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Estimation of Non-Linear Dependence between Exchange Rate and Stock Market through Different Time Scales: Latin Case

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The aim of this paper was to investigate a relationship between stock returns and exchange rate in Brazilian stock market over different time scales. Therefore we used daily quotations of Ibovespa and ratio between the U.S. Dollar and Brazilian Real, from April, 2003 to August, 2011. The initial analysis evidenced that Brazilian stock market is more volatile than Real/Dollar exchange rate. With the time scales coefficients we estimated families of copulas for the relationship between daily log-returns of Dollar/Real exchange rate and Ibovespa. The results of estimated copulas evidenced that dependence between the variables is negative in all time scales, thus confirming the balanced portfolio theory. The coarse scales exhibit great negative dependence, which is reduced in the middle scales and reaches the lower level at the finest scales. This confirms that investors, holding portfolios composed by Brazilian stocks for long time, are more sensitive to Dollar/Real exchange rate. Regarding the shape of joint probability of the variables, there was a predominance of Student's t copula. This result emphasized that there is more dependence in the tails between exchange rate and Brazilian stock markets than the normally expected. It highlights the necessity of risk management in portfolios composed by stocks of Brazilian market in situations of great variation in Dollar/Real exchange rate. No less important is the caution of the exchange rate policy of the Brazilian government to huge variations and turbulence periods in the stock market.

Keywords: *exchange rates; wavelets; copulas; Brazilian market.*

Introduction

Stock market plays a vital role in modern economy since it acts as a mediator between lenders and borrowers. That is, a well-functioning stock market may assist development process in an economy through two important channels: boosting savings and allowing for a more efficient allocation of resources. It can be very helpful to diversify domestic funds and channels into productive investment; however, to perform this important task it is very necessary that stock market has significant relationship with the macroeconomic variables (Mohammad *et al.*, 2009). Investors have a great interest in discovering variables that may help to forecast stock prices. They can manage their positions and portfolios (increase returns and/or lower risk) more appropriately if they can use macroeconomic news as reliable indicators where stock market is headed (Wang, 2011). Corroborating, policymakers pay attention to the situation of the stock market that can be regarded as a leading indicator of future macroeconomic activity. Thus, this situation can control the direction and stability of economy by adjusting macroeconomic variables if the relationship between stock returns and economic activity has predictive power. Well performing exchange market is very helpful for economic activity through growth and saving, efficient allocation of investment and attracting foreign direct investment. With constant growth of the financial and economic globalization, exchange rates assume crucial role in a whole macroeconomic analysis of stock returns.

There is no theoretical consensus in both existence of relationship between stock prices and exchange rates and a direction of the relationship. However, conforming to (Ozbay, 2009), two approaches have been asserted to establish a relationship between exchange rate and stock prices: goods market model, developed by Dornbusch and (Fisher, 1980), and portfolio balance model. The first one states that there is a positive relationship between exchange rates and stock returns, due to the assumption of using direct exchange rate quotation (Stavarek, 2004). The second one puts much more stress on the role of capital account transactions (Tahir & Ghani, 2004). Portfolio balance model assumes a negative relationship between stock prices and exchange rates.

Regarding empirical studies, there is also a lack of consensus. Several researches on this subject have distinct results. (Stavarek, 2004) studies this relationship in developed economies and recent members of the European Union. Obben *et al.*, (2006) verified this question for New Zealand stock market. (Abugri, 2008) found divergent results among Brazilian, Mexican, Argentinean and Chilean markets. Maysami *et al.*, (2004) studied Singaporean market. (Horobet & Ilie, 2007) tested it in Romanian market. (Ozbay, 2009) emphasizes that the relationship between stock returns and exchange rate is not stable over time and that there are differences among countries regardless of either developed or emerging markets.

Thus, it is imminent the necessity of modeling the dependence between exchange rates and stock returns.

Further, the shape and magnitude of this relationship should be known in order investors could protect their positions. Thus, the main objective of this paper is to propose a new approach, wavelet analysis, to investigate the relationship between stock returns and exchange rate in Brazilian market over different time scales. For that we use daily quotations of Ibovespa and ratio between the U.S. Dollar and Brazilian Real, from April, 2003 to August, 2011, totalizing 2049 observations. This approach is based on a wavelet multiscaling method that decomposes a given time series on a scale-by-scale basis.

Accordingly to Gencay *et al.*, (2003), the main advantage of the wavelet analysis is the ability to decompose data into several time scales and the ability to handle nonstationary data, localization in time, and the resolution of the signal in terms of the analysis time scale. Since it is likely that there are different decision-making time scales among traders, the true dynamic structure of the relationship between stock returns and inflation itself will vary over different time scales associated with those different horizons (Kim & In, 2005). Wavelet analysis extends previous investigations that were only comparisons with the short run and the long run.

We first decompose the series of daily log-returns of Ibovespa and Real/Dollar exchange rate through wavelet multiscaling, obtaining the multi-level time scales. Secondly, we estimate families of copulas in each pair of time scales, in order to verify, through a selection criterion, which is the characteristic of the joint probability distribution of this bivariate relationship and its dependence measure.

A copula is a function that links univariate marginal to their multivariate distribution. Since it is always possible to map any vector of random variables into a vector with uniform margins, we are able to split the margins of that vector and a digest of the dependence, which is the copula. Copulas can lead with the variables that do not have Gaussian distribution and with non-linear dependences, being a powerful statistic tool for estimating multivariate relationships.

Wavelets

Wavelets, as is suggested by their name, are little waves. The term wavelet was created in geophysics literature by Morlet *et al.*, (1982). However, evolution of wavelets occurred over a significant time scale and in many disciplines, and their background can be found in studies of Da(ubechies, 1992; Meyer, 1993; Vidakovic, 1999; Heil & Walnut, 2006), among others.

Basic wavelets are described as father and mother wavelets. A father wavelet (scaling function) represents a smooth baseline trend, while mother wavelets (wavelet function) are used to describe all deviations from trends (Kim & In, 2005). Father and Mother Wavelets are represented by formulations (1) and (2), respectively.

$$\phi_{j,k}(x) = 2^{j/2}\phi(2^j x - k) \quad (1)$$
$$\psi_{j,k}(x) = 2^{j/2}\psi(2^j x - k) \quad (2)$$

Where $j, k \in \mathbb{Z}$, for some coarse scale j_0, that will be taken as zero. From these expressions an orthonormal system is generated. Conforming to (Morettin *et al.*, 2010), for any function *f* that belongs to this system we may write, uniquely:

$$f(x) = \sum_k \alpha_{0,k}\, \phi_{0,k}(x) + \sum_{j\geq 0}\sum_k \beta_{j,k}\psi_{j,k}(x). \quad (3)$$

In (3), $\alpha_{0,k} = \int f(x)\, \phi_{0,k} dx$ and $\beta_{j,k} = \int f(x)\, \psi_{j,k} dx$

are the wavelet coefficients. Thus, let's consider a time series, *f(t)*, which we want to decompose into various wavelet scales. Given the father wavelet, such that its dilates and translates constitute orthonormal bases for all the subspaces that are scaled versions of the initial subspace, we can form a Multiresolution Analysis (MRA) for *f(t)* (Burrus *et al.*, 1998).

In economics and finance Wavelet analysis has previously been applied for examination of foreign exchange data using waveform dictionaries (Ramsey and Zhang, 1997), decomposition of economic relationships of expenditure and income (Ramsey & Lampart, 1998), systematic risk in the capital asset pricing model (Gençay *et al.*, 2003), beyond the paper of Ramsey (2002) and the book by Gençay *et al.*, (2002).

Copulas

Definitions and concepts

Dependence between random variables can be modeled by copulas. A copula returns the joint probability of events as a function of the marginal probabilities of each event. This makes copulas attractive, as univariate marginal behavior of random variables can be modeled separately from their dependence (Kojadinovic & Yan, 2010).

The concept of copula was introduced by (Sklar, 1959). However, only recently its applications have become clear. A detailed treatment of copulas as well as of their relationship to concepts of dependence is given by (Joe, 1997; Nelsen, 2006). A review of applications of copulas to finance can be found in Embrechts *et al.*, (2003) and in Cherubini *et al.*, (2004).

For ease of notation we restrict our attention to the bivariate case. The extensions to the *n*-dimensional case are straightforward. A function $C : [0,1]^2 \rightarrow [0,1]$ is a *copula* if, for $0 \leq x \leq 1$ and $x_1 \leq x_2$, $y_1 \leq y_2$, (x_1, y_1), $(x_2, y_2) \in [0,1]^2$, it fulfills the following properties:

$$C(x,1) = C(1,x) = x, \;\; C(x,0) = C(0,x) = 0. \quad (4)$$
$$C(x_2, y_2) - C(x_2, y_1) - C(x_1, y_2) + C(x_1, y_1) \geq 0. \quad (5)$$

Property (4) means uniformity of the margins, while (5), the *n-increasing property*, means that $P(x_1 \leq X \leq x_2, y_1 \leq Y \leq y_2) \geq 0$ for (X, Y) with distribution function *C*.

In the seminal paper of (Sklar, 1959), it was demonstrated that copula is linked to the distribution function and its marginal distributions. This important theorem states that:

(i) Let *C* be a copula and F_1 and F_2 univariate distribution functions. Then (6) defines a distribution function *F* with marginals F_1 and F_2.

$$F(x,y) = C\big(F_1(x), F_2(y)\big), \; (x,y) \in R^2 \quad (6)$$

(ii) For a two-dimensional distribution function *F* with marginals F_1 and F_2, there exists a copula *C* satisfying (3). This is unique if F_1 and F_2 are continuous and then, for every:

$$C(u,v) = F\left(F_1^{-1}(u), F_2^{-1}(v)\right) \quad (7)$$

In (7), F_1^{-1} and F_2^{-1} denote generalized left continuous inverses of F_1 and F_2.

However, as Frees and Valdez (1998) note, it is not always obvious how to identify the copula. Indeed, for many financial applications, the problem is not to use a given multivariate distribution but consists in finding a convenient distribution to describe some stylized facts, for example the relationships between different asset returns.

Families of Copulas

The most frequently used copulas are Elliptical and Archimedean (Yan & Kojadinovic, 2010). Among the elliptical copulas, which are characterized by the class of symmetric copulas, we highlight Normal and Student's *t* Copulas. In the class of Archimedean copulas, which best fit the asymmetric distributions, the families stand out: Frank and Gumbel.

These families of copulas are defined, according to Cherubini *et al.*, (2004), below.

Let $\Phi_{\rho_{XY}}$ be the joint distribution of a bi-dimensional vector, with linear correlation coefficient ρ_{XY}. Normal Copula is defined in (8).

$$C^{Ga}(u,v) = \Phi_{\rho XY}\left(\Phi^{-1}(u), \Phi^{-1}(v)\right) \quad (8)$$

In (8), Φ^{-1} is the inverse of the standard univariate normal distribution function Φ. Gaussian Copula generates the standard Gaussian joint distribution function, whenever the margins are standard normal.

Let ρ be the bivariate linear correlation and υ represents the degrees of freedom of the Student's *t* distribution function, so Student's *t* copula is defined in (9).

$$T_{\rho,\upsilon}(u,v) = t_{\rho,\upsilon}\left(t_\upsilon^{-1}(u), t_\upsilon^{-1}(v)\right) \quad (9)$$

In (9), t_υ^{-1} is the inverse of the univariate Student's *t* distribution function with υ degrees of freedom; and $t_{\rho,\upsilon}$ is the bivariate distribution corresponding to t_υ.

Frank Copula, which appeared in Frank (1979), is represented in (10).

$$C(u,v) = -\frac{1}{\alpha}\ln\left(1 + \frac{(\exp(-\alpha u)-1)(\exp(-\alpha v)-1)}{exp(-\alpha)-1}\right) \quad (10)$$

In (7), α is a generator and its range is $(-\infty, 0) \cup (0, +\infty)$.

Gumbel family, which was introduced by Gumbel (1960), is represented by formulation (11)

$$C(u,v) = exp\left\{-[(-\ln u)^\alpha + (-\ln v)^\alpha]^{1/\alpha}\right\} \quad (11)$$

In (11), the range for the generator α is $[1, +\infty)$

Method

In order to investigate the relationship between stock returns and exchange rate in the Brazilian market over different time scales, we use daily quotations of Ibovespa and ratio between the U.S. Dollar and Brazilian Real, from April, 2003 to August, 2011, totalizing 2049 observations. This index is commonly used in academic papers as proxy for this financial market. It is compounded of stocks that are more representative in terms of liquidity and value. As wavelets work with a number of data which is a power of 2, in order to use more data we would need at least 4096 observations, but before 1999 the exchange rates in the Brazilian economy were fixed by the government, turning unavailable any kind of association with the capital market. Regarding to the Exchange rate, Dollar was chosen because the U.S. are the biggest economics in the world and the more important commercial partner of Brazil. Further, the U.S. Dollar has an important role in the financial activity in the Brazilian market, being used as a hedging instrument, for example.

After, we calculated the log-returns of the variables, conforming the formulation (12).

$$r_t = \ln P_t - \ln P_{t-1} \quad (12)$$

In (12), P_t is a daily quotation in period *t* of the variables present in this study, whereas r_t represents their log-returns at time *t*. Thus we have 20482^{11} observations of each variable. This is necessary because wavelets decomposition works with number of data in form of power of two.

After, we implemented the MRA, proposed in section 2, for both variables. Thus we obtained ten groups of coefficients $(2^0 + 2^1 + \cdots + 2^{10} = 2048))$, which represent the distinct frequency levels of the discrete decomposition. This step was based on the Haar mother wavelet, which is represented by formulation (13).

$$\psi(x) = \begin{cases} 1, x \in [0\,, 1/2) \\ -1, x \in [1/2, 1) \\ 0\ otherwise \end{cases} \quad (13)$$

In that sense, Nason (2008) appoints that the Haar wavelet is a good choice because it exhibits many characteristic features of wavelets. Two relevant characteristics are the oscillation (the Haar wavelet goes up and down), mathematically this can be expressed by the condition that $\int_{-\infty}^{\infty} \psi(x)dx = 0$, a property shared by all wavelets; and the compact support (not all wavelets have compact support, but they must decay to zero rapidly).

Subsequently, through this discrete decomposition, we formed pairs composed by the 0 to 10 levels of log-returns calculated from both variables. Due to the low number of coefficients in the very first five levels (0 to 4 scales), we just calculated its linear correlations. For the remainder, which represents the finest scales, we estimated the families of copulas introduced at section 3. Data were standardized into pseudo-observations $\boldsymbol{U}_j = \left(U_{1j}, \dots, U_{ij}\right)$ through the ranks as $U_{ij} = R_{ij}/(n+1)$. The next step was, to estimate the copula's parameters, there was employed the procedure of inversion of the copula based *Kendall's Tau (τ)*, that serves to measure the monotonic dependence, which is calculated as in (14).

$$\tau(x,y) = 4\int_0^1\int_0^1 C(u,v)dC(u,v) - 1 \quad (14)$$

To determine which copula model best fits the residuals of the markets studied, we applied a rank-based version of the familiar Cramér–von Mises statistic, discussed in (Genest *et al.*, 2009), and extended in (Genest *et al.*, 2011), which made it possible to check the validity of the dependence structure of the margins separately. These authors emphasize that it is a blanket test, i.e., a procedure whose implementation requires neither an arbitrary categorization of data, nor any strategic choice of smoothing method. The goodness-of-fit test employed is defined in (15), and it tests the null hypothesis that data are fitted by $C_{\theta n}$, a copula with vector of parameters θ.

$$S_n = \int_{[0,1]^d} \mathbb{C}_n(\boldsymbol{u})^2 \, dC_n(\boldsymbol{U}) \qquad (15)$$

In (15), $C_n(\boldsymbol{U}) = \frac{1}{n}\sum_{i=1}^{n} \mathbf{1}(U_{i1} \leq u_1;\ U_{i2} \leq u_2)$ is known as an empirical copula; $\boldsymbol{U}_j = (U_{1j}, \dots, U_{ij})$ are the pseudo-observations; $\mathbb{C}_n = \sqrt{n}(C_n - C_{\theta n})$ is an empirical process that assesses the distance between the empirical copula and the estimation $C_{\theta n}$; n is the number of observations. In practice, the limiting distributions of S_n depend on the family of copulas under the composite null hypothesis, and on the unknown parameter value θ in particular.

Results

Initially, in order to avoid non-stationarity problems, we calculated the log-returns of the daily Real/Dollar exchange rate and of the Prices of Ibovespa. In order to illustrate the temporal evolution of the series, Figures 1 and 2 present, respectively, the plot of prices and log-returns of both variables.

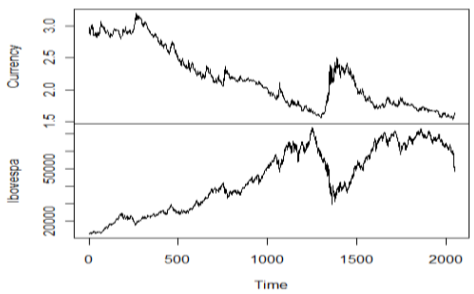

Figure 1. Temporal evolution of daily prices of Real/Dollar and Ibovespa for April 2003 to August 2011

Figure 1 gives us an idea of the relationship between currency and the Brazilian stock market. There is the inverse dependence notable. In periods of upper trend in the stock market, there was a decrease at the level of the ratio Real/Dollar, and the opposite was true. It should be noted that there was a huge fall in the prices of the Ibovespa around the observation 1400. This was the effect of the sub-prime crisis of 2007/2008.

Figure 2 corroborates the results of the previous plot. Again, the vestiges of the North American crisis can be noted by the volatility cluster around the observation 1400. Further, it's notable that the stock market showed more dispersion than the exchange rate. In order to analyze the descriptive behavior of the series, Table 1 presents some statistics.

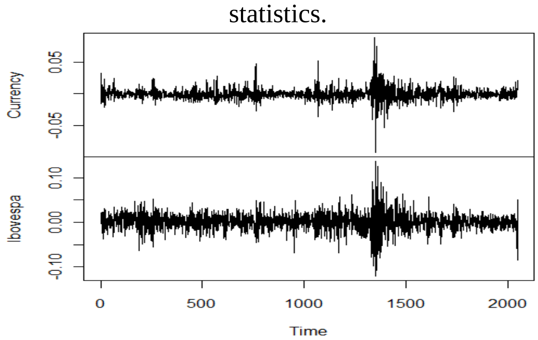

Figure 2. Temporal evolution of daily log-returns of Real/Dollar and Ibovespa for April 2003 to August 2011

The results contained in Table 1 emphasize that Ibovespa had greater central tendency measures than the exchange rate, but too close of zero. Also, the deviation of stock markets was bigger that the exchange rate, denoting that the first is riskier than the second. Beyond, Ibovespa had negative skewness, while the exchange rate presented positive one. Both series are leptokurtic, as evidenced by their kurtosis. The descriptive statistics of Ibovepa are quite common in financial returns, as documented by (Longin & Solnik, 2001).

Table 1

Descriptive statistics of daily log-returns of Real/Dollar and Ibovespa for April 2003 to August 2011

	Exchange	**Ibovespa**
Mean	-0.0003	0.0007
Median	-0.0007	0.0015
St. Deviation	0.0095	0.0191
Skewness	0.6247	-0.1068
Kurtosis	14.4174	5.4567

After this initial empirical analysis, we implemented the MRA, proposed in section 2, for both variables. Figure 3 illustrates the obtained coefficients in 11 levels (2048 observations). Figure 3 emphasizes that at finest scales, there were much more oscillation in both series due to the intense activity existing in the whole financial market in the short-term. Further, the variations in stock market were bigger than those of the exchange rate. This result corroborates with those found in the descriptive analysis.

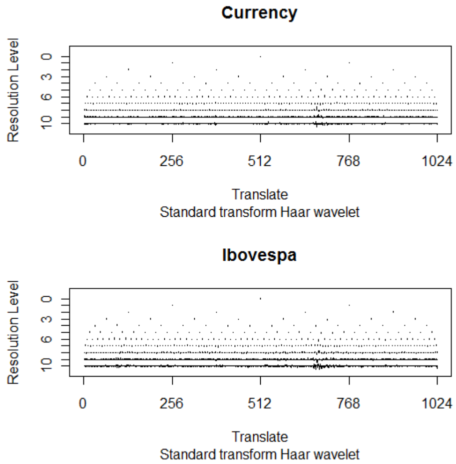

Figure 3. Coefficients of the MRA for 11 scales of daily log-returns of Real/Dollar exchange rate and Ibovespa

Subsequently, we estimated Normal, Student's *t*, Frank and Gumbel copulas for the relationship of the 5 to 10 scales of daily log-returns of Real/Dollar exchange rate and Ibovespa in order to determine their joint probability function to verify the shape and magnitude of their dependence. Table 2 presents the results.

Bold values are significant at 5 % level. * indicates the better fit.

The results contained in Table 2 clearly indicate that during the analyzed period, there was a negative dependence in all frequency scales of the relationship of daily log-returns of Real/Dollar exchange rate and Ibovespa.

Table 2

Estimated parameters of the copulas, values and significance of the S_n tests in the relationships of scales for daily log-returns of Real/Dollar exchange rate and Ibovespa

	Scale 5			**Scale 6**			**Scale 7**		
Copula	Param.	S_n test	p-value	Param.	S_n test	p-value	Param.	S_n test	p-value
Normal	-0.5816	0.0322	0.6558	-0.4056	0.0322	0.1753	-0.6775	0.0170	0.6848
Student's *t*	-0.5816	0.0301	*0.7917	-0.4056	0.0288	*0.3232	-0.6775	0.0142	*0.9146
Frank	-4.0948	0.0343	0.5539	-2.5409	0.0369	0.0914	-5.2813	0.0279	0.1314
Gumbel	1.0000	**0.0586**	0.0145	1.0000	**0.0639**	0.0035	1.0000	**0.5973**	0.0005
	Scale 8			**Scale 9**			**Scale 10**		
Copula	Param.	S_n test	p-value	Param.	S_n test	p-value	Param.	S_n test	p-value
Normal	-0.6891	0.0118	*0.8946	-0.5404	0.0197	0.2862	-0.2719	0.0186	0.3961
Student's *t*	-0.6891	0.0122	0.8796	-0.5404	0.0245	0.1337	-0.2719	0.0115	*0.9256
Frank	-5.4519	0.0202	0.3212	-3.6773	0.0167	*0.5140	-1.6187	0.0290	0.0694
Gumbel	1.0000	**1.3022**	0.0005	1.0000	**1.5597**	0.0005	1.0000	**0.6490**	0.0005

This was evidenced by the estimated parameters of the Normal, Student's t and Frank copulas. Further, the estimated Gumbel copula rejected the null hypothesis of comformity of the data for all frequency scales. This can be explained by the fact that Gumbel family has more joint probability at the positive tail, being unable to fit the negative dependence of the variables. Nevertheless, the parameter for the Gumbel copula was always the unity, which is the smaller possible value. These results corroborate those found by (Tabak, 2006; Abugri, 2008; Adam & Tweneboah, 2008), among others.

Negative dependence between the variables can be explained, in conformity with (Stavarek, 2004), because a rise in domestic stocks prices would attract capital flows, which increase the demand for domestic currency and cause exchange rate to be appreciated. A rising stock market leads to the appreciation of domestic currency through direct and indirect channels. A rise in prices encourages investors to buy more domestic assets simultaneously selling foreign assets to obtain domestic currency indispensable for buying new domestic stocks, leading to a domestic currency appreciation.

Despite the signal of the relationship is negative in all scales, there were changes in its magnitude. Figure 4 presents the evolution of the correlation between the returns of Dollar/Real exchange rate and Ibovespa in the scales level.

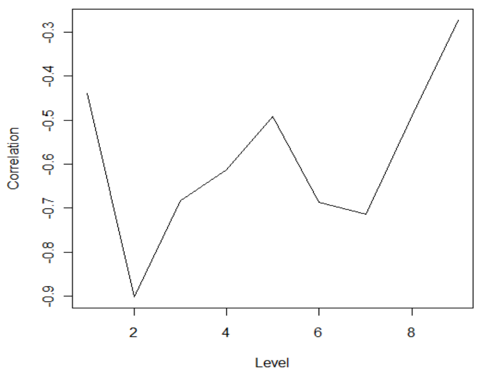

Figure 4. Evolution of the correlation between daily log-returns of Real/Dollar exchange rate and Ibovespa through 11 time scales

Figure 4 shows that there is a change in the magnitude of the dependence. Initially there is a growth of the inverse association on the coarse scales, emphasizing that long-term investors are more sensitive to this relationship. Also, there is a reduction of negative correlation when middle scales are considered. In the finest scales, first there is a growth of the dependence but, after, in the very finest scales (9 and 10), the negative association gets its lower level. This result implies less sensitive dependence of Dollar/Real exchange and Brazilian stock market for the very short-time investor. This result can be associated with the necessary delay for changes in exchange rate impact the stock market and vice-versa.

Further, the very short-time investor is not very concerned with the exchange rate and the future perspective of companies as with risk diversification, for example. Regarding to the shape of joint probability of daily log-returns of Dollar/Real exchange rate and Ibovespa at distinct frequencies, there was a predominance of Student's t copula. The exception was the scales 8 and 9, where the best fit was obtained by Normal and Frank copulas, respectively. This result evidences that there is more dependence in the tails between exchange rate and Brazilian stock markets than normally expected. Thus, there is reinforced the necessity of risk management of portfolios composed by stocks of this market in situations of great variation in Real/Dollar exchange rate. No less important is the caution of the exchange rate policy of the Brazilian government to huge variations and turbulence periods in the national stock market.

Concluding Remarks

The aim of this paper was to investigate the relationship between stock returns and exchange rate in the Brazilian stock market over different time scales. Therefore we used daily quotations of Ibovespa and ratio between the U.S. Dollar and Brazilian Real, from April, 2003 to August, 2011.

The initial analysis evidenced that Brazilian stock market is more volatile that Real/Dollar exchange rate. It was confirmed by temporal evolution plots, descriptive statistics and wavelet based coefficients. After, with the time scales coefficients we estimated families of copulas for the relationship between daily log-returns of Dollar/Real exchange rate and Ibovespa at the scales 5 to 10. The coarse scales were left out because they contain few observations. For these, we just calculated the correlation between the variables.

The results of estimated copulas evidenced that dependence between the variables is negative in all time scales, confirming the balanced portfolio theory. Nevertheless, there were some changes in this association in the distinct frequencies. The coarse scales exhibit great negative dependence, which was reduced in the middle scales and reached the lower level at the finest scales. This confirms that investors that hold portfolios composed by Brazilian stocks for long time are more sensitive to Dollar/Real exchange rate. This is because they are concerned with the future expectative of the companies that they share, which are frequently affected in the long-term. The inverse reasoning explains the results for the short-time investors, which are represented by the finest scales.

Regarding the shape of joint probability of the variables, there was a predominance of Student's t copula. This result emphasized that there is more dependence in the tails between exchange rate and Brazilian stock markets than normally expected. It highlights the necessity of risk management in portfolios composed by stocks of Brazilian market in situations of great variation in Dollar/Real exchange rate. No less important is the caution of the exchange rate policy of the Brazilian government to huge variations and turbulence periods in the stock market, in order not to lose the control of crucial areas, as, for example, the commercial balance.

Finally, we suggest for future researches that similar procedure is utilized to approach the relationship of more economic variables and stock markets in others countries; as well investigate contagion between stock markets in distinct time frequencies. As a limitation of the study, we emphasize the difficulty in isolating the effect of other variables and conditions in the analyzed relationship.

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Valiutos kurso ir fondų biržos netiesioginės priklausomybės įvertinimas panaudojant skirtingas laiko skales: Lotynų Amerikos pavyzdys

Santrauka

Fondų birža atlieka labai svarbų vaidmenį šiuolaikinėje ekonomikoje, nes ji veikia kaip tarpininkas tarp skolintojų ir besiskolinančiųjų. Gerai funkcionuojanti fondų birža gali padėti šalies plėtros procesui dviem būdais: didindama santaupas ir leisdama daug efektyviau panaudoti resursus. Gali būti labai naudinga investuojant, išplečiant vietinius fondus ir kanalus. Tačiau, norint atlikti šią svarbią užduotį, būtina, kad fondų birža turėtų pastebimą ryšį su makroekonomikos kintamaisiais (Mohammad ir kt., 2009).

Investuotojai yra suinteresuoti atrasti kintamuosius, kurie padėtų numatyti akcijų kainas, nes tada jie gali tinkamiau valdyti savo portfelius (didinti grąžą ir/arba mažinti riziką). Jie gali panaudoti makroekonomikos naujienas kaip patikimus rodiklius ten, į kur nukreipia fondų birža (Wang, 2011). Patvirtindami tai, politikai atkreipia dėmesį į tokią fondų biržos situaciją, kurią galima laikyti vyraujančiu būsimos makroekonominės veiklos rodikliu. Tokia situacija sudaro galimybes kontroliuoti ekonomikos kryptis ir stabilumą, (derindama makroekonominius kintamuosius, jei santykis tarp kapitalo grąžos ir ekonominės veiklos gali būti prognozuojamas). Gerai veikianti valiutų rinka labai padeda ekonomikai augti ir taupyti, efektyviai investuoti ir pritraukti tiesiogines investicijas iš užsienio. Vykstant finansinei ir ekonominei globalizacijai, valiutų kursai turi lemiamą įtaką analizuojant kapitalo grąžą.

Nėra vienodos teorijos analizuojant kapitalo kainas ir valiutų kursus. Kaip teigia Ozbay (2009), norint nustatyti santykį tarp valiutos kurso ir kapitalo kainos, buvo pasiūlyti du metodai: prekių rinkos modelis, kurį sukūrė Dornbusch ir Fisher (1980) ir portfelio pusiausvyros modelis. Pirmasis metodas atskleidžia, kad egzistuoja teigiamas santykis tarp valiutos kurso ir kapitalo grąžos (remiantis valiutos kurso kotiravimo tiesioginio panaudojimo prielaida) (Stavarek, 2004). Antrasis metodas labiau išskiria kapitalo sąskaitų sandorių svarbą (Tahir ir Ghani, 2004). Portfelio pusiausvyros modelis numano neigiamą santykį tarp kapitalo kainos ir valiutos kurso.

Taigi, būtinybė sumodeliuoti priklausomybę tarp valiutos kurso ir kapitalo grąžos yra neišvengiama. Toliau reikėtų nustatyti santykio formą ir dydį, kad investuotojai galėtų apsaugoti savo pozicijas. Tokiu būdu, pagrindinis šio *darbo tikslas* yra pasiūlyti naują metodą, t. y. bangelių analizę, kad būtų galima ištirti santykį tarp kapitalo grąžos ir valiutos kurso Brazilijos rinkoje, esant skirtingoms laiko skalėms. Šis metodas yra pagrįstas daugiapakopiu bangelių metodu, kuris suskaido turimas laiko serijas *skalė po skalės*.

Tai nustatyti, mes naudojame kasdieninius *Ibovespa kotiravimus* ir santykį tarp JAV dolerio ir Brazilijos realo laikotarpiu nuo 2003 metų balandžio mėnesio iki 2011 metų rugpjūčio mėnesio (iš viso 2049 stebėjimų). Šis indeksas dažniausiai yra naudojamas ir akademiniuose darbuose kaip finansinės rinkos pakaitalas. Jį sudaro kapitalas, kuris daug tinkamesnis likvidumo ir vertės prasme. Norint panaudoti daugiau duomenų, mums reikia atlikti mažiausiai 4096 stebėjimų. Tačiau iki 1999 metų, Brazilijos ekonomikos valiutos kursą fiksavo vyriausybė. Kalbant apie valiutos kursą, buvo pasirinktas doleris, nes stipriausia pasaulio ekonomika yra JAV. Taip pat JAV yra labai svarbus Brazilijos komercinis partneris. Be to, JAV doleris atlieka svarbų vaidmenį Brazilijos finansų rinkoje, pavyzdžiui, jis naudojamas kaip apsaugos priemonė.

Pirmiausia mes suskaidome kasdienines *Ibovespa* logaritminės grąžos ir *realo/dolerio* keitimo kurso serijas, atlikdami *bangelių* suskaidymą į skales ir gaudami kelių lygių laiko skales. Antra, mes išmatuojame jungčių grupes kiekvienoje laiko skalių poroje, norėdami (per atrankos kriterijų) patikrinti, kuri iš jų yra šio dvimačio santykio jungtinės tikimybės paskirstymo charakteristika ir jo priklausomybės matmuo. *Bangelės*, kaip rodo jų pavadinimas, yra mažos *bangos*. Terminą *bangelė* geofizikoje sukūrė Morlet ir kt. (1982). Anot Gençay ir kt., (2003), pagrindinis bangelių analizės privalumas yra galimybė išskaidyti duomenis į keletą laiko skalių, taip pat galimybė valdyti skirtingus duomenis, lokalizuoti juos laike, signalą suskaidyti analizės laiko prasme. Kadangi yra tikėtina, kad tarp prekiautojų egzistuoja skirtingos sprendimo priėmimo laiko skalės, tikroji santykio tarp kapitalo grąžos ir pačios infliacijos dinaminė struktūra bus nevienoda skirtingose laiko skalėse, susijusiose su skirtingais horizontais (Kim ir In, 2005). *Bangelių* analizė papildo ankstesnius tyrimus, kuriuose buvo lyginama ilgalaikė ir trumpalaikė veikla.

Jungtis yra funkcija, kuri sujungia vieno požymio ribą su jų daugiamačiu paskirstymu. Kadangi bet kurį, atsitiktinių kintamųjų vektorių visada galima pažymėti vektoriuje, turinčiame vienodas maržas, mes galime padalinti to vektoriaus ribas ir priklausomybę, kurie ir yra jungtis. Jungtys, būdamos galinga statistine priemone, skirta įvertinti daug kintamųjų turinčius ryšius, gali pirmauti su tais kintamaisiais, kurie neturi Gauso paskirstymo ir su nelinijinėmis priklausomybėmis,

Pirminė analizė parodė, kad Brazilijos fondų birža yra daug nepastovesnė už *realo/dolerio* valiutos kursą. Tai patvirtino laikinu plėtros planu, aprašomąja statistika ir *bangelėmis* pagrįsti koeficientai. Po to, panaudodami laiko skalės koeficientus, mes įvertinome jungčių grupes dėl ryšių tarp kasdieninės *dolerio/realo* kurso logaritminės grąžos ir *Ibovespa*, skalėse nuo 5 iki 10. Apytikslės skalės buvo praleistos, nes jos apima tik keletą stebėjimų. Mes jomis tik apskaičiavome koreliaciją tarp kintamųjų.

Įvertintų jungčių rezultatai parodė, kad priklausomybė tarp kintamųjų yra neigiama visose laiko skalėse, taip patvirtindama portfelio pusiausvyros teoriją. Nepaisant to, šio susijungimo skirtinguose dažniuose buvo pokyčių. Apytikslės skalės parodo didelę neigiamą priklausomybę, kuri sumažėjo vidutinėse skalėse ir pasiekė žemiausią lygį tiksliausiose skalėse. Tai patvirtina, kad investuotojai, turintys Brazilijos ilgalaikių akcijų portfelius, yra daug jautresni *dolerio/realo* kursui. Taip yra todėl, kad jiems rūpi kompanijų, kurių akcijas jie turi, ateitis, kuri dažnai priklauso nuo laikotarpio. Kitokie argumentai paaiškina rezultatus trumpalaikiams investuotojams, kuriuos vaizduoja tiksliausios skalės.

Dėl jungtinės kintamųjų tikimybių formos, vyravo *Studento t* jungtis. Šis rezultatas parodė, kad egzistuoja didesnė priklausomybė tarp valiutos kurso ir Brazilijos fondų biržos, nei manoma. Tai atskleidžia būtinybę valdyti portfelių riziką, kurią sudaro Brazilijos rinkos akcijos, kai *dolerio/realo* kursas labai kinta. Ne mažiau svarbus yra Brazilijos vyriausybės atsargumas valiutos kurso politikos atžvilgiu fondų biržoje, esant dideliam kitimui ir neramumų laikotarpiais, kad būtų neprarasta kontrolė ekonomiškai svarbiose srityse ir išlaikyta jų pusiausvyra.

Raktažodžiai*: valiutos kursai; bangelės; jungtys Copulas; Brazilijos rinka.*

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