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An Empirical Investigation of the Nexus between Sovereign Bond Yields and Stock Market Returns – A Multiscale Approach

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This paper studies comovement between changes in sovereign bond yields and stock market returns for ten Eurozone countries (Austria, Belgium, Finland, France, Ireland, Italy, Germany, Netherlands, Portugal, and Spain) in the period from January 2000 – end of August 2011, applying the maximal overlap discrete wavelet transform (MODWT) variance and correlation tools.

We found that the for all but one country (namely Portugal) the stock return volatility is higher than the volatility of sovereign bond yield changes across all wavelet scales. The highest volatility in stock indices' returns and bond yield changes is captured by lower level MODWT wavelet coefficients (i.e., level 1 ($j = 1$) MODWT wavelet coefficients, associated with dynamics over 2 to 4 days and level 2 ($j = 2$) MODWT wavelet coefficients associated with dynamics over 4 to 8 days). A practical implication stemming from this finding is that investors with short investment horizons that want to efficiently manage the risk of their portfolio investments have to respond to every fluctuation in realized returns or bond yield changes, while for an investor with a much longer horizon, the need to do this is reduces, as long-run risk is significantly smaller as indicated by the variance.

The results of the paper show also that the estimated wavelet correlation changes with the time scale and is mostly positive for all countries but Portugal. The statistical evidence against hypothesis of no multiscale dependence yet is weak. Thus we cannot statistically claim that the wavelet coefficients at higher scales are significantly different (either higher or lower) that those at lower scales.

For the financial markets of Portugal and Germany we proved that the dependence between stock and sovereign bond market dynamics may not be just a multiscale phenomenon, but may also exhibits time dynamics across scales. The rolling-window wavelet correlation estimates show that at all scales correlation turned negative for Portugal at the start of 2010 – this is when the Eurozone debt crisis started which also hit hard the Portugal sovereign bond market – thus probably causing the ″average″ wavelet correlation for Portugal for the whole observed period to be negative.

Keywords: *Eurozone, stock markets, sovereign bonds, wavelet analysis, Eurozone crisis.*

Introduction

The comovement of returns between different asset classes is of major importance for risk management and portfolio management. The (co)movements of stock prices and sovereign bond yields (or returns) are of particular interest to individual and institutional financial investors as stocks and bonds are the two prime investment asset classes for majority of investors. Changes in comovement patterns call for an adjustment of portfolios (Savva & Aslanidis, 2010). Namely, to achieve optimal balance between risk and return many investors use a tactical asset allocation strategy and reallocate their investments when their expectations about risk and returns of particular assets in their portfolios change.

The co-movement between stock prices and sovereign bond yields (or returns) has received considerable attention in the literature. (Campbell & Ammer, 1993; Stivers & Sun, 2002) argue that stock and bond prices should move in the same direction (thus implying that stock market returns and the changes of sovereign bond yields should move in the opposite direction) due to a common discount rate effect or common movements in future expected returns (Stivers *et al.*, 2002; Dajcman, 2012a). More recent studies, investigating the time-varying comovement, however, reported periods of positive correlations between stock market returns and the dynamics of sovereign bond yields (see Dajcman, 2012a). According to (Campbell & Ammer, 1993), theoretically, positive correlations between stock market returns and the dynamics of sovereign bond yields can be explained by variations in expected inflation, since increases in inflation should negatively affect bonds, but not stocks. Financial market dynamics and changes in market participants' assessments about risk may also have an important impact on the relationship between stock market returns and the dynamics of sovereign bond yields (Gulko, 2002; Baur & Lucey, 2009). In periods of financial market turbulence, the ″flight-to-quality″ phenomena might be observed (Gulko, 2002).

The comovement of stock price and bond yield changes may not only be time-varying but also dependent on the investment horizon of investors who may hold stocks and/or bond for a shorter or longer time periods. (Candelon *et al.*, 2008) argued that comovement analysis should consider the distinction between short- and long-term investors. From the point of view of portfolio diversification, short-term investors are more interested in the comovements of asset classes at higher frequencies (short-term movements), while long-term investors focus on lower frequencies comovements (Dajcman, 2012b). Another reason, why the relationship between dynamics of stock prices and bond yields may be a multiscale phenomenon is that the long-run relationship between the two asset classes may be obscured in the short run by financial market volatility or short-term noise which might derive from investors trading in order to rebalance their portfolio or to satisfy their immediate (unexpected) consumption needs (Kim & In, 2007; Harrison & Zhang, 1999).

To study stock and bond yield changes on a multiscale basis one has to resort to a time-frequency domain analysis to obtain insight about comovements at particular time-frequency (scale) level (Lee, 2004; Pakko, 2004; Rua & Nunes, 2009). In such a context, with both the time horizon of economic decisions and the strength and direction of economic relationships between variables that may differ according to the time scale of the analysis, a useful analytical tool may be represented by wavelet analysis (Pinho & Madaleno, 2009).

There are several studies using wavelet tools (variance, wavelet correlation, and wavelet cross-correlation) to investigate interdependence between economic (or financial) variables on different time scales (Kim & In, 2005; In *et al.*, 2008; In & Kim, 2006; Kim & In, 2007; Gencay *et al.*, 2001a; Gallegati, 2008; Conlon *et al.*, 2009; Zhou 2011). These studies confirm that interdependence between financial (or economic) variables is scale dependent, exhibiting different correlation structures at different time scales.

To our knowledge there is only one study (Kim & In, 2007) examining the multiscale relationship between the stock prices and sovereign bond yield changes applying wavelet analysis. (Kim & In, 2007), examining the relationship between stock prices and bond yields in the G7 countries, found that the correlation between changes in stock prices and bond yields can differ from one country to another and can also depend on the time scale.

In this paper we study comovement between changes in sovereign bond yields and stock market returns for ten Eurozone countries (Austria, Belgium, Finland, France, Ireland, Italy, Germany, Netherlands, Portugal, and Spain) in the period from January 2000 – end of August 2011 (different starting periods are used for different countries due to data availability), applying the maximal overlap discrete wavelet transform (MODWT) variance and correlation tools. Unlike the study of (Kim & In, 2007), who use monthly data, we apply the daily data of stock market returns and bond yield changes and also study the dynamics of comovement on a scale-by-scale basis whereas in the study of (Kim & In, 2007) the analysis is static.

Methodology

Wavelet Analysis

The wavelet analysis[1] involves the projection of the original series onto a sequence of basis functions, known as wavelets. There are two basic wavelet functions: the father wavelet (called also a scaling function), ϕ, and the mother wavelet (called also a wavelet function), ψ, which can be scaled and translated to form a basis for the Hilbert space $L^2(\mathbb{R})$ of square integrable functions. The father and mother wavelets are defined as:

$$\phi_{j,k}(t) = 2^{-\frac{j}{2}}\phi(2^{-j}t - k) \tag{1a}$$

and

$$\psi_{j,k}(t) = 2^{-\frac{j}{2}}\psi(2^{-j}t - k) \tag{1b}$$

where $j = 1, \ldots, J$ is the scaling parameter in a J-level decomposition and k is a translation parameter ($j, k \in \mathbb{Z}$). The long term trend of the time series is captured by the father wavelet, which integrates to 1, while the mother wavelet, which integrates to 0, describes fluctuations from the trend. The continuous wavelet transform of a square integrable time series $X(t)$ consists of the scaling, $\alpha_{J,k}$, and wavelet coefficients, $\beta_{j,k}$, (Craigmile & Percival, 2002):

$$\alpha_{J,k}(t) = \int \phi_{J,k}(t)X(t) \tag{2a}$$

and

$$\beta_{j,k}(t) = \int \psi_{j,k}(t)X(t). \tag{2b}$$

It is possible to reconstruct $X(t)$ from these transform coefficients using:

$$X(t) = \sum_k \alpha_{J,k}\phi_{J,k}(t) + \sum_k \beta_{J,k}\psi_{J,k}(t) + \sum_k \beta_{J-1,k}\psi_{J-1,k}(t) + \cdots + \sum_k \beta_{1,k}\psi_{1,k}(t) \tag{3}$$

In practice we observe a time series at finite number of regularly spaced times, so we can make use of a discrete wavelet transform (DWT)[2] or a maximal overlap discrete wavelet transform (MODWT). The MODWT is a linear filtering operation that transforms a series into coefficients related to variations over a set of scales.

MODWT Wavelet Analysis

Let[3] $\boldsymbol{X}$ be a N dimensional vector whose elements are the real-valued time series $\{X_t : t = 0, \ldots, N-1\}$, where the sample size N is any positive integer. For any positive integer, J_0, the level J_0 MODWT of $\boldsymbol{X}$ is a transform consisting of the $J_0 + 1$ vectors $\widetilde{\boldsymbol{W}}_1, \ldots, \widetilde{\boldsymbol{W}}_{J_0}$ and $\widetilde{\boldsymbol{V}}_{J_0}$, all of which have the dimension N. The vector $\widetilde{\boldsymbol{W}}_J$ contains the MODWT wavelet coefficients associated with changes at scale $\tau_j = 2^{j-1}$ (for $j = 1, \ldots, J_0$)[4], while $\widetilde{\boldsymbol{V}}_{J_0}$ contains MODWT scaling coefficients associated with averages at scale $\lambda_{J_0} = 2^{J_0}$.

Based upon a definition of MODWT coefficients we can write (Percival & Walden, 2000):

$$\widetilde{\boldsymbol{W}}_J = \widetilde{\mathcal{W}}_J\boldsymbol{X} \tag{4a}$$

[1] In description of the wavelet analysis we follow (Dajcman *et al.*, 2012; Dajcman, 2013).

[2] For a presentation of DWT, please refer to (Percival & Walden, 2000).

[3] Concepts and notations as in (Percival & Walden, 2000) are used.

[4] (Percival & Walden, 2000) denote scales of MODWT obtained wavelet coefficients with a letter τ and scales of scaling coefficients with λ. We use the same notations.

$$\widetilde{\boldsymbol{V}}_{J_0} = \tilde{\mathcal{V}}_{J_0}\boldsymbol{X} \quad (4b)$$

where $\widetilde{\mathcal{W}}_j$ and $\tilde{\mathcal{V}}_{J_0}$ are $N \times N$ matrices.

By definition, the elements of $\widetilde{\boldsymbol{W}}_j$ and $\widetilde{\boldsymbol{V}}_{J_0}$ are outputs obtained by filtering $\boldsymbol{X}$, namely:

$$\widetilde{W}_{j,t} = \sum_{l=0}^{L_j-1} \tilde{h}_{j,l} X_{t-l \, modN} \quad (5a)$$

$$\tilde{V}_{j,t} = \sum_{l=0}^{L_j-1} \tilde{g}_{j,l} X_{t-l \, modN}, \quad (5b)$$

for $t = 0, \dots, N-1$, and where $\tilde{h}_{j,l}$ and $\tilde{g}_{j,l}$ are the *j*th-level MODWT wavelet and scaling filters defined in terms of the *j*th-level equivalent wavelet and scaling filters $\{h_{j,l}\}$ and $\{g_{j,l}\}$ for a discrete wavelet transform (DWT):

$$\tilde{h}_{j,l} = h_{j,l}/2^{j/2} \quad (6a)$$

and

$$\tilde{g}_{j,l} = g_{j,l}/2^{j/2} \quad (6b)$$

Each of the MODWT wavelet filters has width $L_j \equiv (2^j - 1)(L-1) + 1$ and can be calculated once basic MODWT wavelet filter $\tilde{h}_{1,l} \equiv \tilde{h}_l = h_l/\sqrt{2}$ and MODWT scaling filter $\tilde{g}_{1,l} \equiv \tilde{g}_l = (-1)^{l+1}\tilde{h}_{L-1-l}$ have been specified.

A DWT filter $\{h_l : l = 0, \dots, L-1\}$ of even width L is called a wavelet filter if:

$$\sum_{l=0}^{L-1} h_l = 0 \quad (7a)$$

and

$$\sum_{l=0}^{L-1} h_l h_{l+2n} = \begin{cases} 1, & \text{if } n = 0 \\ 0, & \text{if } n \text{ is nonzero integer} \end{cases} \quad (7b)$$

A DWT scaling filter is defined in terms of the wavelet filter:

$$g_l \equiv (-1)^{l+1} h_{L-1-l} \quad (8a)$$

and satisfies conditions:

1) $\sum_l g_l = \sqrt{2}$ (8b)

2) $\sum_{l=0}^{L-1} g_l g_{l+2n} = \begin{cases} 1, & \text{if } n = 0 \\ 0, & if\ n \text{ is nonzero integer} \end{cases}$ (8c)

The MODWT treats the series as if it were periodic, whereby the unobserved samples of the real-valued time series $X_{-1}, X_{-2}, \dots X_{-N}$ are assigned the observed values at $X_{N-1}, X_{N-2}, \dots X_0$. The MODWT coefficients are thus given by:

$$\widetilde{W}_{j,t} = \sum_{l=0}^{N-1} \tilde{h}^{\circ}_{j,l} X_{t-l \, modN} \quad (9a)$$

$$\tilde{V}_{j,t} = \sum_{l=0}^{N-1} \tilde{g}^{\circ}_{j,l} X_{t-l \, modN} \quad (9b)$$

for $t = 0, \dots, N-1$; $\tilde{h}^{\circ}_{j,l}$ and $\tilde{g}^{\circ}_{j,l}$ are periodization of $\tilde{h}_{j,l}$ and $\tilde{g}_{j,l}$ to circular filters of length N.

This periodic extension of the time series is known as analyzing $\{X_t\}$ using ″circular boundary condition″ (Percival & Walden, 2000; Cornish *et al.*, 2006). There are $L_j - 1$ wavelet and scaling coefficients that are influenced by the extension (″ the boundary coefficients″). Since L_j increases with j, the number of boundary coefficients increases with scale. Exclusion of boundary coefficients in the wavelet variance, wavelet correlation and covariance provides unbiased estimates (Cornish *et al.*, 2006).

Wavelet Variance and Wavelet Correlation

One of the important uses of the MODWT is to decompose the sample variance of a time series on a scale-by-scale basis. Since the MODWT is energy conserving (Percival & Mojfeld, 1997):

$$\|\boldsymbol{X}\|^2 = \sum_{j=1}^{J_0} \left\|\widetilde{\boldsymbol{W}}_j\right\|^2 + \left\|\widetilde{\boldsymbol{V}}_{J_0}\right\|^2 \quad (10)$$

A scale-dependent analysis of variance from the wavelet and scaling coefficients can be derived (Cornish *et al.*, 2006):

$$\hat{\upsilon}_X^2 = \|\boldsymbol{X}\|^2 - \bar{X}^2 = \frac{1}{N}\sum_{j=1}^{J_0} \left\|\widetilde{\boldsymbol{W}}_j\right\|^2 + \frac{1}{N}\left\|\widetilde{\boldsymbol{V}}_{J_0}\right\|^2 - \bar{X}^2 \quad (11)$$

Wavelet variance is defined for stationary and nonstationary processes with stationary backward differences (Percival & Walden, 2000). Let $\{X_t : t = \cdots, -1, 0, 1, \dots\}$ be a discrete parameter real-valued stochastic process whose *d*th-order differencing (d may take any nonnegative integer values) gives a stationary process:

$$Y_t \equiv (1-B)^d X_t \equiv \sum_{k=0}^{d} \binom{d}{k} (-1)^k X_{t-k}, \quad (12)$$

with spectral density function (SDF) $S_Y(\cdot)$ and mean μ_Y (which may not be zero). Let $S_X(\cdot)$ represent the SDF for $\{X_t\}$, for which $S_X(f) = S_Y(f)/D^d(f)$, where $D(f) \equiv 4sin^2(\pi f)$ (if $\{X_t\}$ is a nonstationary process, then this relationship between $S_X(\cdot)$ and $S_Y(\cdot)$ represents definition for $S_X(\cdot)$). Filtering $\{X_t\}$ with a MODWT Daubechies wavelet filter $\{\tilde{h}_{j,l}\}$ of width $L \geq 2d$, a stationary process of *j*th-level MODWT wavelet is obtained:

$$\bar{W}_{j,t} \equiv \sum_{l=0}^{L_j-1} \tilde{h}_{j,l} X_{t-l}, \quad t = \cdots, -1, 0, 1, \dots, \quad (13)$$

where $\bar{W}_{j,t}$ is a stochastic process obtained by filtering $\{X_t\}$ with the MODWT wavelet filter $\{\tilde{h}_{j.l}\}$ and $L_j \equiv (2^j - 1)(L-1) + 1$

Let us suppose that we are given a time series, which is realization of one segment (segment with values $X_0, \dots, X_{N-1}$) of the process $\{X_t\}$. Under condition $M_j \equiv N - L_j + 1 > 0$ (i.e. by considering only the non-boundary wavelet coefficients, obtained by filtering stationary time series with MODWT) and that either $L > 2d$ or $\mu_X = 0$ (realization of either of these two conditions implies $E\{\bar{W}_{j,t}\} = 0$ and therefore $\upsilon_X^2(\tau_j) = E\{\bar{W}_{j,t}^2\}$), an unbiased estimator of wavelet variance of scale τ_j ($\upsilon_X^2(\tau_j)$) is given by (Percival & Walden, 2000):

$$\hat{\upsilon}_X^2(\tau_j) = \frac{1}{M_j} \sum_{t=L_j-1}^{N-1} \widetilde{W}_{j,t}^2, \quad (14)$$

where $\{\widetilde{W}_{j,t}\}$ are the *j*th-level MODWT wavelet coefficients for time series ($\widetilde{W}_{j,t} \equiv \sum_{l=0}^{L_j-1} \tilde{h}_{j,l} X_{t-l \, modN}, t = 0, 1, \dots, N-1$).

The estimator $\hat{\upsilon}_X^2(\tau_j)$ is a random variable and under assumption that $\{\bar{W}_{j,t}\}$ is a Gaussian stationary process with mean zero and SDF $S_Y(\cdot)$, $S_j^2(f) > 0$ almost everywhere and $A_j \equiv \int_{-\frac{1}{2}}^{\frac{1}{2}} S_j^2(f)df < \infty$ (where $S_j(f)$is the spectrum of the wavelet coefficients of scale j), then the estimator $\hat{\upsilon}_X^2(\tau_j)$ of equation (14) is asymptotically normally distributed with the random interval:

$$\left[\hat{\upsilon}_X^2(\tau_j) - \Phi^{-1}(1-p)(\frac{2A_j}{M_j})^{\frac{1}{2}}, \hat{\upsilon}_X^2(\tau_j) + \Phi^{-1}(1-p)(\frac{2A_j}{M_j})^{\frac{1}{2}}\right] \quad (15)$$

The interval captures the true variance and corresponds to a $100(1-2p)\%$ confidence interval for $\upsilon_X^2(\tau_j)$ (see Percival 1995; Serroukh *et al.*, 2000). However the lower confidence interval given by the equation (15) can be negative. As $\hat{\upsilon}_X^2(\tau_j)$ is proportional to the sum of squares of M_j zero mean Gaussian random variables (each having the same variance), $\hat{\upsilon}_X^2(\tau_j)$ could be renormalized to obey a chi-square distribution with η degrees of freedom ($\eta = max\left\{\frac{M_j}{2^j}, 1\right\}$), from which strictly positive confidence intervals would follow. Using the approximation of (Priestley, 1981; Percival & Walden, 2000) show that an approximate $100(1-2p)\%$ confidence interval for $\hat{\upsilon}_X^2(\tau_j)$ is given by:

$$\left[\frac{\eta\hat{\upsilon}_X^2(\tau_j)}{Q_\eta(1-p)}, \frac{\eta\hat{\upsilon}_X^2(\tau_j)}{Q_\eta(p)}\right], \quad (16)$$

where $Q_\eta(p)$ is the $p \times 100\%$ percentage point for the χ_η^2 distribution, i.e. $P\left|\chi_\eta^2 \leq Q_\eta(p)\right| = p$.

Given two stationary processes $\{X_t\}$ and $\{Y_t\}$, whose *j*th-level MODWT wavelet coefficients are $\{\overline{W}_{X,j,t}\}$ and $\{\overline{W}_{Y,j,t}\}$, an unbiased covariance estimator $\hat{\upsilon}_{XY}(\tau_j)$ is given by (Percival, 1995):

$$\hat{\upsilon}_{X,Y}(\tau_j) = \frac{1}{M_j}\sum_{t=L_j-1}^{N-1} \widetilde{W}_{j,t}^{(X)} \widetilde{W}_{j,t}^{(Y)} = cov\{\overline{W}_{X,j,t}, \overline{W}_{Y,j,t}\} \quad (17)$$

where $M_j \equiv N - L_j + 1 > 0$ is the number of non-boundary coefficients at the *j*th-level.

The MODWT correlation estimator for scale τ_j is obtained by making use of the wavelet covariance and the square root of wavelet variances:

$$\hat{\rho}_{X,Y}(\tau_j) = \frac{\hat{\upsilon}_{X,Y}(\tau_j)}{\hat{\upsilon}_X(\tau_j)\hat{\upsilon}_Y(\tau_j)}, \quad (18)$$

where $\left|\hat{\rho}_{XY}(\tau_j)\right| \leq 1$. The wavelet correlation is analogous to its Fourier equivalent, the complex coherency (Gencay *et al.*, 2002).

Given the inherent non-normality of the wavelet correlation coefficients for small sample sizes, a nonlinear transformation is sometimes required in order to construct a confidence interval (Gencay *et al.*, 2001a). Calculation of confidence intervals of correlation coefficient is based on (Percival, 1995; Percival & Walden, 2000). The random interval:

$$\left[\tanh\left\{h\left[\rho_{XY}(\tau_j)\right] - \frac{\Phi^{-1}(1-p)}{\sqrt{N_j-3}}\right\}, \tanh\left\{h\left[\rho_{XY}(\tau_j)\right] + \frac{\Phi^{-1}(1-p)}{\sqrt{N_j-3}}\right\}\right] \quad (19)$$

Captures the true wavelet correlation and provides an approximate $100(1-2p)\%$ confidence interval. Function $h(p) = tanh^{-1}(\widehat{\rho})$ defines the Fisher's z-transformation. N_j is the number of wavelet coefficients obtained by *j*th-level of DWT and not by the MODWT transformation. This is because the Fisher's z-transformation assumes uncorrelated observations and the DWT is known to approximately de-correlate a wide range of power-law processes (Ranta, 2010). The approximate confidence interval for the estimated wavelet correlation does not utilize any information regarding the distribution of the wavelet coefficients. Hence, no adjustment is made regarding the distribution of the incoming wavelet coefficients; they may be Gaussian or non-Gaussian (Gencay *et al.*, 2001a).

Data and Empirical Results

Comovement between stock market returns and the dynamics of sovereign bond yields were calculated for ten Eurozone countries, listed in Table 1. The stock indices returns ($r_{1,t}$) were calculated as the differences in the logarithms of the daily closing prices of indices ($r_{1,t} = ln(P_t) - ln(P_{t-1})$, where P is an index value). The following stock market indices were considered: ATX for Austria, BEL20 for Belgium, HEX25 for Finland, CAC40 for France, ISEQ for Ireland, FTSEMIB for Italy, DAX for Germany, AEX for Netherlands, PSI20 for Portugal, and IBEX35 for Spain. Yields (y) of the central-government bonds (bullet issues) with 10 year maturity dates were considered. Changes (i.e., dynamics) of sovereign bond yields ($r_{2,t}$) were calculated as $ln(y_t) - ln(y_{t-1})$ as suggested by (Durre & Giot, 2005) or by Kim and In (2007).[5] The period of observation is different for individual countries due to data availability. Days where there was no concurrent trading in the national stock and bond market were left out. The data source for the index prices was Yahoo! Finance and for the central government bond yields the Denmark's central bank. Table 1 presents some descriptive statistics of the data.

The stationarity of time series was tested by the Augmented Dickey-Fuller (ADF) test, Phillips-Perron (PP) and Kwiatkowski-Phillips-Schmidt-Shin (KPSS) tests (Table 2). As the trend was not significant in any of test models, only results for the models with a constant are presented. The rejection of the null hypothesis of ADF and PP test and non-rejection of the null hypothesis of KPSS test leads to the conclusion of no unit-root in the time series. The alternative hypothesis, of stationary time series, can be accepted.

[5] Alternatively, changes (i.e., dynamics) of sovereign bond yields can also be calculated as $ln(1+y_t) - ln(1+y_{t-1})$, as also suggested by (Durre & Giot, 2005). The results of the dynamical conditional correlation analysis also for the later definition of bond yield changes however differ only marginally and lead to the same conclusions as with the chosen definition of sovereign bond yield changes.

Table 1

Descriptive statistics for time series of stock index returns and bond yield changes

		Period of obser.	Min	Max	Mean	Std. deviation	Skewness	Kurtosis	Jacque-Bera statistics
Austria	-Stock index -Bonds	3 Jan. 2000 – 24 Aug. 2011	-0,1025 -0,05899	0,12020 0,06851	0,00020 -0,00025	0,01507 0,01061	-0,3367 0,2562	11,0056 5,5977	7,726.34*** 839,23***
Belgium	-Stock index -Bonds	3 Jan. 2000 – 24 Aug. 2011	-0,08319 -0,04134	0,09334 0,06603	-0,00014 -0,00013	0,01354 0,01027	0,0830 0,3300	9,1672 5,6127	4,678.43*** 892,63***
Finland	-Stock index -Bonds	3 Sep. 2001 – 24 Aug. 2011	-0,08905 -0,06943	0,09286 0,05476	0,00013 -0,00026	0,01479 0,01152	-0,0107 0,0429	6,8725 4,7434	1,559.00*** 316,74***
France	-Stock index -Bonds	3 Jan. 2000 – 24 Aug. 2011	-0,09472 -0,04921	0,1059 0,06003	-0,00021 -0,00023	0,01566 0,0106	0,0340 0.1247	7,8903 4,7838	2,942.10*** 399,03***
Ireland	-Stock index -Bonds	4 Jan. 2000 – 24 Aug. 2011	-0,1396 -0,215	0,09733 0,08457	-0,00024 0,00016	0,01507 0,01246	-0,5799 -1,2966	10,4813 37,7476	6,987.67*** 148,021.78***
Italy	-Stock index -Bonds	3 Nov. 2003 – 24 Aug. 2011	-0,08599 -0,1406	0,1087 0,07492	-0,00029 -0,00006	0,01458 0,01052	-0,0389 -1,0261	11,3326 22,3510	5,711.31*** 31,145.98***
Germany	-Stock index -Bonds	3 Jan. 2000 – 24 Aug. 2011	-0,07433 -0,07596	0,108 0,07637	-0,00006 -0,00031	0,01628 0,01225	0,0414 0,0153	7,1594 6,8570	2,122.30*** 1,824.34***
Netherlands	-Stock index -Bonds	3 Jan. 2000 – 24 Aug. 2011	-0,0959 -0,07631	0,1003 0,05518	-0,00030 -0,00028	0,01597 0,01134	-0,0482 0,0304	8,8213 5,4588	4,169.30*** 744,08***
Portugal	-Stock index -Bonds	11 Feb. 2005 – 24 Aug. 2011	-0,1038 -0,3001	0,102 0,1455	-0,00017 0,00085	0,01262 0,0167	-0,0393 -2,9192	13,7228 76,1499	7,933.89*** 371,564.56***
Spain	-Stock index -Bonds	3 Jan. 2000 – 24 Aug. 2011	-0,09586 -0,1582	0,1348 0,06068	-0,00011 -0,00004	0,01542 0,01112	0,1342 -1,2159	8,6761 23,0986	3,929.97*** 49,884.47***

*Note: *** indicates that the null hypothesis (of normal distribution) is rejected at the 1 % significance.*

Table 2

Results of the test of time series stationarity

		KPSS test (a constant)	PP test(a constant)	ADF test (a constant)
Austria	-Stock index -Bonds	0,352* (1) 0,068 (9)	-50,507***(L=3) -51,427***(L=10)	-50,507***(L=3) -51,471***(L=0)
Belgium	-Stock index -Bonds	0,147 (7) 0,082 (8)	-50,390***(L=5) -51,418***(L=10)	-50,483***(L=0) -51,487***(0)
Finland	-Stock index -Bonds	0,178 (1) 0,085 (4)	-48,570*** (L=2) -49,176*** (L=4)	-48,570*** (L=0) -49,181*** (L=0)
France	-Stock index -Bonds	0,113 (10) 0,051 (6)	-56,768*** (L=9) -53,715*** (L=6)	-26,622*** (L=4) -53,718*** (L=0)
Ireland	-Stock index -Bonds	0,335 (3) 0,416* (3)	-51,587*** (L=5) -45,733*** (L=8)	-51,604*** (L=0) -45,856*** (L=0)
Italy	-Stock index -Bonds	0,409* (4) 0,117 (18)	-44,024*** (L=4) -39,492*** (L=21)	-44,025*** (L=0) -31,707*** (L=1)
Germany	-Stock index -Bonds	0,147 (1) 0,085 (14)	-55,571*** (L=1) -52,729*** (L=15)	-55,571*** (L=0) -52,749*** (L=0)
Netherlands	-Stock index -Bonds	0,111 (9) 0,071 (11)	-55,550*** (L=9) -53,262*** (L=12)	-29,953*** (L=4) -53,274*** (L=0)
Portugal	-Stock index -Bonds	0,295 (4) 0,325 (8)	-38,663*** (L=6) -35,206*** (L=5)	-38,688*** (L=0) -28,624*** (L=1)
Spain	-Stock index -Bonds	0,144 (7) 0,169 (10)	-54,957*** (L=7) -49,213*** (L=13)	-54,780*** (L=0) -38,601*** (L=1)

*Notes: All test of stationarity of time series were performed for a model with a constant and a model with a constant plus trend. As no trend was significant (at the 5 % level) in any of the test model, only results for the models with a constant are reported. For KPSS and PP tests the Bartlet Kernel estimation method is used with Newey-West automatic bandwidth selection. Optimal bandwidth is indicated in parenthesis under the statistics. For ADF test the optimal number of lags to be included (L) for the test was selected by SIC criteria (30 was a maximum lag). Exceeded critical values for rejection of null hypothesis are marked by *** (1 % significance level), ** (5 % significance level) and * (10 % significance level).*
Source: Own calculations.

To obtain MODWT estimates of variance and correlation, a Daubechies least asymmetric filter with a wavelet filter length of 8 (LA8) was chosen. This is a common wavelet filter used in empirical studies on financial market interdependencies (Gencay *et al.*, 2001b; Gallegati 2005; Ranta 2010, Dajcman *et al.*, 2012). The maximum level of MODWT is 6 ($J_0 = 6$) to achieve an optimal balance between sample size and the length of the filter. Scale τ_1 (or scale 1, as $\tau_1 = 2^{1-1} = 1$) measures the dynamics of returns over 2 to 4 days[1]; scale τ_2 (scale 2, as $\tau_2 = 2^{2-1} = 2$) over 4 to 8 days; scale τ_3 (scale 4, as $\tau_3 = 2^{3-1} = 4$) over 8 to 16 days; scale τ_4 (scale 8, $\tau_4 = 2^{4-1} = 8$) over 16 to 32 days; scale τ_5 (or scale 16) over 32 to 64 days; and scale τ_6 (or scale 32) over 64 to 128 days. To obtain unbiased estimates of cross-correlation, only non-boundary wavelet coefficients were considered.

The results of MODWT variance decomposition analysis are presented in Figure 1. Notably, for all but one country (namely Portugal) the stock return volatility is higher than the volatility of sovereign bond yield changes across all wavelet scales.

[1] The jth-level MODWT coefficients are associated with the frequencies in the interval $[\frac{1}{2^{j+1}}, \frac{1}{2^j}]$, i.e. with dynamics or oscillations in the period of 2^j to 2^{j+1} days. See, for example (Gencay *et al.*, 2002, 2003; Lee, 2004; Fernandez, 2005; Percival & Walden, 2000).

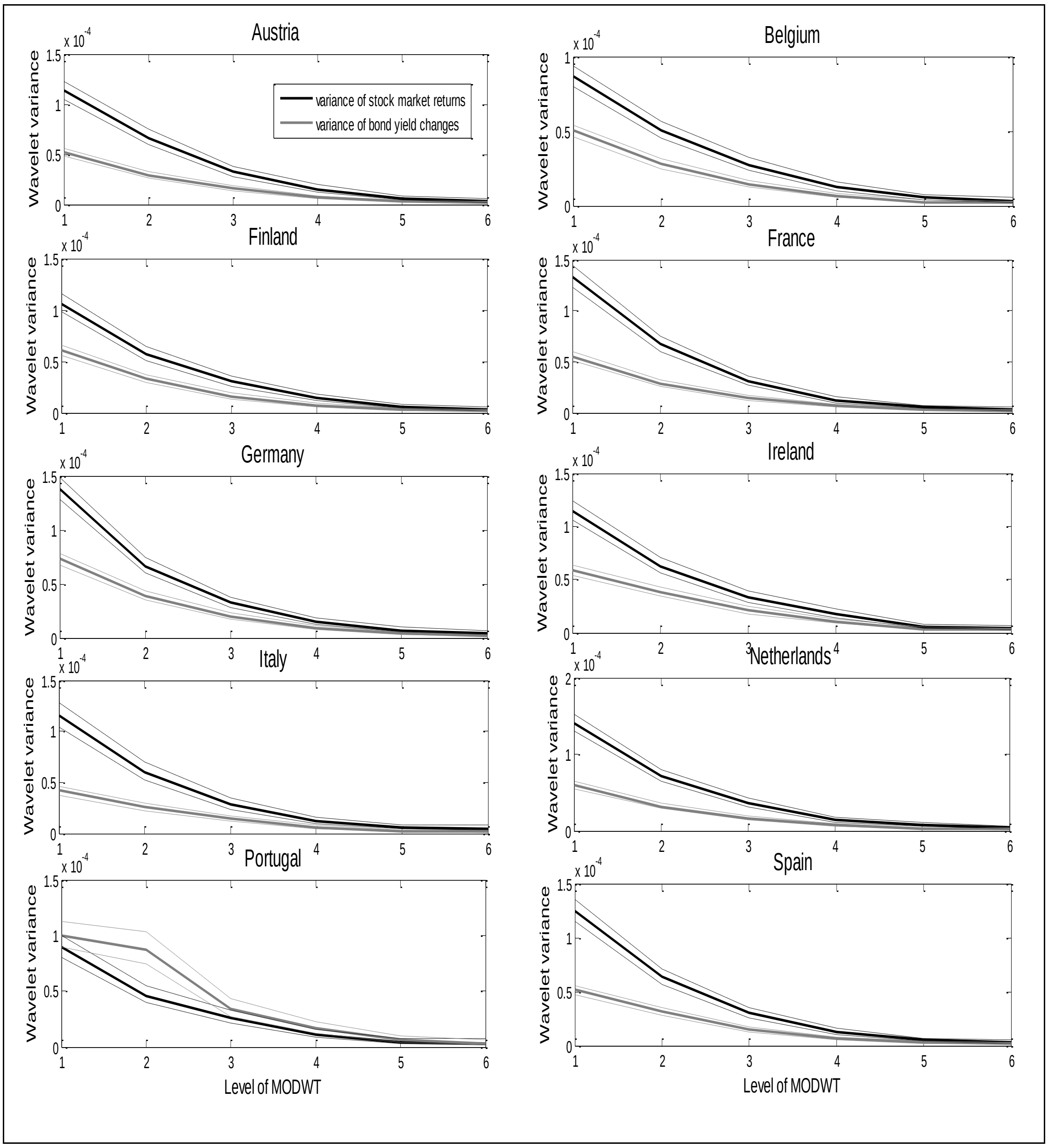

Figure 1. Wavelet variance of stock returns and sovereign bond yield changes

Notes: The MODWT-based wavelet variances have been constructed using the LA8 wavelet filter. Estimates of wavelet variance for different levels ($j = 1,\ldots,6$) of MODWT wavelet coefficients are drawn with a full line. The black line indicates the wavelet variance of changes in stock prices and the gray line indicates the wavelet variance of changes in bond yields. The wavelet variances of stock indices's return series and bond yield changes series are drawn under the assumption of Gaussianity. The 95 % confidence intervals, calculated by equation (16), around the wavelet variance estimates are drawn with dotted lines. Source: Own calculations.

The highest volatility in stock indices' returns and bond yield changes is captured by lower level MODWT wavelet coefficients (i.e., level 1 ($j = 1$) MODWT wavelet coefficients, associated with dynamics over 2 to 4 days and level 2 ($j = 2$) MODWT wavelet coefficients associated with dynamics over 4 to 8 days). The finding that the variance decreases as the wavelet scale increases has, as (Kim & In, 2005, 2007) note, important implications for the investors in these asset classes. Investors with short investment horizons that want to efficiently manage the risk of their portfolio investments have to respond to every fluctuation in realized returns or bond yield changes, while for an investor with a much longer horizon, the need to do this is reduces, as long-run risk is significantly smaller as indicated by the variance.

The strength of comovement between stock market returns and bond yield changes at different time scales as analyzed by MODWT correlation analysis (the results are presented in Figure 2).

We find that the estimated wavelet correlation changes with the time scale and is mostly positive for all countries. As already noted, from the theoretical perspective the positive correlation can be explained by the different impact of inflation on stock and bond prices (Campbell & Ammer, 1993) or by the ″flight-to-quality″ argument. As inflation in the observed countries was not particularly high during the observed period, in our opinion the greater role has played the ″flight-to-quality″, i.e. the financial market dynamics and the changes in market participants' assessments about risk of the stocks and sovereign bonds. The negative correlation of stock market returns and sovereign bond yield changes in Portugal can thus imply that the sovereign bonds on average did not play the ″safe haven″ role in its financial market throughout the whole observed period. To elaborate this issue, we next calculated the rolling-window correlations between stock market returns and bond yield changes for Portugal and Germany, as the sovereign bonds of the later country are regarded as safe haven in international financial markets. Using this approach, correlation between the two stock indices return series at time t was calculated from w observations (where w is size of the window), centered around time t. The window was rolled forward one day at a time, resulting in a time series of wavelet correlation. This way we obtained $N - w$ correlation coefficients. The results are presented in Figure 3.

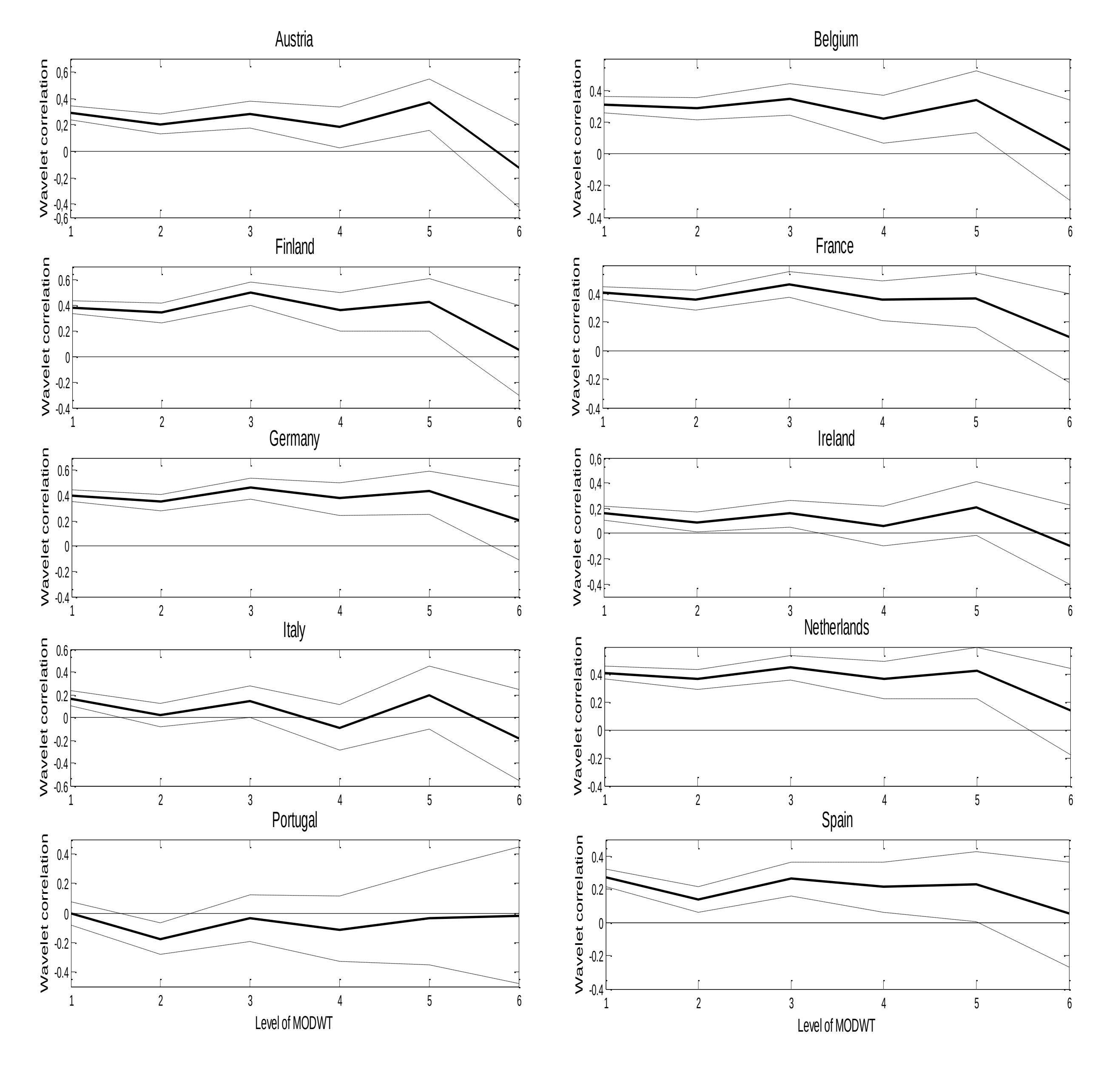

Figure 2. Wavelet correlation between stock returns and sovereign bond yield changes

Notes: The estimated correlation between jth-level of MODWT wavelet coefficients for stock market returns and bond yield changes are drawn with a full line. The 95 % confidence intervals (calculated by equation (19)) around the wavelet correlation estimates are drawn with dotted lines. The approximate confidence interval for the estimated wavelet correlation does not utilize any information regarding the distribution of the wavelet correlation. Therefore, these confidence intervals are robust to non-Gaussianity.
Source: Own calculations

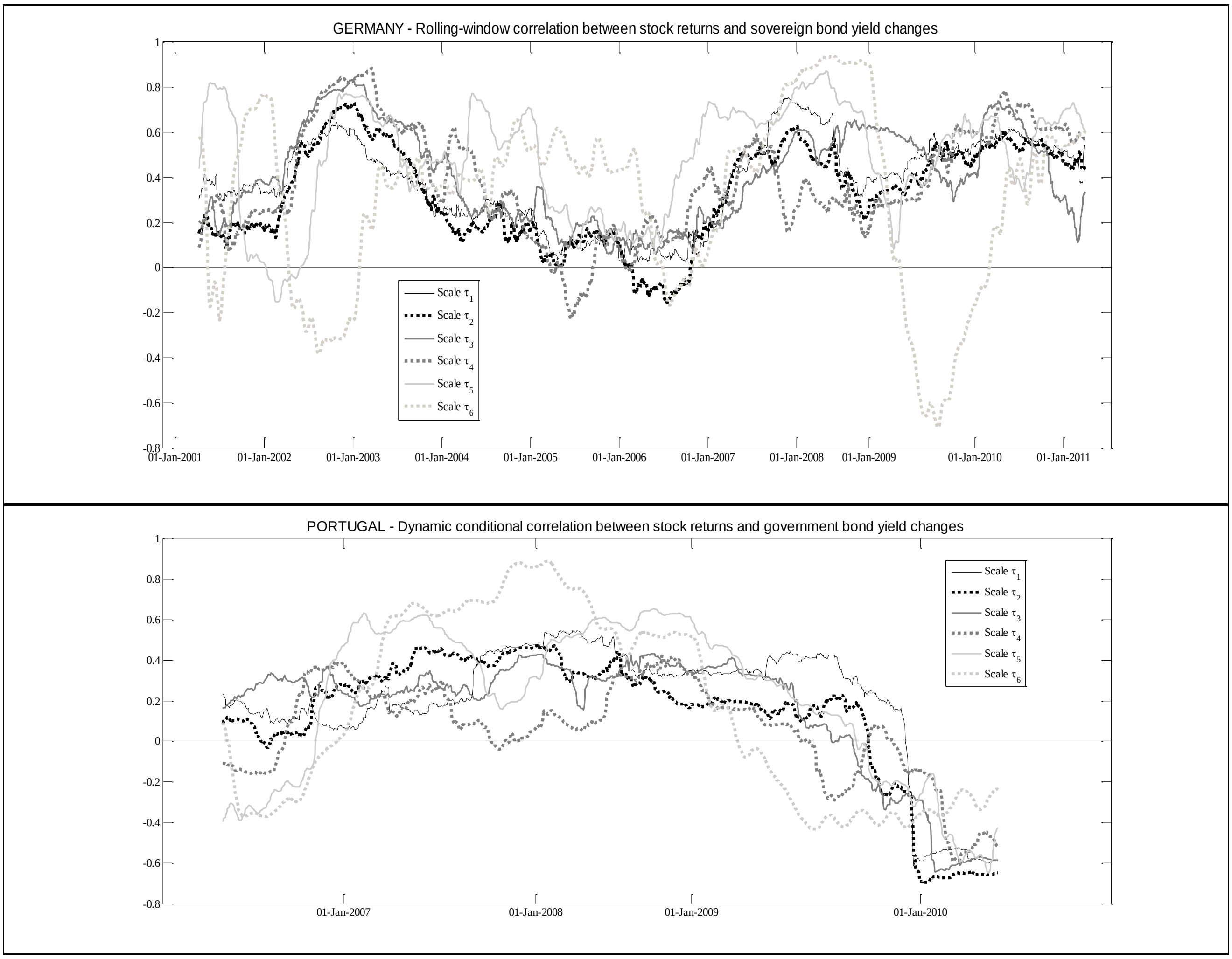

Figure 3. Rolling-window correlation between stock returns and sovereign bond yield changes for Germany and Portugal

Notes: The window size has to capture enough data points to obtain reasonable estimates for higher scales. We chose $w = 200$ *days, as in(Ranta, 2010). The MODWT coefficients not affected by the boundary condition are considered only. The final date for which the rolling-correlation estimate is obtained for Germany and Portugal is not the same as the number of trading days in stock and sovereign bond market for Germany and Portugal are not the same. Due to the construction characteristics of the rolling-window correlation coefficients, the effect of the event should start to show up in the graph 100 days before (half the window length) the actual time of the event that is denoted on the time axis.*

More findings emerge from Figure 3. Firstly, the correlation between stock market returns and sovereign bond yield changes is not just a multiscale phenomenon, but also exhibits time dynamics. Secondly, the time path of lower wavelet scale correlations is similar, whereas the higher wavelet scale (especially scale τ_6) correlation exhibits more specific time dynamics. Thirdly, the rolling-window correlations at all scales turned negative for Portugal at the start of 2010 – this is when the Eurozone debt crisis started which also hit hard the Portugal sovereign bond market – thus probably causing the ″average″ wavelet correlation for Portugal for the whole observed period (as calculated and presented in Figure 2) to be negative.

Conclusion

This paper examined comovement between changes in sovereign bond yields and stock market returns for ten Eurozone countries (Austria, Belgium, Finland, France, Ireland, Italy, Germany, Netherlands, Portugal, and Spain) in the period from January 2000 – end of August 2011 (different starting periods are used for different countries due to data availability), applying the maximal overlap discrete wavelet transform (MODWT) variance and correlation tools.

We found that the for all but one country (namely Portugal) the stock return volatility is higher than the volatility of sovereign bond yield changes across all wavelet scales. The highest volatility in stock indices' returns and bond yield changes is captured by lower level MODWT wavelet coefficients (i.e., level 1 ($j = 1$) MODWT wavelet coefficients, associated with dynamics over 2 to 4 days and level 2 ($j = 2$) MODWT wavelet coefficients associated with dynamics over 4 to 8 days). A practical implication stemming from this finding is that investors with short investment horizons that want to efficiently manage the risk of their portfolio investments have to respond to every fluctuation in realized returns or bond yield changes, while for an investor with a much longer horizon, the need to do this is reduces, as long-run risk is significantly smaller as indicated by the variance.

The results of the paper show also that the estimated wavelet correlation changes with the time scale and is mostly positive for all countries but Portugal. The statistical evidence against hypothesis of no multiscale dependence yet is weak. Thus we cannot statistically claim that the wavelet coefficients at higher scales are significantly different (either higher or lower) that those at lower scales.

For the financial markets of Portugal and Germany we proved that the dependence between stock and sovereign bond market dynamics may not be just a multiscale phenomenon, but may also exhibits time dynamics across scales. The rolling-window wavelet correlation estimates show that at all scales correlation turned negative for Portugal at the start of 2010 – this is when the Eurozone debt crisis started which also hit hard the Portugal sovereign bond market – thus probably causing the ″average″ wavelet correlation for Portugal for the whole observed period to be negative.

References

Baur, D. G., & Lucey, B. M. (2009). Flights and Contagion - An Empirical Analysis of Stock-Bond Correlations. *Journal of Financial Stability*, 5(4), 339–352. http://dx.doi.org/10.1016/j.jfs.2008.08.001

Boudoukh, J., Richardson, M., & Whitelaw, R. F. (2008). The myth of long-horizon predictability. *Review of Financial Studies,* 21(4), 1577–1605. http://dx.doi.org/10.1093/rfs/hhl042

Campbell, J. Y., & Ammer, J. (1993). What moves the stock and bond markets? A variance decomposition for long-term asset returns. *The Journal of Finance,* 48(1), 3–37. http://dx.doi.org/10.1111/j.1540-6261.1993.tb04700.x

Cornish, R. C., Bretherton, C. S., & Percival, D. B. (2006). Maximal Overlap Discrete Wavelet Statistical Analysis with Application to Atmospheric Turbulence. *Boundary-Layer Meteorology,* 119(2), 339–374. http://dx.doi.org/10.1007/s 10546-005-9011-y

Craigmile, F. P., & Percival, D. B. (2002). Wavelet-Based Trend Detection and Estimation. In: El-Shaarawi, A. – Piegorsch, W. W. (eds): Entry in the Encyclopedia of Environmetrics. Chichester, UK: John Wiley & Sons, 2334–2338.

Dajcman, S., Festic, M., & Kavkler, A. (2012). Comovement Dynamics between Central and Eastern European and Developed European Stock Markets during European Integration and Amid Financial Crises – A Wavelet Analysis, *Inzinerine Ekonomika-Engineering Economics*, 23(1), 22–32.

Dajcman, S. (2012a). Comovement between stock and bond markets and the ″flight-to-quality″ during financial market turmoil – a case of the Eurozone countries most affected by the sovereign debt crisis of 2010–2011. *Applied Economics Letters*, 19(17), 1655–1662.

Dajcman, S. (2012b). The dynamics of return comovement and spillovers between the Czech and European stock markets in the period 1997–2010. *Czech Journal of Economics and Finance* (*Finance uver*), 61(4), 368–390.

Dajcman, S. (2013). Interdependence Between Some Major European Stock Markets - A Wavelet Lead/Lag Analysis. *Prague Economic Papers*, 2013(1), 28–49.

Durre, A., & Giot, P. (2005). An international analysis of earnings, stock prices and bond yields. *European central bank working paper* No. 515, 2005, [Online] available at: *www.ecb.int/pub/pdf/scpwps/ecbwp515.pdf*, accessed on January 6, 2012.

Fernandez, V. (2005). Time scale decomposition of price transmission in international markets. *Emerging Markets Finance and Trade,* 41(4), 57–90.

Gallegati, M. (2005). *A Wavelet Analysis of MENA Stock Market.* EconWPA Working paper No. 0512027.

Gallegati, M. (2008). Wavelet analysis of stock returns and aggregate economic activity, *Computational Statistics & Data Analysis,* 52(6), 3061–3074.

Gencay, R., Selcuk, F., & Whitcher, B. (2001a). Scaling properties of foreign exchange volatility. *Physica A: Statistical Mechanics and its Applications*, 289(1–2), 249–266.

Gencay, R., Selcuk, F., & Whitcher, B. (2001b) Differentiating intraday seasonalities through wavelet multi-scaling. *Physica A*, 289(3-4), 543–556.

Gencay, R., Selcuk, F., & Whitcher, B. (2002). An Introduction to Wavelet and Other Filtering Methods in Finance and Economics. San Diego, Academic Press.

Gencay, R., Selcuk, F., & Whitcher, B. (2003). Systematic risk and timescales, *Quantitative Finance,* 3(2), 108-116.

Gulko, L. (2002). Decoupling. *Journal of Portfolio Management*, 28(3), 59–66. http://dx.doi.org/10.3905/jpm.2002.319843

Harrison, P., & Zhang, H. H. (1999). An investigation of the risk and return relation at long horizons, *Review of Economics and Statistics,* 81(3), 399–408.

In, F., & Kim, S. (2006). The hedge ratio and the empirical relationship between the stock and futures markets: A new approach using wavelet analysis. Jou*rnal of Business,* 79(2), 799–820. http://dx.doi.org/10.1086/499138

Kim, S., & In, F. (2005). The relationship between stock returns and inflation: new evidence from wavelet analysis. *Journal of Empirical Finance,* 12(3), 435–444. http://dx.doi.org/10.1016/j.jempfin.2004.04.008

Kim, S., & In, F. (2007). On the relationship between changes in stock prices and bond yields in the G7 countries: Wavelet analysis. *Journal of International Financial Markets, Institutions and Money,* 17(2), 167–179. http://dx.doi.org/10.1016/ j.intfin.2005.10.004

Lee, H. S. (2004) Price and volatility spillovers in stock markets: A wavelet analysis. *Applied Economics Letters,* 11(3), 197–201.

Malkiel, B. G. (2003). A random walk down Wall Street. New York: Norton.

Percival, D. B. (1995). On the Estimation of the Wavelet Variance. *Biometrika,* 82(3), 619–631. http://dx.doi.org/10. 1093/biomet/82.3.619

Percival, D. B., & Mojfeld, H. O. (1997). Analysis of subtidal coastal sea level fluctuations using wavelets. *Journal of the American Statistical Association,* 92(439), 868–880. http://dx.doi.org/10.1080/01621459.1997.10474042

Percival, B. D., & Walden, A. T. (2000). Wavelet Methods fot Time Series Analysis. New York: Cambridge University Press. http://dx.doi.org/10.1017/CBO9780511841040

Pinho, C., & Madaleno, M. (2009). Time frequency effects on market indices: world comovements. Paper presented at Paris December 2009 Finance International Meeting AFFI - EUROFIDAI.

Priestley, M. M. (1981). Spectral analysis and Time Series, London: Academic Press.

Savva, C. S., Aslanidis, N. (2010). Stock market integration between new EU member states and the Euro-zone. *Empirical Economics*, 39(2), 337–351. http://dx.doi.org/10.1007/s00181-009-0306-6

Serroukh, A., Walden, A. T., & Percival, D. B. (2000). Statistical Properties and Uses of the Wavelet Variance Estimator for the Scale Analysis of Time Series. *Journal of the American Statistical Association*, 95(449), 184–196. http://dx.doi.org/10.1080/01621459.2000.10473913

Strivers, C., & Sun, L. (2002) Stock market uncertainty and the relation between stock and bond returns. Federal Reserve Bank of Atlanta Working Paper 2002-3.

Stivers, C., Sun, L., & Connoly, R. (2002). Stock implied volatility, stock turnover and the stock-bond return relationship. Federal Reserve Bank of Atlanta Working Paper 2002-3a.

Zhou, J. (2011). Multiscale analysis of international linkages of REIT returns and volatilities. *Journal of Real Estate Financial Economics*. [online first]. http://dx.doi.org/10.1007/s11146-011-9302-7

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