z_j = S_{+j} + \frac{S_{-\min} \sum_{j=1}^n S_{-j}}{S_{-j} \sum_{j=1}^n \frac{S_{-\min}}{S_{-j}}}, \quad (8)$$

where

$$S_{+j} = \sum_{i=1}^m \omega_{+i} \rho_{+ij}^0 \quad (9)$$

is the sum of maximizing weighted criteria values  $\tilde{r}_{+ij}$ , normalized by formula (6) for each  $j$ -th alternative;

$$S_{-j} = \sum_{i=1}^m \omega_{-i} \rho_{-ij}^0 \quad (10)$$

is the sum of minimizing weighted normalized criteria values  $\rho_{-ij}^0$ ;  $j=1,2,\dots,n$ ;  $n$  is the number of the compared alternatives;  $S_{-\min} = \min_j S_{-j}$  is minimal  $S_{-j}$  value of minimizing criteria of all the alternatives. The sign ‘+’ shows that only normalized values of  $j$ -th alternative’s maximizing criteria  $\tilde{r}_{+ij}$ , multiplied by their weights  $\omega_{+i}$ , are summed up. Similarly, the sign ‘-’ applies to minimizing criteria and their weights  $\omega_{-i}$ .

The formulas (8) and (10) also show the inherent inconsistency of *COPRAS*: the value of the most important alternative of a minimizing criterion  $\rho_{-ij}^0$  is the smallest, however, the largest criterion weight  $\omega_{-i}$  matches it, while the sum of these weighted values  $S_{-j}$  is in the denominator of the criterion (8). This may lead to incorrect evaluation of the alternatives.

In order to use the same notation in all multicriteria evaluation methods, which would be different from the original (Zavadskas, Kaklauskas 1996), we will denote the criterion values by  $r_{ij}$  (instead of  $x_{ij}$ ) and their weights –

by  $\omega_i$ , as well as differently denoting maximizing and minimizing weights by  $\omega_{+i}$  and  $\omega_{-i}$ , respectively (rather than by  $q_i$  in the original). The criteria values normalized by formula (6) will be denoted by  $\rho_{ij}^0$  (the authors use normalization with the weights

$$d_{ij} = \frac{x_{ij} \cdot q_i}{\sum_{j=1}^n x_{ij}},$$

i.e., in our case,  $d_{ij} = \omega_i \rho_{ij}^0$ ), while the criterion of the method will be denoted by  $Z_j$  rather than by  $Q_j$ , used in the method *VIKOR* (Opricovic, Tzeng 2004). If only maximizing criteria are used,  $Z_j = S_{+j} = S_j$ .

The concept underlying the method *COPRAS* is quite clear: the estimate of the  $j$ -th alternative  $Z_j$  is directly proportional to the effect produced by maximizing criteria  $S_{+j}$  and inversely proportional to the sum of the weighted normalized values of minimizing criteria – the component  $S_{-j}$ .

The criterion  $Z_j$  (8) may be expressed in a more compact form as follows:

▪ The same constant  $S_{-\min}$  (Ginevicius *et al.* 2004) in the numerator and denominator of the formula (8) can be cancelled, and the formula will be of the form:

$$Z_j = S_{+j} + \frac{\sum_{j=1}^n S_{-j}}{S_{-j} \sum_{j=1}^n \frac{1}{S_{-j}}}, \quad (11)$$

or

$$Z_j = Z_{+j} + Z_{-j}, \quad (12)$$

where  $Z_{+j} = S_{+j}$  is the component of the effect of all maximizing criteria (9), while

$$Z_{-j} = \frac{\sum_{j=1}^n S_{-j}}{S_{-j} \sum_{j=1}^n \frac{1}{S_{-j}}}. \quad (13)$$

Is the component of the effect of all minimizing criteria.

The numerator and denominator of the formula may be divided by the expression  $S_- = \sum_{j=1}^n S_{-j}$ . Thus, the

normalized value  $\rho_j^0 = \frac{S_{-j}}{S_-} = \frac{S_{-j}}{\sum_{j=1}^n S_{-j}}$ , rather than the sum  $S_j$

of the weighted normalized values of the  $j$ -th alternative’s all minimizing criteria, will be found in the numerator of the formula (11).

Then, formula (8) will be rearranged into the formula:

$$Z_j = S_{+j} + \frac{1}{\sum_{j=1}^n \frac{1}{S_{-j}}} \quad (14)$$

This formula can hardly facilitate the calculation of the criterion  $Z_j$ , but it better reflects the concept underlying the method *COPRAS*.

The normalized value  $S_{-j}^0$ , replacing the sum  $S_j$  in the numerator, reduces the influence of minimizing criteria on the total evaluation result  $Z_j$  (by increasing the numerator of the second summand), particularly, if the number of minimizing criteria is small. The sum of inverse minimizing criteria  $\sum_{j=1}^n \frac{1}{S_{-j}}$  in the numerator can also reduce the influence of the result obtained: the values of  $S_{-j}$  may be about zero, i.e. their inverse value in the numerator of the formula (13-14) may become very large.

Therefore, the results yielded by *COPRAS* may be sensitive to slight data variation, and the ranks assigned may differ from those obtained by using other methods. Let us demonstrate the calculation results of two data variants by methods *COPRAS*, *SAW* and *TOPSIS*. *SAW* is based on classical normalization (6), with minimizing criteria converted to the maximizing ones (2). Two sets of data are given in Table 7.

Two versions of the alternatives  $A_1$ ,  $A_2$  and  $A_3$ , described by four criteria  $R_1$ ,  $R_2$ ,  $R_3$  and  $R_4$  are considered. The data referring to them differ to a small extent. In particular, the values of the first three criteria are the same, while the value of the fourth criterion of the first alternative  $r_{41}=105$  is replaced by  $r_{41}=110$  and  $r_{42}=215$  is substituted for  $r_{42}=200$ .

Table 7

**Illustration of instability of *COPRAS* application**

Criteria	Direction	Weights $\omega_i$	Variant I			Variant II		
			$A_1$	$A_2$	$A_3$	$A_1$	$A_2$	$A_3$
$R_1$	max	0.26	42	71	53	42	71	53
$R_2$	max	0.23	19	18	20	19	18	20
$R_3$	min	0.24	13	11	12	13	11	12
$R_4$	min	0.27	105	215	149	110	200	149

The data referring to two variants of the third alternative have completely matched. The calculation

results yielded by *COPRAS* compared to the data obtained by *SAW* and *TOPSIS* are given in Table 8.

Table 8

**Calculation results yielded by *COPRAS*, *SAW* and *TOPSIS***

Method	Variant I			Variant II		
	$A_1$	$A_2$	$A_3$	$A_1$	$A_2$	$A_3$
<i>COPRAS</i> $S_{+j}$	0.142	0.184	0.164	0.142	0.184	0.164
$S_j$	0.147	0.199	0.165	0.151	0.191	0.168
$Z_j$	0.332	0.334	0.335	0.332	0.334	0.335
Rank	1	3	2	3	2	1
<i>TOPSIS</i>	0.575	0.423	0.539	0.526	0.471	0.494
Rank	1	3	2	1	3	2
<i>SAW</i>	0.337	0.327	0.336	0.340	0.336	0.331
Rank	1	3	2	1	2	3

As it has been shown in the Table, the ranks assigned to the alternatives by all three methods – *COPRAS*, *TOPSIS* and *SAW* have matched for the first variant. The values of the 3rd criterion have slightly changed in the 2nd variant, however, the ranks obtained by *COPRAS* (Table 8) have changed places (3-2-1 instead of 1-3-2), The ranks given by *TOPSIS* have not changed, and the rank assigned to the best 3rd alternative by *SAW* has not changed either, though the 1st and 2nd alternatives have changed places. The results obtained show that *COPRAS* may be less stable than other methods in the case of data variation, while the ranks of the alternatives given by *COPRAS* may differ to great extent from those yielded by other methods. This not only reveals the particular problems associated with *COPRAS* application, but also demonstrates common

approaches to evaluating multicriteria methods. Thus, each multicriteria method has its advantages and disadvantages and, therefore, simultaneous use of several methods and the analysis of causes of estimates' variation may be recommended.

The method *COPRAS* also has some advantages and valuable features. The criterion of the method  $Z_j$  and properties of its components allow us to easily compare and check the results of calculations and to compare the methods *COPRAS* and *SAW*. However, these properties have not been profoundly investigated and described in the literature.

**Property 1.**

The sum  $S_+ = \sum_{j=1}^n S_{+j}$  of the evaluation components of all  $n$  alternatives of maximizing criteria  $S_{+j}$  is equal to the sum of maximizing criteria weights  $\sum_{i=1}^m \omega_{+i}$ :

$$S_+ = \sum_{j=1}^n S_{+j} = \sum_{i=1}^m \omega_{+i}. \quad (15).$$

A similar result is obtained for minimizing criteria:

$$S_- = \sum_{j=1}^n S_{-j} = \sum_{i=1}^m \omega_{-i}. \quad (16)$$

Proof.

$$S_+ = \sum_{j=1}^n S_{+j} = \sum_{j=1}^n \sum_{i=1}^m \omega_{+i} \rho_{+ij} = \sum_{i=1}^m \omega_{+i} \sum_{j=1}^n \rho_{+ij} = \sum_{i=1}^m \omega_{+i}$$

and

$$S_- = \sum_{j=1}^n S_{-j} = \sum_{j=1}^n \sum_{i=1}^m \omega_{-i} \rho_{-ij} = \sum_{i=1}^m \omega_{-i} \sum_{j=1}^n \rho_{-ij} = \sum_{i=1}^m \omega_{-i}.$$

This result was interpreted (Zavadskas, Kaklauskas 1996) so that the sum of the components of maximizing (and minimizing) criteria  $S_+ = \sum_{j=1}^n S_{+j}$  is equal to the sum of normalized weighted values:

$$S_+ = \sum_{j=1}^n S_{+j} = \sum_{i=1}^m \sum_{j=1}^n \omega_{+i} \frac{r_{+ij}}{\sum_{j=1}^n r_{+ij}} = \sum_{i=1}^m \sum_{j=1}^n d_{+ij}$$

(as mentioned above, the authors denoted  $d_{+ij} = \omega_{+i} \frac{r_{+ij}}{\sum_{j=1}^n r_{+ij}}$ ).

**Conclusion.** The sum of normalized weighted values of all maximizing and minimizing criteria of the compared alternatives is equal to the sum of the criteria weights  $\sum_{i=1}^m \omega_i$  (in the considered case, to unity):

$$S = S_{+j} + S_{-j} = \sum_{i=1}^m \omega_i.$$

Actually,

$$S = S_{+j} + S_{-j} = \sum_{i=1}^m \omega_{+i} + \sum_{i=1}^m \omega_{-i} = \sum_{i=1}^m \omega_i.$$

**Property 2.** The sum of minimizing criterion components  $Z_{-j}$  (13) of *COPRAS* evaluation criterion  $Z_j$  is equal to the sum of weights of minimizing criteria  $\sum_{i=1}^m \omega_{-i}$ :

$$\sum_{j=1}^n Z_{-j} = \sum_{i=1}^m \omega_{-i}. \quad (17)$$

Proof.

$$\begin{aligned} Z_- &= \sum_{j=1}^n Z_{-j} = \sum_{j=1}^n \frac{\sum_{i=1}^m S_{-j}}{S_{-j} \sum_{i=1}^m \frac{1}{S_{-j}}} = \\ &= \frac{\sum_{j=1}^n S_{-j}}{\sum_{j=1}^n \frac{1}{S_{-j}}} \cdot \sum_{j=1}^n \frac{1}{S_{-j}} = \sum_{j=1}^n S_{-j} = S_- = \sum_{i=1}^m \omega_{-i}. \end{aligned} \quad (18)$$

Explanation. The constant  $\sum_{j=1}^n S_{-j}$  is placed in the numerator of formula (18), while the constant  $\sum_{j=1}^n \frac{1}{S_{-j}}$  is placed in the denominator. Then, the constant  $\sum_{j=1}^n \frac{1}{S_{-j}}$  is reduced in the fraction and the equation (16) is used.

**Conclusion.** When the method *COPRAS* is used, the sum of the evaluation components  $Z_{+j}$  and  $Z_{-j}$ , i.e. the sum of the criterion's  $Z_j$  values is equal to the sum of the criteria weights  $\sum_{i=1}^m \omega_i$  (in the considered case, to unity):

$$\sum_{j=1}^n Z_{+j} + \sum_{j=1}^n Z_{-j} = \sum_{j=1}^n Z_j = Z = \sum_{i=1}^m \omega_i. \quad (19)$$

In particular, formulas (12), (15) and (14) will be used:

$$\sum_{j=1}^n Z_j = \sum_{j=1}^n Z_{+j} + \sum_{j=1}^n Z_{-j} = \sum_{i=1}^m \omega_{+i} + \sum_{i=1}^m \omega_{-i} = \sum_{i=1}^m \omega_i.$$

As proved above (see (7)), this result is also valid for the sum of the criteria of *SAW*  $S_j$ , using classical normalization technique (6):

$$\sum_{j=1}^n S_j = \sum_{i=1}^m \omega_i = 1.$$

By using formulas (15), (18) and (19), the calculation results yielded by *COPRAS* may be validated. For

example, in the monograph of the authors of *COPRAS* approach ((Zavadskas, Kaklauskas, Banaitiene 2001), the calculation results of the evaluation process are provided (267-268 pp. Table 6.20). The values of the *COPRAS* evaluation components  $S_{+j}$ ,  $S_{-j}$  of maximizing and

minimizing criteria and the evaluation criterion  $Z_j$  are given in Table 9, while the sums of all these values are presented in the last column of this table.

Table 9

**The results obtained in multicriteria evaluation of the alternatives of plots**

	1	2	3	4	5	
$S_{+j}$	0.1374	0.1292	0.1109	0.1116	0.0496	$S_+ = \sum_{j=1}^5 S_{+j} = 0.5385$
$S_{-j}$	0.1405	0.0898	0.0855	0.0983	0.0470	$S_- = \sum_{j=1}^5 S_{-j} = 0.4611$
$Z_j$	0.1908	0.2129	0.1988	0.1880	0.2093	$Z = \sum_{j=1}^5 Z_j = 0.9998$

The rounded off sum of 12 maximizing criteria weights  $\sum_{i=1}^{12} \omega_{+i} = 0.5387$ , as well as the sum of the weights of two minimizing criteria  $\sum_{i=1}^2 \omega_{-i} = 0.4611$  and the sum of weights of all 14 criteria  $\sum_{i=1}^{14} \omega_i = \sum_{i=1}^{12} \omega_{+i} + \sum_{i=1}^2 \omega_{-i} = 0.9998$ , calculated based on the data presented in Table 6.20 of the above-mentioned monograph, agreed with the respective values of  $S_+$ ,  $S_-$  and  $Z$  given in Table 9.

Let us consider another example, representing one of the recent cases of *COPRAS* applications (Sliogeriene *et al.* 2009). Thirty two criteria, including 23 maximizing and 9 minimizing criteria, were used. The calculations also demonstrated the validity of the results obtained by using the method *COPRAS*:

$$S_+ = \sum_{j=1}^{14} S_{+j} = 1.1341 \approx \sum_{i=1}^{23} \omega_{+i} = 1.134;$$

$$S_- = \sum_{j=1}^{14} S_{-j} = 0.8693 \approx \sum_{i=1}^9 \omega_{-i} = 0.869;$$

$$Z = \sum_{j=1}^5 Z_j = 2.0396 \approx \sum_{i=1}^{32} \omega_i = 2.003.$$

Property 2 and the conclusion made allow us to compare the methods *COPRAS* and *SAW*: the sum of the criteria values of all  $n$  alternatives obtained by using these two methods is equal to the sum of the criteria weights  $\sum_{i=1}^m \omega_i$  (in the considered case, to unity):

$$S = \sum_{j=1}^n S_j = \sum_{j=1}^n Z_j = Z = \sum_{i=1}^m \omega_i \quad (20)$$

where  $S_j$  is *SAW* criterion,  $Z_j$  is *COPRAS* criterion (*SAW* is based on ‘classical’ normalization (6)),  $S$  and  $Z$  are the

respective general estimates of all the alternatives evaluated by these methods.

The equality (20) allows *COPRAS* (and *SAW*) to be applied to the evaluation of hierarchically structured complex values of the same level (Ginevicius, Podvezko 2006, 2007c) to obtain normalized values of all higher level alternatives (if  $\sum_{i=1}^m \omega_i = 1$ ).

As mentioned above, when only maximizing criteria are used, the results of calculation by *COPRAS* agree with the data yielded by *SAW*, i.e. for each  $j$ -th alternative we get:

$$S_{+j} = S_j = Z_j = Z_{+j} \quad (j=1,2,\dots,n).$$

In a common case, the evaluation components of maximizing criteria, describing the alternatives by both methods, match each other, i.e.  $S_{+j} = Z_j$ , with all  $j=1,2,\dots,n$ .

Their sums,  $S_+ = \sum_{j=1}^n S_{+j}$  and  $Z_+ = \sum_{j=1}^n Z_{+j}$ , also agree.

Due to this, the sums of evaluation components of all minimizing criteria

$$Z_- = \sum_{j=1}^n Z_{-j} = \sum_{j=1}^n \frac{\sum_{i=1}^n S_{-i}}{S_{-j} \sum_{i=1}^n \frac{1}{S_{-i}}}$$

and

$$S_- = \sum_{j=1}^n S_{-j} = \sum_{j=1}^n \sum_{i=1}^m \omega_{-i} \frac{\omega_j}{\omega_j},$$

obtained by *COPRAS* and *SAW*, will also agree.

This shows that the values of the components of minimizing criteria evaluation as well as general estimates, obtained by these methods, should not differ considerably. The main evaluation principle shared by quantitative multicriteria methods and stating that a more important alternative correlates with a larger value of the criterion of

the method, also accounts for the result obtained. A great number of the performed real case calculations confirms that the difference between the values of the criteria of the above two methods is insignificant. Thus, the evaluation of the development rate of Lithuanian regions (Ginevicius,

Podvezko 2009), based on these two methods (see Table 10), yielded practically the same results (with the difference being  $10^{-4}$ ) because only two out of fourteen criteria were minimizing.

Table 10

**An example illustrating the evaluation results yielded by COPRAS and SAW: the data obtained in multicriteria evaluation of social and economic development of Lithuanian regions in 2006 – 2007**

Method Criterion	Region									
	Alytus	Kaunas	Klaipėda	Marijampolė	Panevėžys	Šiauliai	Tauragė	Telšiai	Utena	Vilnius
2006										
SAW $S_i$	0.0931	0.1016	0.1259	0.0906	0.0849	0.0870	0.0732	0.0929	0.0878	0.1628
COPRAS $Z_i$	0.0931	0.1017	0.1260	0.0906	0.0849	0.0871	0.0732	0.0930	0.0878	0.1627
2007										
SAW $S_i$	0.0928	0.0990	0.1149	0.0854	0.0913	0.0834	0.0742	0.1077	0.0867	0.1646
COPRAS $Z_i$	0.0928	0.0991	0.1149	0.0854	0.0912	0.0834	0.0742	0.1077	0.0868	0.1646

The solution of the problem associated with the comparative analysis of five different building technologies (Ginevicius *et al.* 2008b), where more than a

half (five) of the nine evaluation criteria were minimizing, has also shown that the evaluation results yielded by the above two methods differ insignificantly (Table 11).

Table 11

**The results obtained in multicriteria evaluation of wall insulation alternatives for the main building of VGTU**

Method	Wall insulation alternative No				
	Ltd1	Ltd2	Ltd3	Ltd4	Ltd5
SAW	0.2188	0.2050	0.1977	0.1884	0.1901
COPRAS	0.2186	0.2051	0.1978	0.1891	0.1909

However, other evaluation results (Ginevicius, Podvezko 2008b) demonstrated a more significant

difference (Table 12), though, in that case, only 3 out of 15 criteria were minimizing.

Table 12

**The results obtained in comparing the reliability of Lithuanian commercial banks by multicriteria methods**

Method		Banks									
		1	2	3	4	5	6	7	8	9	10
SAW	Value	0.1034	0.1475	0.1682	0.1609	0.0605	0.0730	0.0759	0.0695	0.0740	0.0699
	Rank	4	3	1	2	10	7	5	9	6	8
COPRAS	Value	0.1052	0.1512	0.1673	0.1622	0.0646	0.0763	0.0833	0.0708	0.0626	0.0563
	Rank	4	3	1	2	8	6	5	7	9	10

The calculation results show that the values of the criteria of the methods COPRAS and SAW usually agree, while the evaluation results may differ. The calculation results obtained by using COPRAS depend on the number of minimizing criteria and their values. However, these problems require further investigation.

**Conclusions**

The methods SAW and COPRAS are widely used for multicriteria evaluation. Though they may seem to be different, both methods have a number of common features and properties.

Some important COPRAS properties, allowing us to more accurately evaluate and validate the calculation results, are defined and proved mathematically.

The cases, when COPRAS may be unstable due to data variation, and the results obtained may differ from the data, yielded by other multicriteria evaluation methods, are described.

Common properties of the methods SAW and COPRAS allow them to be used for comparison and evaluation of criteria describing hierarchically structured complex magnitudes, which are of the same hierarchical level.

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**SAW ir COPRAS metodų analizė ir palyginimas**

Santrauka

Praktikoje dažnai sprendimą priimančiam asmeniui iš keleto pasiūlytų variantų, galimų veiklos alternatyvų reikia pasirinkti geriausią. Tai gali būti geriausio technologinio arba investicinio projekto pasirinkimas, geriausios pagal finansinę, komercinę veiklą įmonės nustatymas arba jų strateginio potencialo palyginimas, šalies regionų ir atskirų šalių plėtros įvertinimas ir daug kitų uždavinių. Nė vieno iš jų neįmanoma aprašyti ir išreikšti vienu dydžiu, rodikliu, nes sunku išskirti tokią jo savybę, kuri jungtų reiškinio visus esminius aspektus.

Pastaruoju metu socialinių ir ekonominių reiškinų, sudėtingų procesų kiekybiniam vertinimui vis plačiau taikomi daugiakriteriai metodai. Pasaulyje sukurta dešimtys daugiakriterių metodų, kurie vienas nuo kito skiriasi savo logika, sudėtingumu, specifika, ypatumu. Nėra geriausio daugiakriterio metodo. Vertinant sudėtingą procesą galima rekomenduoti kartu taikyti keletą metodų, analizuoti vertinimų nesutapimo priežastį ir priimant sprendimą taikyti rezultatų vidurki.

Daugiakriteriai metodai iš esmės skiriasi nuo kitų optimizavimo metodų. Daugiakriterių metodų pagrindą sudaro rodiklių, apibūdinančių lyginamus objektus (alternatyvas)  $A_j$  ( $j=1, 2, \dots, n$ ), statistinių duomenų arba ekspertų vertinimų matrica  $\mathbf{R} = \left\| r_{ij} \right\|$  ir rodiklių reikšmingumai (svoriai)  $\omega_i$  ( $i=1, 2, \dots, m$ ), čia  $m$  – rodiklių skaičius,  $n$  – lyginamų objektų (alternatyvų) skaičius. Vertinimo tikslas – taikant kiekybinius daugiakriterius metodus, atrinkti geriausią alternatyvą, ranguoti lyginamus objektus  $A_j$  tyrimo tikslo atžvilgiu, t. y. išdėstyti juos svarbumo eilės tvarka. Nei vieno metodo negalima pritaikyti formaliai, iš karto. Kiekvienas metodas turi savo pranašumą, ypatumą. Taikant kiekybinius daugiakriterius vertinimo metodus, nustatoma, kokio pavidalo – maksimizuojamas arba minimizuojamas yra kiekvienas rodiklis. Maksimizuojamųjų rodiklių geriausios reikšmės didžiausios, minimizuojamųjų rodiklių geriausios reikšmės mažiausios. Kiekybinių daugiakriterių metodų kriterijai dažniausia sujungia rodiklių bedimenses (normalizuotas) reikšmes ir rodiklių svorius.

Lietuvoje praeito šimtmečio devintajame dešimtmetyje, siekdamas vertinti skirtingų statybos etapų efektyvumą ir ieškodamas jų optimalių technologinių sprendimų, daugiakriterius metodus pradėjo taikyti VGTU profesorius E. K. Zavadskas. Tuo pačiu laiku ir pasaulyje buvo sukurta nemažai naujų daugiakriterių metodų, kurie buvo taikomi skirtingose mokslo ir praktikos srityse. Lietuvoje vėliau šiuos metodus plačiai taikė prof. E. K. Zavadsko mokiniai, jo kolegos, taip pat ir skirtingų sričių mokslininkai.

Iš taikomų Lietuvoje daugiakriterinių metodų – vietų sumos, geometrinio vidurkio, *SAW* (*Simple Additive Weighting*), *TOPSIS* (*Technique for Order Preference by Similarity to an Ideal Solution*), kompromisinio klasifikavimo metodas *VIKOR* (*Vlsekriterijumsko KOMPromisino Rangiranje serbiškai*), *COPRAS* (*Complex Proportional Assessment*), *PROMETHEE* (*Preference Ranking Organisation Method for Enrichment Evaluation*) – dažniausiai taikomi *SAW*, *COPRAS* ir *TOPSIS*: du pirmieji turi daug bendrų savybių, nors nemažai pranašumų ir skirtumų.

*SAW* (*Simple Additive Weighting*) yra pats seniausias, tipinis, labiausiai žinomas ir dažniausiai praktikoje taikomas metodas. Metodo kriterijus tiksliai atspindi kiekybinių daugiakriterių metodų idėją – rodiklių reikšmių ir jų svorių sujungimą į vieną dydį. Tai atitinka ir metodo pavadinimas. Skaičiuojama visų rodiklių pasvertų normalizuotų (bedimensių) reikšmių suma kiekvienam  $j$ -m objektui. Geriausią variantą atitinka didžiausia kriterijaus reikšmė. Lyginamuosius variantus reikia išdėstyti (ranguoti) metodo kriterijaus suskaičiuotų reikšmių mažėjančia tvarka.

*SAW* metodą galima taikyti, jeigu visi rodikliai maksimizuojamieji. Tai yra *SAW* metodo apribojimas ir nepatogumas. Tačiau minimizuojamuosius rodiklius nesunku pertvarkyti į maksimizuojamuosius.

1996 metais Vilniaus Gedimino technikos universiteto mokslininkai E. K. Zavadskas, A. Kaklauskas (1996) sukūrė kompleksinį proporcingo įvertinimo metodą *COPRAS* (*Complex Proportional Assessment*). *COPRAS* metodas taikomas daugiakriterio vertinimo procese, taikomi ir maksimizuojamieji, ir minimizuojamieji rodikliai. Tuo *COPRAS* metodas pranašesnis už *SAW* metodą. Metodą plačiai taiko autoriai, jų mokiniai ir sudėtingų procesų kiekybinių daugiakriterių vertinimų specialistai. *COPRAS* metode atskirai vertinama maksimizuojamųjų ir minimizuojamųjų kriterijų (rodiklių) įtaka vertinimo rezultatui. Maksimizuojamųjų rodiklių  $j$ -os alternatyvos vertinimo komponentė sutampa su *SAW* metodo pasvertų normalizuotų reikšmių suma, t. y. jei taikomi tik maksimizuojamieji rodikliai ir rodiklių reikšmių klasikinė normalizacija (kiekvieno rodiklio normalizuotų reikšmių suma lygi vienetui), tai *COPRAS* metodo skaičiavimo rezultatai sutampa su *SAW* metodo rezultatais.

*SAW* ir *COPRAS* metodai plačiai taikomi, tačiau į daugelį jų savybių ir ypatumų dar nebuvo atkreiptas dėmesys. Šiame darbe plačiai išnagrinėti *SAW* ir *COPRAS* metodai, jie išanalizuoti ir palyginti, suformuluotos ir įrodytos *SAW* ir *COPRAS* metodo savybės bei ištirtas *COPRAS* metodo stabilumas duomenų svyravimo atžvilgiu. Suvokiant šias savybes, galima motyvuoti metodų taikymą, prognozuoti minimizuojamųjų rodiklių įtakos laipsnį bendram vertinimo rezultatui, operatyviai patikrinti skaičiavimo rezultatų teisingumą. *SAW* ir *COPRAS* metodus galima taikyti praktikoje, vertinant hierarchiškai struktūrizuotų sudėtingų dydžių vieno lygmens rodiklius. Visi teoriniai rezultatai iliustruoti pavyzdžiais ir skaičiavimais.

Raktažodžiai: *daugiakriteriai vertinimai, MCDA, SAW metodas, COPRAS metodas, metodų palyginimas.*

The article has been reviewed.

Received in February, 2010; accepted in April, 2011.

DOI: 10.5755/j01.ee.22.2.310