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A Multi-Objective Robust Optimization Model for a Facility Location-Allocation Problem in a Supply Chain under Uncertainty

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The final purpose of this study is presentation a mathematical model for a facility location-allocation problem so as to design an integrated supply chain. A supply chain with multiple suppliers, multiple products, multiple plants, multiple transportation alternatives and multiple customers is taken into account for this purpose. The problem is to specify a number and capacity level of plants, allocation of customers demand, and selection and order allocation of suppliers. A scenario approach is considered to deal effectively with the uncertainty of demand and cost parameters. The formulation is a robust multi-objective mixed-integer linear programming (MOMILP), in the context of which two conflicting objectives is taken into account simultaneously: (1) minimizing the total costs of a supply chain including raw material costs, transportation costs and establishment costs of plants, and (2) minimizing the total deterioration rate occurred by transportation alternatives. Then, the problem can be reduced to a linear one. Finally, by applying the LP-metric method, the model is solved as a single objective mixed-integer programming model. An experiment study corroborates that this procedure can be proposed to design an effective supply chain.

Keywords: *Supply chain design, Facility location-allocation, Robust multi-objective optimization, Uncertainty.*

Introduction

Many companies struggle with justifying the costs of supply chain (SC) in order to increase the profit achieved by the market share. One of the areas in designing a supply chain is logistics. What many companies fail to see is the coordination and cooperation between logistic issues and supply chain management. In order to create a profitable final product, the company must address all aspects of supply chain, including facility location-allocation.

The area of SCM is a significant issue among authorities which has stimulated a lot of debates. SCM as the process of planning, implementing and controlling the operations of the supply chain desires to meet customer requirements as efficiently and effectively as possible. It spans all movements and storage of raw materials, work-in-process (WIP) inventory, and finished goods from the point-of-origin to the point-of-consumption (Simchi-Levi *et al.*, 2004).

A facility location-allocation problem that has been a well-established research area within operations research (OR), as one of the most noted examples of a supply chain problem, comprises a series of potential facility plants where a facility can be opened and a series of demand points that must be serviced. The goal is to pick a subset of facilities to open in such a way that the sum of distances from each demand point to its nearest facility and the sum of opening costs of the facilities are minimized. If a supply chain still wants to stay profitable, logistic issues should be regarded as part of a decision making process. The purpose of this article is to model the situation that location and transportation decisions can use to design the optimized supply chain.

Background

The goal of facility location-allocation problems in supply chain is to identify the best locations of various nodes and allocation of demands to them. The study of facility location–allocation problems stretches back to 1960s when (Cooper, 1963) proposed the basic facility location–allocation problem. Since then, this problem has become a contentious issue of debate among scientists and caused quite a stir in numerous circles and camps. A number of studies in the realm of facility location–allocation problems have been conducted in the literature; for instance, a dynamic multi-period location–allocation problem (Manzini & Gebennini, 2008; Torres-Sotoa & Halit, 2011) a continuous site location problem (Jiang and Yuan, 2008) a joint facility location–allocation and production problem (Kanyalkar & Adil, 2005; Liu & Lin, 2005) a capacitated facility location–allocation problem (Liu & Lin, 2005; Amiri, 2006; Torres-Sotoa & Halit, 2011;) and a multi-objective facility location–allocation problem (Bashiri & Hosseininezhad, 2009; Singh & Singh, 2011; Jolai *et al.,* 2011; Zarandi *et al.,* 2011).

Considering the increasing competition between companies, more demanding customers and reduction in profit margin, SCM became an important practice for companies that not only want to stay in business but also have their results optimized and meet the clients' expectations. Literature about facility location-allocation in supply chain design is extensive and diverse. Historically, researchers have focused relatively early on the design of distribution systems (Lehtonen & Salonen, 2006; Stasiskiene & Sliogeriene, 2009), but without enough considering the supply chain as a whole. Almost 80 percent of the surveyed papers refer to one or two parts of SCM and among these papers; about two thirds model location decisions in only one part (Melo *et al*., 2009). However, there are some studies considered the whole supply chain in facility location-allocation problem (Sabri & Beamon, 2000; Salema *et al.,* 2006; Yeh, 2006; Manzini & Gebennini, 2008; Samaranayake *et al.*, 2011).

Uncertainty in facility location models

Another characteristic of a facility location-allocation model comes into the equation when various parameters vary throughout the time in a predictable way. They can be placed in the model to get a network design in order to deal effectively with the possible future changes. There are large numbers of deterministic models in comparison with stochastic ones (Melo *et al.*, 2009). According to (Sabri & Beamon, 2000), uncertainty is regarded as one of the most controversial, but significant problems in SCM. Nevertheless, a stochastic environment in combination with location decisions in the context of SCM is still infrequent in the literature. Different sources of uncertainty, such as customer demands, exchange rates, travel times, amount of returns in reverse logistics, supply lead times, transportation costs, and holding costs, have been discussed throughout the literature. (Bar-Lev *et al.,* 1993; Halidias & Michta, 2007; Abiri & Yousefli, 2011; Bartke, 2011; Carmichael & Balatbat, 2011).

Multi-objective optimization with single/multiple periods and products in facility location models

Most of the existing literature about facility location is focused on the optimization of only one objective; usually cost or profit and other important factors such as deterioration rate with various transportation alternatives (TAs) are left outside the analysis.

There are many techniques for multi-objective optimization such as *ε*-constrained and LP-metric methods. (Guillen *et al.,* 2005) used the *ε*-constrained method to make trade-off between three conflicting objectives, profit, demand satisfaction and financial risk cost in a three echelon supply chain. (Mirzapour Al-e-Hashem *et al.*, 2011) utilized LP-metric method to solve a multi-objective aggregation planning in supply chain under uncertainty with multiple products and multiple periods which minimize the production related costs and total losses of supply chain. This method provides a set of objectives that are Pareto efficient, thus forming a Pareto frontier.

Single/multiple periods and single or multiple products are the other issues that separate a facility location-allocation model, in which most of researches considered single period. Although, a single period facility location model may be enough to find a ''robust" network design (Melo *et al*., 2009), there are some papers considered multiple periods in production horizon (Ko & Evans, 2007; Hinojosa *et al.,* 2008; Srivastava, 2008; Manzini & Gebennini, 2008; Jolai *et al.,* 2011). In addition, some researchers have taken into account multiple products (Sabri & Beamon, 2000; Ko & Evans, 2007; Hinojosa *et al.,* 2008; Srivastava, 2008; Pati *et al.,* 2008; Manzini & Gebennini, 2008; Kazemi *et al.,* 2009).

For the detailed literature review on facility location–allocation problems, readers are referred to (Drezner & Hamacher 2002; Klose & Drexl, 2005; Melo *et al.,* 2009; Farahani *et al.,* 2010).

Problem statement

Companies must decide on conflicting decisions in supply chain which maximizes the benefit of the whole supply chain as well as minimizing deficiencies. Evaluation of the recent researches on facility location-allocation models in supply chain proves that logistic issues and effects of them are rarely considered in supply chain design.

In this paper, we develop a robust optimization program to a facility location-allocation problem to design a supply chain under uncertain customer demands and cost parameters. We consider several suppliers, several plants, and several customer zones with different transportation alternatives (TA). The supply chain produces two kinds of different products to fulfill the customers' demand, in which the information is given for one period (i.e., planning period). Two conflicting objectives are considered, simultaneously. The first objective aims to minimize the total cost of a supply chain including raw material costs, transportation costs and establishment costs of plants. This objective determines which plants at which a capacity level be opened, allocation of the customers' demand to the plants and supplier selection and order allocation problem. The second objective tries to minimize the deterioration rate caused by different TAs. Using the LP-metric method, these two objectives are combined, and then the single objective programming is solved. While there is a vast literature devoted on this type of problem, to the best of our knowledge, the majority of researchers consider some of these aspects individually or not considered some of them at all. Furthermore, to enable the model to deal with real situations, different TAs are considered in the whole supply chain. The results show that the proposed model enables decision makers to design an effective supply chain and provide them a global insight to plan for a whole supply chain.

The remainder of the paper is organized as follows. Section 2 describes the robust optimization framework. The problem description and formulation are presented in Section3. Then, solution procedure is presented in Section

4. Section 5 discusses the results by representing an experiment study. Finally, conclusions and future work are presented in Section 6.

Robust optimization

Two kinds of robustness, namely solution robustness (the solution is nearly optimal in all scenarios) and model robustness (the solution is nearly feasible in all scenarios) were proposed as a framework for robust optimization by (Mulvey *et al.*, 1995). There are two distinct forms of constraints in robust optimization; structural constraint and control constraint. The former is formulated following the concept of linear programming and its input data are free of any noise, while the latter is taken as an auxiliary constraint affected by noisy data (Leung *et al.*, 2007). Moreover, two groups of variables are defined; the design variable which cannot be adjusted once a specific realization of the data, and the control variable which is subject to adjustment once uncertain parameters.

The framework of robust optimization is briefly described as following (Mirzapour Al-e-Hashem *et al.*, 2011):

$$\text{Min } c^T x + d^T y \quad (1)$$

s.t.

$$Ax = b, \quad (2)$$

$$Bx + Cy = e, \quad (3)$$

$$x, y \geq 0, \quad (4)$$

where *x* denotes the vector of decision variables that is determined under the uncertainty of model parameters. *B*, *C* and *e* represent random technological coefficient matrices and right hand side vector, respectively.

Assume a finite set of scenarios $S = \{1, 2, \dots, s\}$ to model the uncertain parameters. Under each scenario $s \in S$, we associate the subset $\{d_s; B_s; C_s; e_s\}$ with the probability of scenario $P_s (\sum_s P_s = 1)$. Eq. (2) is the structural constraint whose coefficients are fixed and free of noise, whilst Eq. (3) is the control constraint whose coefficients are subject to noise. Also, control variable *y*, which is subject to adjustment when one scenario is realized, can be denoted as y_s for scenario *s*. δ_s presents the infeasibility of the model under scenario s in a condition that model is infeasible. A robust optimization model is formulated by:

$$\text{Min } \sigma(x, y_1, y_2, \dots, y_s) + \vartheta \rho(\delta_1, \delta_2, \dots, \delta_s) \quad (5)$$

s.t.

$$Ax = b, \quad (6)$$

$$B_s x + C_s y_s + \delta_s = e_s \qquad \forall s \in S \quad (7)$$

$$x \geq 0, \; y_s \geq 0, \; \delta_s \geq 0 \qquad \forall s \in S \quad (8)$$

There are two terms in the objective function representing solution robustness and model robustness. The first term of the objective function becomes a random variable taking the value $\psi_s = c^T x + d^T y$ with the probability of P_s under scenario *s*. The second term is a feasibility penalty function, which is used to penalize infeasible solutions under some of the scenarios. (Mulvey *et al.*, 1995) used the following equation to represent solution robustness:

$$\sigma(\text{o}) = \sum_{s \in S} P_s \psi_s + \lambda \sum_{s \in S} P_s \left(\psi_s - \sum_{s' \in \Omega} P_{s'} \psi_{s'} \right)^2 \quad (9)$$

where λ denotes the weight placed on a solution variance, in which the solution is less sensitive to change in the data under all scenarios as λ increases. However, the expression in Eq. (9) involves a complicated term, generating a quadratic form in formulation.

Yu & Li (2000) pointed out that dealing with such problems requires a great deal of computations due to the quadratic term and proposed an absolute deviation instead of the quadratic term, which has the following form:

$$\sigma(\text{o}) = \sum_{s \in S} P_s \psi_s + \lambda \sum_{s \in S} P_s \left| \psi_s - \sum_{s' \in S} P_{s'} \psi_{s'} \right| \quad (10)$$

Converting objective (10) from a non-linear to a linear programming model with linear constraints by introducing two non-negative deviational variables, we can solve the problem with less computational efforts (Wagner, 1975). Based on (Leung *et al.*, 2007), instead of minimizing the sum of the absolute deviations in Eq. (10), two deviational variables is minimized subject to the original constrains and additional soft constraints that give positive values of the difference inside the absolute functions. However, (Yu & Li, 2000) stated that this direct linearization approach is largely restricted due to many non-negative deviational variables and constraints introduced. The framework of their model is designed to minimize the objective function as follows:

$$\sigma(\text{o}) = \sum_{s \in S} P_s \psi_s + \lambda \sum_{s \in S} P_s \left[\left(\psi_s - \sum_{s' \in S} P_{s'} \psi_{s'} \right) + 2\theta_s \right] \quad (11)$$

s.t.

$$\psi_s - \sum_{s' \in S} P_{s'} \psi_{s'} + \theta_s \geq 0 \qquad \forall s \in S \quad (12)$$

$$\theta_s \geq 0, \qquad \forall s \in S \quad (13)$$

It is shown the transformation from quadratic programming in Eq. (9) to the mean absolute deviation minimization problem in Eq. 10 and thus changes from the latter to its linear programming formulation in Eqs. (11) to (13). Using the weight ϑ the trade-off between solution robustness and model robustness can be modeled by the MCDM process. According to the above discussions, the objective function can be formulated by (Mirzapour Al-e-Hashem *et al.*, 2011):

$$\text{Min} \sum_{s \in S} P_s \psi_s + \lambda \sum_{s \in S} P_s \left[\left(\psi_s - \sum_{s' \in S} P_{s'} \psi_{s'} \right) + 2\theta_s \right] + \vartheta \sum_{s \in S} P_s \delta_s \quad (14)$$

Model description

The proposed multi-objective mathematical model can be described as follows. There are *I* potential plants, *J* suppliers and *C* customers (see Figure 1). Products are produced by plants, which are produced by raw materials supplied by suppliers regarding to the consumption rates. The production cost of a certain item at different plants and the raw material cost in different suppliers can be different. All plants, suppliers and customers are spread geographically. Then considering TAs, the transportation cost from suppliers to plants and from plants to customers can vary. The problem is to determine the set of plants to be opened and the capacity level of these plants. Also, the quantity of raw materials r provided by supplier j to fulfill requirement of plant i and quantity of end products m shipped to customers are determined in a way that the total cost and the deterioration rate of transportation are minimized simultaneously. It is worth note that different TAs are allowed in the whole supply chain network.

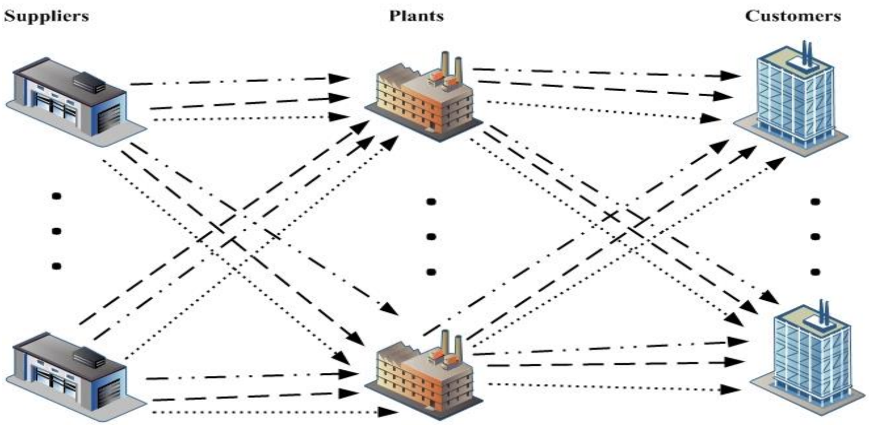

Figure 1. General schema for a supply chain with different TAs

Indices

i Index for plants ($i = 1, \dots, I$);
j Index for suppliers ($j = 1, \dots, J$);
c Index for customer zones ($c = 1, \dots, C$);
r Index for raw materials ($r = 1, \dots, R$);
m Index for end products ($m = 1, \dots, M$);
q Index for TAs ($q = 1, \dots, Q$);
n Index for the capacity level of plants ($n = 1, \dots, N$);

Parameters

P_{mc}^{s} Selling price of product m in customer zone $c$in scenario s;
TCS_{jirq}^{s} Transportation cost of raw material r from supplier j to plant i using ; TA q in scenarion s;
TCC_{icmq}^{s} Transportation cost of product m from plant i to customer zone c; using TA q in scenarion s;
CM_{rj}^{s} Purchasing cost of raw material r supplier j in scenarion s;
ES_{in}^{s} Establishing cost of plant i in capacity level n in scenarion s;
D_{mc}^{s} Demand for product m in customer zone c in scenarion s;
Y_{rm} Number of units of raw material r required for each unit product m;
C_{jr} Maximum number of raw material r supplier $j$could produce;
b_{in} Capacity level n for plant i;
α_{rq}^{s} Deterioration rate for raw material r using TA q in scenaro s;
β_{mq}^{s} Dterioration rate for product m using TA q in scenaro s;
n_q Available quantity of TA q;
a_q Capacity of TA q;
t_m Production time of product m;
p_r Space required for transporting raw material r;
l_m Space required for transporting product m;
ρ_s Occurence probability of scenario$s$.

Decision variables

SUP_{jirq} Number of units of raw material r shipped from supplier j to plant i; using TA q;
CUS_{icmq} Number of units of product m shipped from plant i to customer zone c; using TA q
X_{mi} Number of product m produced in plant i;
S_{in} 1, if plant i established in capacity level n.

Mathematical model

$$\text{Min } Z_1 = \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{r=1}^{R}\sum_{q=1}^{Q} CM_{rj} \times SUP_{jirq} + \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{r=1}^{R}\sum_{q=1}^{Q} TCS_{jirq}^{s} \times SUP_{jirq} + \sum_{i=1}^{I}\sum_{c=1}^{C}\sum_{q=1}^{Q}\sum_{m=1}^{M} TCC_{icmq}^{s} \times CUS_{icmq} + \sum_{i=1}^{I} ES_{in}^{s} \times S_{in} - \sum_{i=1}^{I}\sum_{c=1}^{C}\sum_{q=1}^{Q}\sum_{m=1}^{M} P_{ck}^{s} \times CUS_{icmq} \quad (15)$$

$$\text{Min} Z_2 = \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{r=1}^{R}\sum_{q=1}^{Q} \alpha_{rq}^{s} \times SUP_{jirq} + \sum_{i=1}^{I}\sum_{c=1}^{C}\sum_{m=1}^{M}\sum_{q=1}^{Q} \beta_{mq}^{s} \times CUS_{icmq} \quad (16)$$

s.t.

$$\sum_{j=1}^{J}\sum_{q=1}^{Q} SUP_{jirq} = \sum_{m=1}^{M} X_{mi} \times Y_{rm} \qquad \forall i, r \qquad (17)$$

$$\sum_{i=1}^{I}\sum_{q=1}^{Q} CUS_{icmq} = D_{mc}^{s} \qquad \forall C, m \qquad (18)$$

$$X_{mi} = \sum_{c=1}^{C}\sum_{q=1}^{Q} CUS_{icmq} \qquad \forall i, m \qquad (19)$$

$$\sum_{i=1}^{I}\sum_{q=1}^{Q} SUP_{jirq} \le C_{jr} \qquad \forall j, r \qquad (20)$$

$$\sum_{m=1}^{M} t_m \times X_{mi} \le \sum_{n=1}^{N} b_{in}.S_{in} \qquad \forall i \qquad (21)$$

$$\sum_{m=1}^{M} t_m \times X_{mi} \ge \sum_{n=1}^{N} S_{in} \qquad \forall i \quad \text{m} \qquad (22)$$

$$\sum_{n=1}^{N} S_{in} \le 1 \qquad \forall i \quad \text{m} \qquad (23)$$

$$\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{r=1}^{R} p_r \times SUP_{jirq} + \sum_{i=1}^{I}\sum_{c=1}^{C}\sum_{m=1}^{M} l_m \times CUS_{icmq} \le n_q.a_q \qquad \forall q \qquad (24)$$

The first objective function (Eq. 15) aims to minimize the costs of a supply chain including raw material purchasing cost, raw material transportation cost, end product transportation cost and establishment cost of plants, from which the total sell is deducted. The second objective function (Eq. 16) tries to minimize the total deterioration rates of different TAs. Constraints 17–19 are balance equations for the raw materials, demand of customers and end products. Eq. (20) specifies the maximum available raw material that can be produced by supplier *j*. Eqs. (21) and (22) defines the relationship between product quantities and capacity level of plants. Eq. (23) ensures that each plant can only have one capacity level. Finally, Eq. (24) specifies the capacity constraint of TAs to transport raw materials and end products.

Robust optimization formulation

The uncertain nature of environment makes the facility location-allocation problems more complex. Incorporating uncertainty into the planning decisions necessarily entails providing overwhelming answers to the following questions respectively (Mirzapour Al-e-Hashem *et al.*, 2011). Firstly, what are the proper approaches to deal effectively with the uncertain parameters? Different scholars are of the conviction that the main approaches are stochastic programming, fuzzy programming, stochastic dynamic programming and robust optimization (Ben-Tal & Nemirovski, 2000; Bertsimas & Sim, 2006). Secondly, how should the appropriate representation of the uncertain parameters be determined? According to (Gupta & Maranas, 2003), in order to handle the uncertainty inherent in the real world problems, three distinct methods were frequently stated. First, the distribution-based approach, where the normal distribution with specified mean and standard deviation is widely raised for modelling uncertain demands and/or parameters; second, the fuzzy-based approach, there in the forecast parameters are considered as fuzzy numbers with accompanied membership functions; and third, the scenario-based approach, in which several discrete scenarios with associated probability levels are used to describe the expected occurrence of particular outcomes.

According to the model presented by (Mulvey *et al.*, 1995), uncertainty is presented by a set of discrete scenarios (s). Therefore, the proposed robust multi-objective model is presented as follows:

$$TC^{s}(transportationcosts): \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{r=1}^{R}\sum_{q=1}^{Q} TCS_{jirq}^{s} \times SUP_{jirq} + \sum_{i=1}^{I}\sum_{c=1}^{C}\sum_{q=1}^{Q}\sum_{m=1}^{M} TCC_{icmq}^{s} \times CUS_{icmq} \qquad (25)$$

$$PC^{s}(purchasingcosts): \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{r=1}^{R}\sum_{q=1}^{Q} CM_{rj} \times SUP_{jirq} \qquad (26)$$

$$EC^s(eatablishingcosts): \sum_{i=1}^{I} ES_{in}^s \times S_{in} \tag{27}$$

$$SR^s(sellrevenue): \sum_{i=1}^{I}\sum_{c=1}^{C}\sum_{q=1}^{Q}\sum_{m=1}^{M} P_{ck}^s \times CUS_{icmq} \tag{28}$$

$$RD^s(rawmaterialdeterioration): \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{r=1}^{R}\sum_{q=1}^{Q} \alpha_{rq}^s \times SUP_{jirq} \tag{29}$$

$$PD^s(productdeterioration): \sum_{i=1}^{I}\sum_{c=1}^{C}\sum_{m=1}^{M}\sum_{q=1}^{Q} \beta_{mq}^s \times CUS_{icmq} \tag{30}$$

The above terms are defined to ease formulation of robust optimization. In the following, the discussed formulation is presented.

$$\text{Min } Z_1 = \textstyle\sum_s \rho_s (PC^s + TC^s + EC^s - SR^s) + \lambda_1 \sum_s \rho_s \left[(PC^s + TC^s + EC^s - SR^s) - \sum_{s'} \rho_{s'} \left(PC^{s'} + TC^{s'} + EC^{s'} - SR^{s'}\right) + 2\theta_1^s\right] + \vartheta \sum_{c,m,s} \rho_s \delta_{1mc}^s \tag{31}$$

$$\text{Min} Z_2 = \sum_s \rho_s (RD^s + PD^s) + \lambda_2 \sum_s \rho_s \left[(RD^s + PD^s) - \sum_{s'} \rho_{s'} \left(RD^{s'} + PD^{s'}\right) + 2\theta_2^s\right] + \vartheta \sum_{c,m,s} \rho_s \delta_{2mc}^s \tag{32}$$

s.t.

$$(PC^s + TC^s + EC^s - SR^s) - \sum_s \rho_s (PC^s + TC^s + EC^s - SR^s) + \theta_1^s \geq 0 \tag{33}$$

$$(RD^s + PD^s) - \sum_s \rho_s (RD^s + PD^s) + \theta_3^s \geq 0 \qquad \forall s \tag{34}$$

$$\sum_{i=1}^{I}\sum_{q=1}^{Q} CUS_{icmq} = D_{mc}^s + \delta_r^s \quad \forall\, m, c, s \tag{35}$$

Constraints (17–24).

where ρ_s is the probability of scenario s. The first and second terms in Eq. (31) and (32) are the mean value and variance of the objective functions, respectively. The last term in Eq. (31) and (32) measures the model robustness with respect to infeasibility associated with control constraint in Eq. (35) under scenario s. Constraints (33) and (34) are auxiliary constraints for linearization defined in Eq. (14). Eq. (35) is a control constraint that its violation is penalized in the objective function. If CUS_{icmq} is greater than D_{mc}^s, then $\delta_r^s = 0$, while as the amount of CUS_{icmq} is less than D_{mc}^s, then $\delta_r^s = \sum_{i,q} CUS_{icmq} - D_{mc}^s$.

Solution procedure

The LP-metric method is one of the well-known MCDM methods for solving multi-objective problems with conflicting objectives simultaneously. We use this method to solve the proposed multi-objective mixed-integer linear programming (MOMILP) model with two inconsistent objective functions. First, each objective function is solved separately and then a single objective is reformulated that aims at minimizing the summation of the normalized differences between each objective and the optimal values of them. In our presented model, it is assumed that two objective functions are named Z_1 and Z_2. Based on the LP-metric method, each objective function is solved once separately. Assume that the optimal values are Z_1^* and Z_2^*. Now, the LP-metric objective function can be formulated by:

$$\text{Min } Z_3 = \left[W \times \frac{Z_1 - Z_1^*}{Z_1^*} + (1 - W) \times \frac{Z_2 - Z_2^*}{Z_2^*}\right] \tag{36}$$

where $0 \leq W_i \leq 1$ is the relative weight of components of the objective function (36) given by the decision makers. The above single objective mixed-integer programming model can be efficiently solved by a linear programming solver.

Computational results

In this section, a hypothetical case is considered to illustrate the applicability of the proposed model to practical problems.

Case description

To describe the proposed model, the data are included in Tables 1 to 6. A number of products, customer zones, potential plants, raw materials, TAs and suppliers are 2, 5, 3, 5, 2 and 4, respectively. Table 1 shows the demand and selling price of each customer zone. The transportation cost of products and raw materials are depicted in Tables 2 and 3. Table 4 shows the purchasing cost of raw materials and maximum capacity of suppliers. Also, the consumption rate of raw materials in unit products is presented. Table 5 presents establishment costs and capacity level of plants. Finally, the deterioration rate of products and raw materials, processing time of products, available quantity and capacity of TAs are shown in Table 6.

Table 1

Demand of customer zones and sales price

Scenarios	Products	Demand					Sales Price				
		Customer zonec					Customer zonec				
		1	2	3	4	5	1	2	3	4	5
1	1	600	300	200	800	700	35000	30000	31000	32000	30000
	2	700	800	700	300	600	40000	41000	42000	42000	42000
2	1	700	500	400	800	700	36000	33000	32000	35000	33000
	2	800	900	900	600	800	43000	43000	43000	43000	43000
3	1	900	500	400	900	1000	37000	36000	34000	37000	34000
	2	800	1100	900	900	800	45000	45000	45000	44000	45000

Table 2

Transportation cost of products

Scenarios	Plants	Products	Customer zones									
			1		2		3		4		5	
			q1	q2	q1	q2	q1	q2	q1	q2	q1	q2
1	1	1	1375	1848	1485	1903	1199	1749	1232	1760	1353	10000
		2	1375	1848	1485	1903	1199	1749	1232	1760	1353	10000
	2	1	1474	1892	1595	1749	1419	10000	1485	1760	10000	1595
		2	1474	1892	1595	1749	1419	10000	1485	1760	10000	1595
	3	1	1342	1925	1309	1815	1485	1639	10000	1870	1210	10000
		2	1342	1925	1309	1815	1485	1639	10000	1870	1210	10000
2	1	1	1250	1680	1350	1730	1090	1590	1120	1600	1230	10000
		2	1250	1680	1350	1730	1090	1590	1120	1600	1230	10000
	2	1	1340	1720	1450	1590	1290	10000	1350	1600	10000	1450
		2	1340	1720	1450	1590	1290	10000	1350	1600	10000	1450
	3	1	1220	1750	1190	1650	1350	1490	10000	1700	1100	10000
		2	1220	1750	1190	1650	1350	1490	10000	1700	1100	10000
3	1	1	1125	1512	1215	1557	981	1431	1008	1440	1107	10000
		2	1125	1512	1215	1557	981	1431	1008	1440	1107	10000
	2	1	1206	1548	1305	1431	1161	10000	1215	1440	10000	1305
		2	1206	1548	1305	1431	1161	10000	1215	1440	10000	1305
	3	1	1098	1575	1071	1485	1215	1341	10000	1530	990	10000
		2	1098	1575	1071	1485	1215	1341	10000	1530	990	10000

Table 3

Transportation cost of raw materials

Scenarios	Suppliers	Raw materials	Plants					
			1		2		3	
			*q*1	*q*2	*q*1	*q*2	*q*1	*q*2
1	1	1	1023	2750	957	3025	1067	2904
		2	1023	2750	957	3025	1067	2904
		3	1023	2750	957	3025	1067	2904
		4	1023	2750	957	3025	1067	2904
		5	1023	2750	957	3025	1067	2904
	2	1	1232	10000	1023	2310	1177	1958
		2	1232	10000	1023	2310	1177	1958
		3	1232	10000	1023	2310	1177	1958
		4	1232	10000	1023	2310	1177	1958
		5	1232	10000	1023	2310	1177	1958
	3	1	1078	2442	1177	10000	1001	2530
		2	1078	2442	1177	10000	1001	2530
		3	1078	2442	1177	10000	1001	2530
		4	1078	2442	1177	10000	1001	2530
		5	1078	2442	1177	10000	1001	2530
	4	1	10000	2200	1221	3025	1023	2805
		2	10000	2200	1221	3025	1023	2805
		3	10000	2200	1221	3025	1023	2805
		4	10000	2200	1221	3025	1023	2805
		5	10000	2200	1221	3025	1023	2805

For scenarios 2 and 3, the estimations are multiplied by 0.9 and 1.1, respectively.

Table 4

Purchasing cost and capacity of suppliers for raw materials

Suppliers	Scenarios	Raw material (cost)					Raw material (capacity)				
		1	2	3	4	5	1	2	3	4	5
1	1	16500	11000	5390	3960	6160	9000	8000	7000	12000	9000
	2	15000	10000	4900	3600	5600					
	3	13500	9000	4410	3240	5040					
2	1	16335	11550	4950	3520	5390	8000	8000	9000	10000	9000
	2	14850	10500	4500	3200	4900					
	3	13365	9450	4050	2880	4410					
3	1	16412	11253	5500	3850	7150	8000	8000	8000	10000	6000
	2	14920	10230	5000	3500	6500					
	3	13428	9207	4500	3150	5850					
4	1	14520	8470	5720	3740	4950	8000	8000	8000	10000	9000
	2	13200	7700	5200	3400	4500					
	3	11880	6930	4680	3060	4050					
		Units of raw materials required for products									
Products		1	2	3	4	5					
1		3	1	2	1	2					
2		2	1	1	1	2					

Table 5

Establishment cost and capacity of plants

Scenarios	Plants					
	1		2		3	
	$n1$	$n2$	$n1$	$n2$	$n1$	$n2$
1	170000000	230000000	160000000	240000000	160000000	220000000
2	160000000	220000000	150000000	230000000	150000000	210000000
3	150000000	210000000	140000000	220000000	140000000	200000000
Capacity	2000	5000	2000	5000	2000	5000

Table 6

Deterioration rate of raw materials and products, capacity of TAs and process time

Products	Scenario s	TAs 1	TAs 2	Volume	Process time
1	1	0,26	0,10	0,9	0,7
	2	0,22	0,09		
	3	0,20	0,08		
2	1	0,15	0,08	1,0	0,6
	2	0,11	0,05		
	3	0,10	0,04		
Raw materials					
1	1	0,12	0,05	0,2	
	2	0,10	0,03		
	3	0,09	0,02		
2	1	0,14	0,05	0,5	
	2	0,12	0,04		
	3	0,11	0,03		
3	1	0,12	0,06	0,4	
	2	0,10	0,05		
	3	0,09	0,04		
4	1	0,15	0,06	0,3	
	2	0,13	0,05		
	3	0,12	0,04		
5	1	0,12	0,02	0,3	
	2	0,11	0,01		
	3	0,10	0,01		
Capacity					
Available vehicle		200	40		
Capacity of vehicle		1000	4000		

Based upon the above-mentioned data and taking into account the three scenarios, namely optimistic, realistic and pessimistic with associated probabilities of 0,2, 0,6 and 0,2 respectively, the model is optimally solved three times, each time with one of the objective functions Z_1, Z_2 and Z_3, in which the first intends to minimize the expected value in addition to the weighted variance and the infeasibility penalty of the total costs of the supply chain network, the second aims to minimize the expected value and weighted variance of the deterioration rates of products and raw materials, and the last, as the LP-metric objective function, is the best values of the above-mentioned objective functions (Z_1^* and Z_2^*).

Computational results

All computations are run using the branch-and-bound algorithm accessed via LINGO 11,0 on a PC Pentium IV-3 GHz and 4 GB RAM DDR under Windows 7. We illustrate the resulted solution, for which we rely on a set of the above-mentioned records in respect of the presented data. Tables 7 to 9 represent the output data characteristics by setting the relative weight (W) of each objective function component to 0,8 and 0,2 respectively, and the model robustness (ϑ) to 1000.

Table 7

Production quantity in planning period

Products	Plant1		Plant2		Plant3	
	$n1$	$n2$	$n1$	$n2$	$n1$	$n2$
1			2133			1567
2			800			3700

Table 8

Supplier selection and order allocation

Suppliers	Raw materials	Plants					
		1		2		3	
		$q1$	$q2$	$q1$	$q2$	$q1$	$q2$
1	1						
	2			200			
	3					2900	
	4						
	5						
2	1			7999			
	2						
	3			5066		3934	
	4			2933			5267
	5			5866			1534
3	1					4101	
	2						
	3						
	4						
	5						
4	1					8000	
	2			2733		5267	
	3						
	4						
	5					9000	

Table 9

Market share for each plant

Plants	Products	Customer zones									
		1		2		3		4		5	
		$q1$	$q2$	$q1$	$q2$	$q1$	$q2$	$q1$	$q2$	$q1$	$q2$
1	1										
	2										
2	1				233				900		1000
	2										800
3	1		900		267		400				
	2		800		1100		900		900		

Table 7 presents the set of the selected plants with their relative capacity level and the quantity that should be produced during the planning period. As shown, plant 2 with a capacity of level 1 (i.e., 2000) and plant 2 with a capacity of level 2 (i.e., 5000) are established. Blank cells are equal to 0 in this table and other similar data. The selected suppliers and allocated orders are provided in Table 8. Supplier 2 has the highest share to supply raw materials for these two plants, while supplier 1 has the least share. Table 9 presents the market share of each plant regarding the customer zones.

As stated before, to present the importance of considering three total costs and deterioration rates simultaneously, three following models are extracted for a further analysis.

(1) Model 1 consists of the total costs of the supply chain (Z_1) subject to the relevant constraints.

(2) Model 2 consists of the sum of the deterioration rates considering TAs (Z_2) subject to the relevant constraints.

(3) LP-metric model, which is a combination of Model1 and Model2, is calculated by (Z_3) subject to the relevant constraints.

Thus, changing values of W, different solutions for multi-objective optimization is obtained. Figure 2 illustrates the Pareto-optimal frontier for different values of W from 0 to 1. It should be noted when $W = 1$, the LP-metric model is equivalent to Model 1, and when $W = 0$, the LP-metric model is equivalent to Model 2.

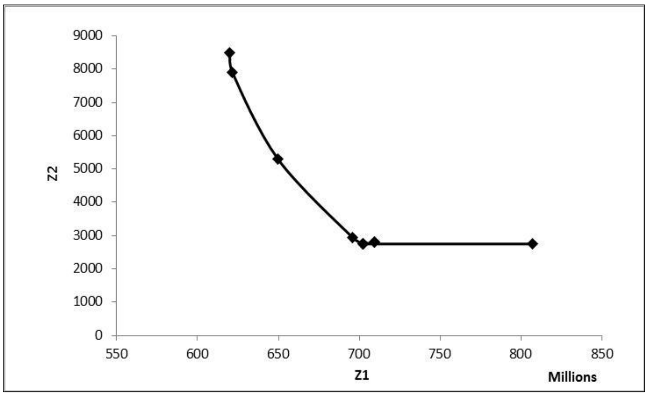

Figure 2. Trade-off between Z_1 and Z_2

Considering just one objective may sacrifice the other. Comparison of the results shows that the LP-metric model makes a trade-off between these two objective functions and it is up to the decision maker to select the suitable W from his/her prospective. In Figs. 3 and 4, we discuss the effect to the analysis resulting from changing the value of λ. The following analysis is based on the presented numerical example. The expected cost increases as the value of λ_1 is constant and the value of λ_2 is increased (see Figure 3). A larger value of λ_2 represents a greater importance of deterioration rate variability at the possible expense of the increase of the expected cost. Therefore, the decision maker can get a lower deterioration rate but a higher expected cost results. Comparison of Figs. 3 and 4 also shows the trade-off between the deterioration rate and the expected cost. We can see that with the same value of λ, the lower expected deterioration rate is achieved by increasing the expected cost.

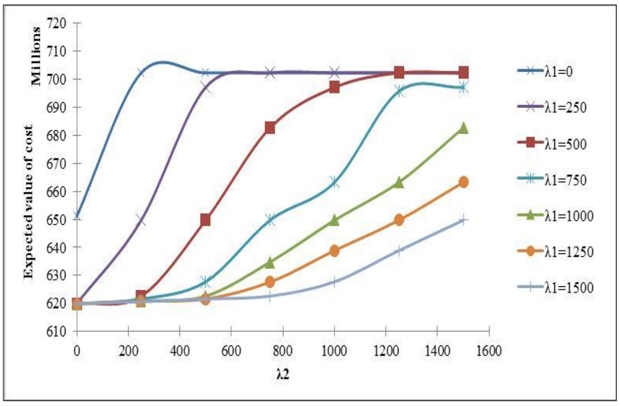

Figure 3. Trade-off between the solution robustness and Z_1

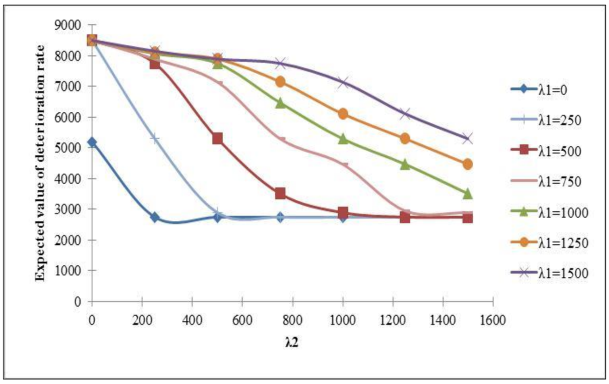

Figure 4. Tradeoff between the solution robustness and Z_2

It is worth noting that the analysis for a specific example provides some guidance for deciding values of λ for this numerical example. In real cases, some trial experiments can help the decision maker in determining the value of λ.

Conclusion

In this paper, we considered the facility location-allocation problem under the stochastic customer demand and cost parameters to design a supply chain. To handle the uncertainty, we adopted the scenario approach. We integrated the design of a supply chain and production planning for members of the supply chain. A robust optimization formulation was developed for two conflicting mixed-integer linear programming. The first objective was to minimize the costs of supply chain while the second objective intended to minimize the deterioration rate of transportation alternatives. The LP-metric method then was utilized to solve the problem and achieve compromising solution between two objectives. The practicability of the model was demonstrated using an experiment study. The results indicated that the proposed model could provide a promising result to design an efficient supply chain. Selecting optimal location and capacity of the sites, this study also provided selection of the best suppliers and distributors and their allocated order in the supply chain. Moreover, transportation alternatives between the members of the supply chain were selected with regard to minimize costs and failure rates.

The limitations of the proposed model are as follows:

- Meta-heuristic algorithms are needed to be developed to solve the model for large-scale problems.
- In the proposed model, possibility of shortage and surplus are not considered.
- The presented model does not consider planning periods. The optimal solution is based on the data of the current period.

In terms of future work, some other issues can be considered to extend the proposed model, such as scheduling issues. Furthermore, using other approaches to incorporating uncertainty (e.g., fuzzy programming) seems to be interesting. Other extensions for this research work can be considered global issues, such as taxes, tariffs and exchange rates in multiple periods. Employing meta-heuristic algorithms to solve problems in large sizes can also be useful.

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