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Portfolio Choice Using Additional Information from Financial Statements - Evidence from the Frankfurt Stock Exchange

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Classical portfolio construction models consider only the information contained in the market prices of stocks, but ignore the financial performance of companies. Typically, the variance is used as the dispersion parameter. This symmetric measure of risk may be inadequate if the distribution of returns differs significantly from the normal or symmetric distribution. With this in mind, we introduce some additional portfolio selection criteria based on companies' financial performance. In addition, we consider semi-variance as an alternative risk measure to variance. The main research objective is to develop a classical portfolio theory by incorporating firms' financial indicators and using semi-variance as a measure of investment risk. In the paper a novel method for portfolio selection has been proposed and we have developed an original computer code to find the efficient three criteria portfolios. A set of 13 types of portfolios was constructed, differing in terms of fundamental values and risk measures. The proposed models were evaluated under the condition of an economic crisis caused by the COVID-19 pandemic. The research sample consisted of the largest companies listed on the Frankfurt Stock Exchange, the DAX 30 index. The research showed that the highest returns were observed for portfolios based on indicators related to the financial condition of the companies. These types of portfolios were also characterized by the highest level of risk, as measured by variance and semi-variance. The analysis thus confirms the usefulness of corporate financial indicators in portfolio decisions and points to the need for further research in this area.

Keywords: *Portfolio Analysis; Fundamental Value; Multicriterial Choice; Market Multiple; Downside Risk.*

Introduction

Classic methods for selecting an investment portfolio were developed by Markowitz (1952, 1959) and then extended by Sharpe (1963). These methods take into account only the market prices of the companies. Investment portfolios are evaluated using to two criteria: potential level of profitability from an investment) and investment's risk. The first criterion is measured with the expected rate of return and the second one with the variance or standard deviation of returns. One considers no other criteria that could provide additional information on the financial position and prospects of a company that could influence the prices of its shares.

However, recently there has been a growing interest in portfolio analysis methods with alternative methods for portfolio construction. The overview of these methods can be found in the article by Kolm, Tutuncu and Fabozzi (2014). The book by Doumpos and Zopounidis (2014) contains a review of multicriteria methods used in this field. In most cases new approach depends on using some other criteria of risk (for example, semi-variance or conditional value at risk) instead of variance or standard deviation of returns. In the article by Fabozzi, Focardi and Jonas (2007) one can find many example of risk measures that are currently used in practice. In other works, one uses some other characteristics of the returns distribution as additional criteria in portfolio evaluation. As an examples of such characteristics we can mention skewness or kurtosis. Expanded portfolio analysis is presented by Briec, Kerstens and Jokund (2007) or Rodriguez, Luque and Gonzalez (2011).

There exist only several works that include criteria not based on assets' returns. There is a branch of the literature which takes into account ethical, social, or environmental criteria in portfolio construction. This can be illustrated with the so-called "socially responsible investments approach" described in Steuer, Qi and Hirschberger (2007). Other examples of this approach are the articles of Ballestero et al. (2012) and Bilbao-Terol et al. (2013). The article of Burchi and Wlodarczyk (2020) investigates the realized performance of socially responsible investments (SRIs) in US and in European financial markets.

There are only a few papers that also take into consideration fundamental values of companies. For example, in the article of Xidonas, Mavrotas and Psarras (2010) dividends paid by the companies were taken into account. Jacobs and Levy (2013) considered the risk associated with leverage. In their approach, the utility function of an investor includes the costs of margin calls. In such situation the investor may be forced to liquidate securities at adverse prices due to their illiquidity. The problem of liquidity was also considered by Lo, Petrov and Wierzbicki (2003). In their approachiquidity of stocks as an additional criterion in the portfolio construction.

According to the theory of finance fundamental factors are important in determining returns on capital markets. The impact of fundamental information concerning companies on their rates of returns was revealed in many financial research, for example in Fama & French (1992), Fama & French (2015), Fama & French (2017), Lam (2002) or Zaremba, Czapkiewicz (2017). Therefore, it seems rational to include them in the process of constructing a portfolio.

There were several works, in which the authors tried to combine a portfolio analysis with a fundamental analysis of companies from the Polish stock markets. Tarczynski (2002) proposed a synthetic measure to evaluate the economic and financial standing of a company, which he called the taxonomic measure of attractiveness of investment (TMAI) and applied this measure as an additional criterion in the portfolio analysis. A portfolio constructed with the use of TMAI was called "a fundamental portfolio". This model has been modified, for example, by replacing variance with semi-variance as a risk measure (Rutkowska-Ziarko & Garsztka, 2014). Rutkowska-Ziarko (2013) used the Mahalanobis distance was used to determine the TMAI. In the article of Pospiech (2019) market indicators were applied to help in the initial selection of companies which can be included in the portfolio.

This study is a continuation and extension of the previous research (Rutkowska-Ziarko & Kliber, 2023) on the application of multi-criteria models and lower risk measures in selecting an investment portfolio. The optimization computer code was improved, and new market indicators, based on cash flow and dividend yield, have been added to the set. Moreover, the timeframe of the study has been extended to incorporate more recent observations. Previous studies have focused on the local European market, specifically the Warsaw Stock Exchange. This article examines investment portfolios consisting of the largest companies listed on the Frankfurt Stock Exchange.

In this study, we extend beyond the conventional risk metric of variance by incorporating semi-variance as an additional measure of risk. The employment of this measure, focusing on downside risk, appears to be especially advantageous during significant market downturns, as in the periods of February and March 2020, precipitated by diminishing investor confidence due to the outbreak of the Covid-19 pandemic. Semi-variance uniquely considers only the negative discrepancies in returns that fall beneath a predefined threshold, disregarding positive discrepancies that result in returns surpassing expectations. A further merit of semi-variance is its independence from assumptions regarding the distribution of return rates and the utility function of investors, as outlined by Harlow and Rao (1989). This is important given the limitations of the quadratic utility function, which, after reaching its peak at a certain return rate of return, starts to decline with returns, which contradicts the fundamental investor preference for greater over lesser returns. The formulation of an optimized portfolio based on semi-variance presents more complexity than when using variance alone as the risk metric. The utilization of standard solver tools is not feasible for identifying portfolios with minimal semi-variance. This is due to the necessity of determining the periods when the portfolio's return rates were beneath the desired level, which depends on the portfolio's specific composition.

The article is organized as follows. In Section 2 we present a brief description of commonly used market multiples and give some reasons why they could be used as an additional criterion for portfolio choice. Section 3 contains a description of the downside risk measures. A mathematical formulation of portfolio optimization problems and a description of the algorithms that were used to solve them are presented in Section 4. Section 5 presents results of empirical research concerning the Frankfurt Stock Exchange during the crisis of the Covid-19 pandemic. The general conclusions are presented in Section 6.

Market Multiples

Market multiples give insights into the valuation of publicly listed corporations by the market. The computation of these multiples requires the utilization of market data alongside the financial outcomes of the firm. Investors rely on having timely and trustworthy financial as well as non-financial statements at their disposal. A highly valuable resource for achieving this is the extensive audit procedure, which effectively reduces the likelihood of errors in the reports (Bartoszewicz & Rutkowska-Ziarko 2022).The results of various studies indicate a connection between the values of market multiplies and the future performance of companies, measured by the rates of returns of their stock prices. For example, Barbee, Jeong and Mukherji (2008) made a research on the impact of market multiples on future prices of stocks. They have studied the profitability of equally weighted portfolios that contained the stocks of companies with different values of particular market multiples.

The most popular indicator of how a firm is evaluated by the market is the P/E multiple (price-to-earnings ratio), which relates earnings per ordinary share to its market price. This ratio informs how many times the market price of one share exceeds the net profit per share. Thus, it determines the number of years after which the capital invested in shares will be returned, if the company generates profits at the current level. The value of the ratio depends on many factors. One of them is the profit of the company, but there are also factors connected with the market conditions. The indicator also reflects the degree of risk associated with the given company. This is due to the fact that investors seek to compensate for uncertainty with higher investment profitability. Most often the ratio is higher for relatively fast-growing companies because investors are willing to pay higher amounts expecting higher returns in the future. The formula for the ratio is presented below:

$$P/E = \frac{stock\ price\ per\ share}{earnings\ per\ share}$$

Breen (1968) and Basu (1977) examined the influence of price-to-earnings (P/E) multiples on the future profitability of firms. Their findings suggest that portfolios comprised of shares from companies with lower P/E multiples yielded a superior annual return in the subsequent year compared to those assembled from firms with higher P/E multiples. Basu's (1977) study is often cited as the pioneering work in assessing the effects of market multiples on future company profitability. Nonetheless, preceding studies, such as that conducted by William Breen (1968), also inquired into this area. Breen's research focused on S&P 500 companies during 1953–1966, utilizing data from the COMPUSTAT database[1], which provides comprehensive fundamental and market data on both active and inactive corporations, representing approximately 99 % of global market capitalization. In specific years, portfolios comprising 10 and 50 companies were created based on the lowest and highest P/E ratios. The outcomes demonstrated that portfolios constituted of shares from companies with lower P/E ratios experienced greater annual returns in the following year than those composed of companies with higher P/E ratios.

The price of a share can be linked not only to different categories of earnings but also to an additional indicators that reflect the company's economic health. For investors, it may be crucial to compare the market value of a company's equity to its assets' net value. The price-to-book value per share ratio is the ratio that determines the number of monetary units that investors have to pay for each unit of the company's book value. The book value is represented by the net assets, that is, the difference between the total assets and the total liabilities. Therefore, if the value of the ratio is greater than one, it means that the market values the company is above its book value. The exact opposite is the case when this indicator is less than one. In general, investors perceive this ratio as an indicator of a company's investment ability. Its high level proves the company's significant investment activity. It also proves that the company generates a significant return on its assets, which may attract investors' interest. In turn, the low value of the ratio indicates ineffective use of the company's assets and their inadequate structure, which can consequently lead to future deterioration of financial results. The formula for the ratio is presented below:

$$P/BV = \frac{stock\ price\ per\ share}{book\ value\ per\ share}$$

where:

$$book\ value\ per\ share\ = \frac{assets - liabilities}{number\ of\ shares}.$$

Rosenberg, Reid and Lanstein (1985) have found a positive link between book-to-market ratio of a company and average returns of its stocks. This was also observed for Japanese stocks (Chan, Hamao, Lakonishok, 1991). These results were utilized by Fama and French (1992), who assumed that book to market can be an important factor explaining the variability of stock returns.

A modification of the price-to-earnings ratio is the price-to-cash flow per share ratio (P/CF). In the definition of this indicator, in the denominator, instead of profit, one uses a net cash flow from operating activities. The reason to introduce this indicator is the fact that cash flow presents a wider range of information and is considered by many authors to be a much better measure of a company's profitability. In addition to the information included in the income statement, cash flows take into account the different times of settlement of individual transactions, as well as non-expenditure costs. There is also an opinion that profit may be distorted by various types of accounting "tricks", such as, for example, write-downs updating the value of individual assets.

Operating cash flows are connected with changes in such positions of the balance sheet as inventories, receivables, and short-term prepayments. Therefore, they are related to gross working capital. They also take into account current liabilities. The increase in inventories, liabilities and accruals increases the demand for net working capital. The decline in these positions has the opposite effect. This has a significant impact on maintaining financial liquidity.

Thus, they directly relate to financial liquidity, which cannot be seen when analyzing only profits. In terms of interpretation, the ratio represents the number of monetary units an investor has to pay for each monetary unit of operating flows. The formula of the ratio is presented below:

$$P/CF = \frac{stock\ price\ per\ share}{operating\ cash\ flow\ per\ share}$$

The last indicator used in this paper is the dividend yield ratio (DY), which is the ratio of the dividend per share to the market price of a share. Its value depends not only on the generated profits, but also on the company's dividend policy. In general, the higher the dividend yield, the more attractive the company is to an investor. This means that investors expect greater benefits through profit payouts. On the other hand, the low level of the ratio is typical for companies that introduce a limited dividend policy. In practice, this means that the profit is retained, increasing equity, and thus the company's development capacity. The reciprocal of this ratio gives additional interpretative possibilities, as it informs how much money investors are willing to pay for each monetary unit of dividend. The formula for the indicator is presented below:

$$DY = \frac{annual\ dividends\ per\ share}{stock\ price\ per\ share}$$

There is in the finance literature discussion regarding the relationship between dividend yield and stock returns. For instance Maio and Santa Clara (2015) found the positive relationship between these variables for aggregate stock market. In turn, Kim (2021) claim that this relationship depend on firm’s dividend reputation.

In this paper, we use the financial indicator in portfolio analysis. Thus, we need to calculate the indicators values for the whole portfolio based on the values of its components. The indicators used should be additive (that is, the indicator of a ‘sum’ of two assets should be equal to the sum of the indicators of individual assets). This condition is fulfilled when the indicators are expressed per price of a share. It is so in the case of dividend yield (DY). As for P/PE, P/CF and P/BV, we will use their reciprocals instead of original values. Thus, we will use the following indicators:

$$EP = \frac{1}{P/E} = \frac{earnings\ per\ share}{stock\ price\ per\ share}$$

[1]*This database was established in 1962 and comprised the data since 1950.*

$$CFP = \frac{1}{P/CF} = \frac{operating\ cash\ flow\ per\ share}{stock\ price\ per\ share}$$

$$BVP = \frac{1}{P/BV} = \frac{book\ value\ per\ share}{price\ of\ the\ share}$$

Downside Risk and Portfolio Choice

In the portfolio theory, from its very beginning until the present day, variance has been a commonly used risk measure (Markowitz, 1952). At the same time, for almost so long there were doubts concerning the validity of using this risk measure (Markowitz, 1959). The main disadvantage of variance is the fact that it treats in the same way negative and positive deviations from a mean rate of return. But returns lower than the mean return are undesirable, while returns higher than the mean offer the potential for increased gains. To exclusively quantify only negative deviations, Markowitz (1959) introduced the concept of semi-variance, which calculates the average of deviations falling beneath a specified threshold. Both semi-variance and lower partial moments focus solely on the distribution's left-side. Although the expected value often serves as the benchmark, similar to variance, an alternative reference point may also be employed. The application of semi-variance as a risk metric aligns with investors' instinctual understanding of risk, as noted by Boasson, Boasson, and Zhou (2011).

It is assumed that variance can serve as a good measure of risk if the returns are normally distributed (or at least their distribution is symmetric) or if we consider, an investor with a quadratic utility function. The classical Markowitz model is ineffective if we consider a portfolio of stock with with skewed returns (Foo, Eng; 2000). The classical mean-variance model, which treats deviations above and below target returns equally, to overestimates risk and eliminates the portfolios that are downside efficient.

Pla-Santamaria and Bravo (2013) analyzed portfolios composed of blue chip stocks from the Dow Jones Industrial Average, uncovering notable discrepancies between the outcomes yielded by the mean-semi-variance optimization model and those derived from the traditional Markowitz mean-variance framework.

The problem was also researched by Klebaner and Landsman (2017) who claimed that investors aiming to prevent their portfolio returns from dropping below a desired threshold are inclined to favor the portfolios that minimize measures of downside risk.

While it is considered that variance as a measure of risk is no worse than semi-variance if the distributions of returns are symmetrical (Estrada, Serra, 2005; Galagedera, Brooks, 2007), empirical studies on capital markets reveal that the return distributions for numerous companies are neither normal nor symmetrical (Adcock, Shutes, 2005; Estrada, Serra, 2005; Markowski, 2001; Post, van Vliet, 2006; Sun Q., Yan, 2003). This non-normality underscores the significance of adopting lower-order risk measures. Particularly in right-skewed return distributions, the bulk of the variance stems from the right-tail of the distribution, i.e. higher returns, while the influence of lower deviations is minimal. Consequently, investors show a preference for firms exhibiting right-skewed return distributions (Galagedera, Brooks, 2007; Peiro, 1999), highlighting the crucial role of skewness in risk analysis even when some companies exhibit symmetrical return distributions. The adoption of semi-variance over variance as a risk metric also finds support from prospect theory (Kahneman, Tversky, 1979), further emphasizing its relevance in financial risk assessment.

The arguments put forth advocate for the adoption of lower-risk measures when comparing them with traditional risk metrics. Measures like semi-variance enable a more universally applicable methodology for risk analysis and the construction of portfolios, independent of the empirical distribution of returns. Furthermore, there's no necessity to assume a particular analytical form for the utility function. The basic and intuitive premise that an investor seeks to maximize gains over losses, thereby favoring higher returns over lower ones, suffices.

A semi-variance, defined by Markowitz (1959), is a lower counterpart of a variance. This lower risk measure is a sum of squares of lower deviations from the target rate of return γ. It is calculated with the following formula:

$$dS^2(\gamma) = \frac{\sum_{t=1}^{m} d_t{}^2(\gamma)}{m-1},\ t = (1,2,\dots,m),$$

where $d_t(\gamma)$ are negative deviations from the target:

$$d_t(\gamma) = \begin{cases} 0 & for\ z_t \geq \gamma \\ z_t - \gamma & for\ z_t < \gamma \end{cases}$$

z_t denotes the rate of return of the company i in the period t and m is the number of time periods for which we have observations.

Bawa (1975) and Fishburn (1977) proposed lower partial moment as extensions of the semi-variance. Lower partial moment of order n is defined as:

$$LPM_i^n = \frac{1}{m-1}\sum_{t=1}^{m} lpm_{it}^n,$$

where

$$lpm_{it} = \begin{cases} 0 & for\ z_t \geq \gamma \\ z_t - \gamma & for\ z_t < \gamma \end{cases}.$$

Notice that for $n = 2$ lower partial moment is equal to semi-variance.

The semi-variance of an investment portfolio $dS_P^2(\gamma)$ is given by:

$$dS_P^2(\gamma) = \sum_{i=1}^{k}\sum_{j=1}^{k} x_i x_j d_{ij}(\gamma)$$

where x_i is the share of the stock i in the portfolio and $d_{ij}(\gamma)$ is the semi-covariance of the rate of return for the i-th and j-th share, which is defined by

$$d_{ij}(\gamma) = \frac{1}{m-1}\sum_{t=1}^{m} d_{ijt}(\gamma),$$

where:

$$d_{ijt}(\gamma) = \begin{cases} 0 & for\ z_{pt} \geq \gamma \\ (z_{it} - \gamma)(z_{jt} - \gamma) & for\ z_{pt} < \gamma \end{cases},$$

$$z_{pt} = \sum_{i=1}^{k} x_i z_{it}, \qquad t = (1,2,\dots,m).$$

It should be noted that the semi-variance of a portfolio depends on the assumed rate γ and it should be recalculated with each change of the parameter γ. If the reference level γ is equal to the mean rate of return, this parameter depends on the structure of the portfolio and should be calculated again each time when this structure changes.

Problems for Portfolio Choice

We consider a portfolio of k different assets. Let μ_i be a mean return of an asset i, estimated from the last m observations:

$$\mu_i = \frac{\sum_{t=1}^{m} z_{it}}{m} .$$

By σ_{ij} we denote a covariance between returns of asset i and that of asset j:

$$\sigma_{ij} = \frac{1}{m-1} \sum_{t=1}^{m} (z_{it} - \mu_i)(z_{jt} - \mu_j).$$

By x_i we denote the proportion of wealth invested in the asset i. The mean return of the portfolio is then given by

$$\mu_P = \sum_{i=1}^{k} x_i \mu_i$$

and the variance of the portfolio rate of return is given by

$$S_P^2 = \sum_{i=1}^{k} \sum_{j=1}^{k} x_i x_j \sigma_{ij}.$$

Semi-variance of the portfolio is given by

$$dS_P^2(\gamma) = \sum_{i=1}^{k} \sum_{j=1}^{k} x_i x_j d_{ij}(\gamma),$$

where semi-covariances of assets' returns are given by (1).

In portfolio selection we also can consider some market multiple. It can one of the four multiples described in Section 2 (EP, CFP, BVP or DY). By β_i we denote the value of this criterion for the asset i. The value of this multiple for the entire portfolio is given by

$$\beta_P = \sum_{i=1}^{k} x_i \beta_i.$$

We consider the following optimization problems:

(a) Variance-minimizing portfolio, i.e. a portfolio that is the solution of

$$\min_{x_1,\dots,x_k} S_P^2 = \sum_{i=1}^{k} \sum_{j=1}^{k} x_i x_j \sigma_{ij} \qquad (2)$$

with the restriction that

$$\sum_{i=1}^{k} x_i = 1, \qquad (3)$$

$$x_1, x_2, \dots x_k \geq 0. \qquad (4)$$

(b) A portfolio that minimizes variance with a restriction on mean return. We require that the mean return of a portfolio should not be lower than some rate μ_0. The optimization problem is given by (2)-(4) with the additional restriction

$$\mu_P = \sum_{i=1}^{k} x_i \mu_i \geq \mu_0 \qquad (5)$$

(c) A portfolio that minimizes variance with a restriction on mean return and on its fundamental value. We assume that the fundamental value of a portfolio (measured with one of the market multiples) should be no lower than its required value β_0. This gives a problem (2)-(5) with an additional restriction.

$$\beta_P = \sum_{i=1}^{k} x_i \beta_i \geq \beta_0 \qquad (6)$$

Mathematically, problems (a)-(c) are problems of quadratic optimization with linear restrictions. They can be solved using standard software. To find the solutions, we used the *quadprog* package in R. The optimization algorithm uses the dual method of Goldfarb and Idnani (1983).

The second set of problems are the ones in which we use semi-variance as a risk measure. The optimization problems were defined as follows:

(d) Minimizing semi-variance, i.e. the optimization problem

$$\min_{x_1,\dots,x_k} dS_P^2(\gamma) = \sum_{i=1}^{k} \sum_{j=1}^{k} x_i x_j d_{ij}(\gamma), \qquad (7)$$

with restrictions (3)-(4).

(e) Minimizing semi-variance with a restriction on mean return. The restrictions in this optimization problem is given by equations (6) and (3)-(5) and the goal function is given by (7).

(f) Minimizing semi-variance with restrictions on mean return and on fundamental value. The optimization problem is defined by equations (7) and (3)-(6).

Problems (d)-(f) are not standard optimization problems because semi-covariances $d_{ij}(\gamma)$ in (7) change with the changes in the portfolio structure $x_1, \dots, x_k$ (which changes a mean return γ). Semi-variance as a risk measure creates considerable difficulties because, while calculating semi-covariances $d_{ij}(\gamma)$ it is required know in which periods the return of the portfolio was lower than the target value γ, which depends on the portfolio structure. This makes calculations of effective portfolios with semi-variance as a risk measure more complicated than is in the case when variance is used. With every proposed portfolio the target rate of return γ changes. Therefore, the there is a need to recalculate semi-covariances $d_{ij}(\gamma)$.

We have used the following numerical algorithm. We started with some initial portfolio (in this case, it was a portfolio minimizing variance). Then we solved each of the problems (d)-(e) as a quadratic programming problem, using the Goldfarb and Idnani (1983) algorithm. After each iteration, we the target return γ was recalculated. Then we recalculated covariances $d_{ij}(\gamma)$ and solved the problem with the new input data. We had repeated this process until we obtained a convergence, i.e. until the changes in portfolio structure between subsequent iterations were sufficiently small. To perform the calculations we have prepared own software written in R.

Data and Empirical Results

The studies covered 21 companies traded on the Frankfurt Stock Exchange. We took companies from the DAX30 index. After excluding financial companies and companies for which quotation series were not long enough to estimate the parameters for portfolio analysis (especially correlation coefficients), we were left with a set of 21 companies[2]. To our analysis of portfolios performance, we took close prices of shares for the period from 12 July 2019

[2]*The companies were: Adidas, Beiersdorf, Bayerische Motoren Werke, Brenntag, Daimler, Deutsche Post, Deutsche Telekom, E.ON, Fresenius Medical Care, Fresenius, Henkel, Infineon Technologies, Merck, MTU Aero Engines, Porsche Automobil Holding, Puma, SAP, Sartorius, Siemens, Symrise and Volkswagen.*

to 19 November 2021. In the estimation, we considered portfolios assessed by their monthly rate of return. Parameters (mean returns, variances and semi-variances) of the stocks were estimated using the last 500 trading days before the starting moment of investment. Therefore, for parameter estimation, we took data for the period from 25 July 2017 to 19 November 2021. Market multiples and close prices have been taken from Thomson Reuters database, Refinitiv Eikon. The length of all investments was 1 month, i.e. we assumed that all portfolios created on a given day were sold after a month (four weeks). The Frankfurt Stock Exchange was selected due to its sizable market capitalization, significance in European stock trading, and the reliable accessibility of financial data. Companies listed on the FSE are required to release financial statements every quarter, setting it apart from other exchanges such as the London Stock Exchange (the biggest European financial market) where financial data is only published annually. This indicates that data on issuers' financial condition is updated annually, while on the FSE it is updated quarterly.

In the case of economics, it is not possible to carry out repetitive experiments, as in experimental sciences, as physics or chemistry. The paper uses the Covid-19 epidemic as a natural experiment. The pandemic period witnessed a significant downturn in stock markets, providing an opportunity to evaluate the effectiveness of various risk diversification strategies during pronounced price drops in financial markets.

In the paper, we consider 13 types of portfolios. Table 1 contains descriptions of these types and symbols used to refer to them.

Table 1

Types of Portfolios Considered

Portfolio	Description
Equally weigted	A portfolio with an equal share of each asset
MinV	Variance-minimizing portfolio
MinV-E	Variance-minimizing portfolio with a restriction on mean return
MinV-E-BVP	Variance-minimizing portfolio with restrictions on mean return and BVP
MinV-E-CFP	Variance-minimizing portfolio with restrictions on mean return and CFP
MinV-E-EP	Variance-minimizing portfolio with restrictions on mean return and EP
MinV-E-DY	Variance-minimizing portfolio with restrictions on mean return and DY
MinSV	Semi-variance-minimizing portfolio with restrictions
MinSV-E	Semi-variance-minimizing portfolio with a restriction on mean return
MinSV-E-BVP	Semi-variance-minimizing portfolio with restrictions on mean return and BVP
MinSV-E-CFP	Semi-variance-minimizing portfolio with restrictions on mean return and CFP
MinSV-E-EP	Semi-variance-minimizing portfolio with restrictions on mean return and EP
MinSV-E-DY	Semi-variance-minimizing portfolio with restrictions on mean return and DY

In the portfolio with restrictions on mean return and/or on one of the fundamental values (BVP, CFP, EP, DY), the following restrictions were assumed. Regarding the fundamental values, we required that the indicator value for the entire portfolio be no lower than the average value for all companies. This ensures that the financial results of the portfolio will be no lower than the average in the market. As for the expected return, we required that the mean return of the portfolio should be no lower than the average of the top 50 % companies. Thus, we were looking for portfolios that provide sufficiently high returns. The whole set of 13 different kinds of portfolios (see Table 1) was constructed every treading day between 12 July 2019 and 19 November 2021. We created total 6318 portfolios for 486 trading days.

Throughout the study timeframe, four distinct research intervals of varying durations were specified based on changes in capital market conditions, as evidenced by fluctuations in the DAX index value (the main index on the Frankfurt Stock Exchange that represents the situation on the whole market). Figure 1 illustrates the trajectory of this index over the research period, with market conditions at the time of purchasing of a portfolio acquisition being a crucial factor in defining these subperiods.

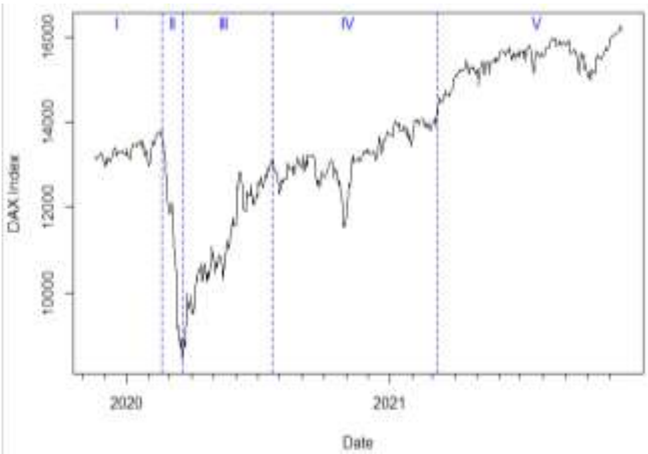

Figure 1. DAX Index and Subperiods

I. a subperiod of low and stable growth, from 18 November 2019 (official confirmation of the Covid-19 case in China) to 19 February 2020;
II. a subperiod of market collapse – from 20 February 2020 to 18 March 2020;
III. a subperiod of very strong growth – from 19 March 2020 to 21 July 2020;
IV. a subperiod of market stabilization – from 22 July 2020 to 5 March 2021 (this subperiod had lasted until the index returned to its level before the market collapse);

V. a subperiod of increased growth lasting from 8 March 2021 to 19 November 2021 (this subperiod was the longest in our analysis).

The analysis carried out allowed for the identification of significant phenomena related to the distribution of the rates of return and risk, depending on the method of building a portfolio and on the period in which securities were purchased. First, attention should be paid to the tendency of changes in the analyzed index. Due to the significant differences in this respect, the analyzed period was divided into five subperiods (Table 2):

Table 2

Research Subperiods

Number of subperiod	The time of buying a portfolio	The situation on the capital market
I	18 November 2019 – 19 February 2020	before market collapse
II	20 February 2020 – 18 March 2020	market collapse
III	19 March 2020 – 21 July 2020	rapid growth
IV	22 July 2020 – 5 March 2021	market stabilization
V	8 March 2021 – 19 November 2021	further growth

The results of the calculations for each subperiod, as well as for the entire research period, are presented in Tables 3–7. Given the extensive number of portfolios analyzed (6318), this paper does not detail the composition of these portfolios or other ex-ante analysis aspects, such as expected portfolio risk, expected rate of return, or average market ratios. The return distribution characteristics detailed in Tables 3–7 pertain exclusively to actualized returns, and reflect the actual outcomes and associated risks. For each of the defined subperiods as well as the entire research period, key metrics were computed, including the mean and median rates of return, standard deviation, minimal rate of return, Value-at-Risk (VaR, here 10 % quantile of returns), semi-deviation, and skewness (asymmetry).

According to the presented results, worth noting that the rates of return on investment portfolios quite strongly reflected the trends of changes in the analyzed index. This applies to all types of investments, regardless of the method of selecting the portfolio. The only exceptions were the portfolios from IV subperiod, created on the basis of the risk minimization method, as well as portfolios created using the price-to-earnings indicator, but only when variance as a criterion was used as a risk criterion.

In the subperiods of growth (that is, subperiods III and V), the highest rates of return were generated by the portfolios built on the basis of the analysis of the market indicators described in the theoretical part. This can be explained by the fact that the use of price-index analysis gives investors a much wider range of information about financial standing of the companies. Due to this, they can optimize the rate of return by focusing on investments with the highest potential of growth. In this context, we should recall that one of the criteria for selecting companies was based on relatively low values of individual market indicators. Therefore, the focus was on the so-called 'undervalued' companies, which created an additional potential for upward changes in their valuation and, consequently, a higher portfolio return rate.

The risk-minimizing portfolios and the portfolios based on the standard Markowitz model had lower, but also positive, rates of return. This can be explained by the limited scope of information used in the selection of companies. Information on the financial results of companies, reflected in the analysis of market indicators, was not used in the portfolio selection. Undoubtedly, this makes the selection of portfolio companies less efficient, which in turn reduces the profitability of the investment. It is also worth noting that the risk-minimization portfolios were created using shares of companies with the lowest volatility of share prices. In periods of growth (III and V), this means that one invests relatively less in the companies with the most dynamically growing share prices, which in turn reduces the possibility of maximizing the rate of return.

We should also add that in the periods of growth, as well as in the periods of decline, portfolios built on the basis of information on market indicators better indicated the volatility of the market index.

The results presented in Tables 5 and 7 show that in the periods of growth (III, V) , building a portfolio based on an analysis of market indicators is a more effective solution compared to other methods. It is also worth noting that, in most cases, there were no significant differences in the rates of return of portfolios built with the use of different indicators. This can mean that all indicators provide similar information on the fundamental values of companies. It also proves their usefulness and importance as a tool for supporting investors' decisions.

Table 3

Summary Statistics of the Realized Rates of Return for the Portfolios Bought in the Time between 18.11.2019-19.2.2020 – Subperiod I, before the Market Collapse

Portfolio	Mean	Median	Std. Dev.	Min	VaR 0.1	Semi-dev.	Asymmetry
Equally weighted	-0.62%	1.51%	8.08%	-39.06%	-6.50%	7.29%	-2.86
MinV	-1.03%	0.33%	6.46%	-30.56%	-5.77%	5.71%	-2.68
MinV-E	-0.92%	0.64%	8.46%	-44.05%	-5.50%	7.52%	-2.87
MinV-E-BVP	-0.70%	1.11%	8.75%	-46.53%	-6.09%	7.84%	-2.96
MinV-E-CFP	-0,82%	0,44%	6,45%	-30,20%	-5,85%	5,73%	-2,72
MinV-E-EP	-0.99%	0.85%	8.84%	-47.08%	-6.45%	7.92%	-2.93
MinV-E-DY	-1.95%	1.99%	12.06%	-44.31%	-18.76%	10.53%	-1.90
MinSV	-1.23%	0.44%	7.35%	-33.24%	-7.49%	6.32%	-2.27
MinSV-E	-1.06%	0.62%	7.98%	-40.18%	-6.76%	6.94%	-2.55
MinSV-E-BVP	-0.74%	0.78%	8.13%	-45.37%	-5.48%	7.26%	-3.08
MinSV-E-CFP	-1.08%	0.90%	7.50%	-37.53%	-7.80%	6.69%	-2.86
MinSV-E-EP	-1.47%	0.55%	8.93%	-49.24%	-6.46%	7.97%	-2.99
MinSV-E-DY	-2.24%	1.45%	11.97%	-45.28%	-17.54%	10.34%	-1.86

Table 4

Summary Statistics of the Realized Rates of Return for the Portfolios Bought in the Time between 20.2.2020-18.3.2020 – Subperiod II, Market Collapse

Portfolio	Mean	Median	Std. Dev.	Min	VaR 0.1	Semi-dev.	Asymmetry
Equally weighted	-10.39%	-17.33%	18.24%	-36.10%	-31.65%	10.98%	0.58
MinV	-9.40%	-14.47%	14.69%	-29.84%	-28.14%	9.08%	0.44
MinV-E	-12.86%	-20.99%	20.62%	-39.20%	-35.86%	12.37%	0.56
MinV-E-BVP	-11.86%	-21.21%	22.96%	-41.91%	-38.40%	13.92%	0.53
MinV-E-CFP	-8,54%	-13,89%	14,54%	-29,82%	-28,10%	9,23%	0,38
MinV-E-EP	-13.44%	-22.64%	23.53%	-42.21%	-38.87%	13.79%	0.66
MinV-E-DY	-13.02%	-21.03%	21.46%	-39.20%	-35.91%	12.52%	0.68
MinSV	-9.19%	-13.25%	14.18%	-30.78%	-26.89%	8.83%	0.40
MinSV-E	-8.84%	-14.14%	18.10%	-36.36%	-33.02%	11.60%	0.28
MinSV-E-BVP	-10.27%	-17.89%	21.74%	-41.24%	-37.77%	13.64%	0.39
MinSV-E-CFP	-10.68%	-16.76%	17.04%	-34.91%	-30.76%	10.39%	0.56
MinSV-E-EP	-12.17%	-20.89%	23.36%	-44.32%	-40.78%	14.57%	0.41
MinSV-E-DY	-11.71%	-19.95%	21.98%	-40.52%	-37.42%	13.40%	0.50

Table 5

Summary Statistics of the Realized Rates of Return for the Portfolios Bought in the Time between 19.3.2020-21.7.2020– Subperiod III, Rapid Growth

Portfolio	Mean	Median	Std. Dev.	Min	VaR 0.1	Semi-dev.	Asymmetry
Equally weighted	6.61%	5.85%	5.31%	-2.93%	0.15%	3.46%	0.51
MinV	3.03%	3.25%	3.18%	-3.86%	-1.07%	2.21%	0.16
MinV-E	4.37%	3.95%	3.35%	-2.13%	0.53%	2.03%	1.12
MinV-E-BVP	7.55%	6.68%	6.19%	-3.28%	0.36%	4.06%	0.41
MinV-E-CFP	3,44%	2,75%	4,50%	-5,82%	-1,96%	2,14%	0,11
MinV-E-EP	6.67%	6.43%	5.29%	-3.18%	0.58%	3.39%	0.65
MinV-E-DY	7.46%	5.94%	5.77%	-1.23%	0.85%	3.66%	0.54
MinSV	2.97%	3.14%	2.99%	-3.31%	-1.05%	2.09%	0.11
MinSV-E	4.08%	3.65%	3.41%	-2.59%	0.01%	2.08%	1.07
MinSV-E-BVP	7.82%	6.35%	7.15%	-5.73%	-0.72%	4.78%	0.25
MinSV-E-CFP	6.60%	6.35%	4.05%	-3.74%	-1.09%	2.21%	0.14
MinSV-E-EP	6.99%	6.22%	6.11%	-5.12%	-0.55%	4.11%	0.27
MinSV-E-DY	7.48%	6.16%	6.19%	-2.78%	0.50%	3.95%	0.49

Table 6

Summary Statistics of the Realized Rates of Return for the Portfolios Bought in the Time between 22.7.2020-5.3.2021–Subperiod IV, Market Stabilization

Portfolio	Mean	Median	Std. Dev.	Min	VaR 0.1	Semi-dev.	Asymmetry
Equally weighted	2.37%	2.24%	2.78%	-6.27%	-0.72%	1.96%	-0.04
MinV	-1.38%	-1.58%	3.58%	-8.66%	-6.19%	2.47%	0.14
MinV-E	0.95%	0.49%	6.46%	-13.03%	-6.40%	4.24%	0.51
MinV-E-BVP	1.99%	0.57%	7.40%	-12.29%	-5.70%	4.54%	0.85
MinV-E-CFP	-1,68%	0,19%	3,56%	-7,74%	-5,56%	2,44%	-0,18
MinV-E-EP	1.56%	-0.28%	7.31%	-13.96%	-6.43%	4.74%	0.51
MinV-E-DY	1.73%	0.32%	8.34%	-15.02%	-7.05%	5.16%	0.80
MinSV	0.26%	-0.32%	6.24%	-12.45%	-6.50%	3.93%	0.74
MinSV-E	1.38%	0.31%	7.26%	-13.63%	-6.63%	4.58%	0.71
MinSV-E-BVP	2.46%	1.32%	8.27%	-13.14%	-5.96%	5.05%	0.83
MinSV-E-CFP	1.47%	1.18%	7.11%	-12.00%	-5.85%	4.34%	0.87
MinSV-E-EP	1.99%	1.03%	7.93%	-14.74%	-6.56%	5.16%	0.42
MinSV-E-DY	2.19%	0.08%	9.01%	-15.54%	-7.24%	5.51%	0.83

One can also notice that in the period of decline (II), the risk-minimizing portfolios generated the lowest negative rates of return, while the portfolios built with the use of market indicators generated the highest (but also negative) returns. It should be emphasized that it was a period of very dynamic decline, and the market index decreased by more than 38%. In this subperiod, the uncertainty was very high, due to the unpredictability of the development of the pandemic situation and its economic consequences. From an investor's perspective, this unpredictability increased uncertainty about companies' future earnings, cash flows, goodwill, or their ability to pay dividends. Usually, such a state of affairs causes an increase in the expected risk premium, which reduces the prices of the shares. In such a situation, it is quite common for investors to close their positions in the stock markets and invest their capital in assets with a much lower level of risk. Thus, the supply of shares grows, resulting in a decrease in market prices. The analysis of empirical data reveals that this scenario took place. It is worth mentioning that the observed decline in share prices was not reflected in the deterioration of the financial results of the companies in the portfolio.

It should also be noted that all portfolios created in subperiod II, regardless of the strategy of choosing a portfolio, generated negative rates of return. Thus, the selection of a specific strategy should be seen more as an attempt to minimize losses. In this situation, building a portfolio based on the principle of risk minimization turned out to be a more effective strategy. This is due to the aforementioned fact that choosing a risk minimization strategy, both in periods of decline and growth, leads to selecting companies with the lowest volatility of share prices. These are the companies with stable financial performance, thus with relatively high resistance to recessions or crises. Therefore, the declines in share prices for this type of companies are lower, compared to other groups of companies. This, in turn, explains the reason why investors treat these companies as entities with a lower level of risk, especially in unfavorable economic and stock market conditions. Investments in this type of stock increase the likelihood of capital protection or at least minimize losses, which is confirmed by the results of the analysis.

Table 7

Summary Statistics of the Realized rates of Return for the Portfolios Bought in the Time between 8.3.2021-19.11.2021–Subperiod V, Further Growth

Portfolio	Mean	Median	Std. Dev.	Min	VaR 0.1	Semi-dev.	Asymmetry
Equally weighted	1.50%	2.32%	3.49%	-7.50%	-3.24%	2.66%	-0.48
MinV	2.05%	3.23%	4.48%	-11.37%	-5.82%	3.73%	-1.32
MinV-E	3.51%	4.54%	7.65%	-12.83%	-8.36%	5.88%	-0.49
MinV-E-BVP	3.38%	4.51%	7.33%	-11.89%	-8.75%	5.66%	-0.53
MinV-E-CFP	2,78%	3,71%	6,19%	-11,73%	-6,36%	4,81%	-0,60
MinV-E-EP	3.53%	4.60%	7.76%	-12.47%	-9.18%	5.96%	-0.50
MinV-E-DY	3.43%	4.73%	8.25%	-13.05%	-10.08%	6.36%	-0.52
MinSV	2.48%	3.82%	5.69%	-12.03%	-5.75%	4.50%	-0.74
MinSV-E	3.59%	4.19%	8.10%	-13.56%	-9.04%	6.14%	-0.41
MinSV-E-BVP	3.43%	4.48%	7.86%	-12.91%	-9.22%	6.00%	-0.47
MinSV-E-CFP	3.08%	4.13%	6.43%	-11.85%	-6.29%	4.96%	-0.56
MinSV-E-EP	3.52%	4.50%	8.61%	-14.09%	-10.40%	6.56%	-0.46
MinSV-E-DY	3.84%	4.56%	8.98%	-13.90%	-10.59%	6.77%	-0.39

Table 8

Summary Statistics of the Realized Returns for the Portfolio Bought in the Time between 18.11.2019-19.11.2021 – the Entire Research Period

Portfolio	Mean	Median	Std. Dev.	Min	VaR 0.1	Semi-dev.	Asymmetry
Equally weighted	1.52%	2.33%	6.91%	-39.06%	-3.63%	5.79%	-2.28
MinV	0.05%	0.89%	5.91%	-30.56%	-6.28%	4.93%	-1.95
MinV-E	1.21%	2.26%	8.55%	-44.05%	-7.86%	6.91%	-1.59
MinV-E-BVP	2.01%	2.47%	9.30%	-46.53%	-7.83%	7.32%	-1.33
MinV-E-CFP	0,38%	0,60%	6,52%	-30,20%	-6,03%	5,09%	-1,22
MinV-E-EP	1.68%	2.50%	9.41%	-47.08%	-8.61%	7.51%	-1.43
MinV-E-DY	2.14%	3.04%	10.26%	-44.31%	-10.06%	8.07%	-1.09
MinSV	0.57%	1.45%	6.91%	-33.24%	-6.65%	5.49%	-1.31
MinSV-E	1.40%	1.91%	8.34%	-40.18%	-8.12%	6.45%	-1.07
MinSV-E-BVP	2.24%	2.04%	9.43%	-45.37%	-7.18%	7.16%	-0.97
MinSV-E-CFP	0.35%	1.27%	7.07%	-31.96%	-7.13%	5.65%	-1.26
MinSV-E-EP	1.76%	2.07%	9.82%	-49.24%	-8.97%	7.71%	-1.30
MinSV-E-DY	2.44%	2.85%	10.65%	-45.28%	-10.13%	8.21%	-0.93

In the subperiod of market collapse (II), all portfolios generated a negative rate of return. Portfolios built with the use of market indicators achieved lower rates of return compared with the risk-minimizing portfolios (Min V, Min SV) and the portfolios from the risk-return effective frontier (Min V-E, Min SV-E). This situation shows a relatively high supply of shares of companies with low values of market indicators. This means that in the periods of dynamic decline, companies that can be seen as some kind of investment opportunity (low values of fundamental ratios) were perceived as entities with a higher level of risk, and therefore they were not the main object of investors' interest. They instead sought instruments that provide capital protection. Therefore, apart from over-the-counter investments (treasury bonds, real estate, precious metals, etc.), stocks of larger companies, with a relatively low level of price volatility, (also in the periods of decline) were preferred.

Quite an interesting phenomenon can be seen in subperiods I and IV, which, incidentally, are very similar in terms of the dynamics of index changes (respectively: increase by 4.4 % and by 6.2 %). At that time, the highest returns were generated by portfolios created with the use of equal shares of all companies (simple diversification). When analyzing the reasons for this phenomenon, attention should be paid to the fact that in both periods the market grew, but only slightly. As was previously said, under such conditions investors are less interested in relatively safe companies with a relatively low level of volatility of stock prices. Therefore, there are quite significant differences between the return on portfolios built using the equal share method and the risk minimization method. Interestingly, the rates of return on portfolios built on the basis of market indicators were also lower. This phenomenon may indicate that in times of slight growth, the search for companies that can be seen as an investment "opportunity" was not the main investment strategy. This may be due to the fact that a relatively low market growth rate limits the potential for an increase in share prices of undervalued companies and thus does not provide sufficient opportunities to maximize the rate of return. This is mainly justified in case of short-term investments when the buyers of securities expect relatively high rates of return in a relatively short time horizon.

It is also worth noting that in the vast majority of the analyzed subperiods, portfolios based on the Markowitz model usually did not generate returns that would maximize investors' profits. The only exceptions were the portfolios in subperiod V, built on with the use of the price-earnings ratio (and variance) or dividend yields (and variance or semi-variance). The reasons for this can be seen in the phenomena described earlier. Namely, during periods of growth, the Markowitz model, due to the limited information scope of the analysis (not including the information contained in market indicators), does not allow to identify "undervalued" companies. This limits the rate of return on the portfolio. However, in the period of decline (II), it does not prefer entities with the lowest volatility of share prices, which, unlike the risk minimization method, limits the possibilities of protecting capital or minimizing losses.

The conducted analyses allow also to compare the effectiveness of portfolios built with the use of the analysis of variance and semi-variance. In most cases, the "semi-variance" portfolios generated lower rates of return compared to the "variance" portfolios. There are, however, two exceptions. The first one was for risk-minimizing portfolios. In this case, in almost all subperiods (except for subperiod III – rapid growth), as well as in the whole research period, the portfolios created with the use of the semi-variance generated higher rates of return than the "variance" portfolios. The second exception was investments in subperiod II (market collapse) when all types of portfolios built with the use of semi-variance had lower negative rates of return in comparison with the "variance" portfolios. The above observations may prove the advantage of semi-variance as a tool for creating a portfolio, both in specific market conditions (crisis and significant drops in indices), and in relation to strictly defined investment preferences (a conservative approach consisting in building portfolios with a limited risk of price volatility).

One can also draw interesting conclusions from the analysis of the standard deviation of rates of return, which is a source of information about the volatility of rates of return, and thus – about the level of risk. The highest standard deviations were recorded for portfolios created with the use of market indicators. This was the case in all the subperiods analyzed. Thus, such portfolios should be treated as having a relatively high level of risk, which confirms the observations made earlier. On the other hand, the standard deviation was the lowest for the portfolios with shares of companies with a relatively low level of price volatility. However, the exceptions were subperiods IV and V, in which the portfolios built on the basis of a simple diversification had the lowest standard deviation. The results of the analysis confirm to a large extent the positive relationship between the rate of return and the level of investment risk.

Taking into account the general approach, i.e. the entire period covered by the study, the results of the analysis largely correspond to the phenomena described above. The highest rates of return were observed for portfolios created with the use of market indicators. A similar tendency has been observed for Warsaw Stock Exchange (Anna Rutkowska-Ziarko & Kliber 2023). This is undoubtedly due to the fact that the vast majority of the research period consisted of subperiods of growth. Therefore, these were favorable conditions for seeking investment opportunities (low values of market indicators) and discounting the fact that these companies were underpriced in the form of a higher rate of return. It is worth noting that the investment decisions in these subperiods to a greater extent corresponded to the fundamental factors: financial results of companies, the general economic situation, the situation in the industry.

In turn, the lowest rates of return were achieved by the portfolios built on the basis of the risk minimization principle. Here, attention should be paid to the trends of the market index throughout the entire research period. There was only one subperiod with a clear decline in the index, and it was the shortest of all subperiods. In such market conditions, investing funds in shares with relatively low price volatility (also in periods of increases on the market) significantly reduces the possibility of maximizing the rate of return. However, the portfolios with equal shares of all assets and the portfolios based on the Markowitz model generated rates of return between other types of portfolios, as was the case in most of the subperiods analyzed.

The conclusions regarding the standard deviation of the rates of return are also confirmed. Its highest level was recorded for portfolios built with the use of market indicators. On the other hand, it was the lowest for portfolios consisting of companies with relatively low price volatility. This phenomenon was also observed in the analysis of the semi-deviation.

Conclusions

In the paper, fundamental portfolios both for variance and semi-variance approach were built. To construct effective portfolios in the semi-variance framework we used software written in R specially prepared for this task.

When developing portfolios with an additional fundamental criterion, in our approach we looked for efficient outcomes across three dimensions: profitability (evaluated by expected return), risk (quantified through the variance or semi-variance of returns), and the market ratios of the constituent companies (assessed via one of four market multiples). We have evaluated the efficiency of various portfolio selection models throughout the Covid-19 epidemic. A timeline was segmented into five distinct phases to accommodate the evolving dynamics on the Frankfurt Stock Exchange.

The portfolios were built in subperiods of different stock market situations, and the selection methods of the portfolio components were also different. To analyze the performance of portfolios we used realized rates of return, based on which we calculated overall assessments of different types of portfolios.

In the entire analyzed period, the highest rates of return were generated by portfolios built on the basis of market indicators. Similar trends were observed in the subperiods with the highest DAX index growth dynamics. On the other hand, in the periods of mild growth, the highest rates of return were characteristic of portfolios built on the basis of the principle of equal shares. In the period of collapse, the highest (negative but the lowest, looking at absolute value) rates of return generated risk-minimizing portfolios.

The individual portfolios also differed in terms of risk. Its highest level was observed for portfolios built on the basis of market indicators. This phenomenon was characteristic of all analyzed subperiods. On the other hand, the lowest level of risk was observed for portfolios unconditionally minimizing risk (measured with variance or semi-variance).

Empirical findings concerning leading firms listed on the Frankfurt Stock Exchange demonstrate that :

- investors can enhance their financial results by incorporating additional criterion related to market ratios, like book-to-market, earnings-to-market, or others. This improvement is particularly evident during phases of pronounced upward market trends. Conversely, in times of moderate growth or downturns, the traditional Markowitz framework and other models examined showed superior efficacy;
- using semi-variance instead of variance tends to yield more favorable outcomes for investors, and this trend is predominantly observed during market downturn periods;
- fundamental portfolios minimizing the semi-variance seem to be a useful tool of choosing investment strategy during periods of significant market uncertainty. This is undoubtedly the situation for investors during the Covid 19 pandemic.

The results of the research indicate a further need to explore the problem and look for new possibilities for the development of classic models of portfolio construction. It is particularly important to include information on the company's current financial position in the portfolio analysis. This involves overcoming some difficulties, such as the availability of reliable and up-to-date financial data. Companies listed on the FSE are required to publish quarterly reports. However, there are exchanges, such as the London Stock Exchange, where this requirement only applies to annual reports.

Including different types of data (in this case share prices and financial ratios) in one model poses a methodological difficulty. We believe that the model proposed in this paper is the next step in the development of multi-criteria stock portfolio selection models, particularly those that combine capital market information with information on the financial condition of the public company.

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